Argument-Sequence Fragments of First-Order Logic

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The quest to find fragments of first-order logic for which satisfiability is algorithmically decidable has been a central undertaking of mathematical logic since the appearance of D. Hilbert and W. Ackermann's "Grundzüge der theoretischen Logik" almost a century ago. One family of first-order fragments for which satisfiability is decidable, but which has languished in relative obscurity, consists of those defined by restricting the allowed sequences of variables occurring as arguments in atomic formulas, an idea dating back W. Quine's homogeneous m-adic formulas. These "argument-sequence fragments", as I call them, include the "fluted fragment", the "ordered fragment" and the "forward fragment". In this talk, I survey known results on the above-mentioned fragments, and describe a recently defined argument-sequence fragment, the "adjacent fragment", which includes the fluted, ordered and forward fragments. Indeed, the adjacent fragment is expressively maximal among the argument-sequence fragments whose satisfiability and finite satisfiability problems are decidable.

The talk consists of two parts. The first is a (relatively) non-technical overview of argument-sequence fragments, with an explanation of the principal theorems and open problems. The second part is more technical, and features outline proofs of some of the theorems mentioned in the first part.