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XXVIII CONFERENCE

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APPLICATIONS OF LOGIC IN PHILOSOPHY  
AND THE FOUNDATIONS OF MATHEMATICS

SZKLARSKA POREBA

POLAND

5-9 MAY 2025

XXVIII Conference  
*Applications of Logic in Philosophy  
and the Foundations of Mathematics*

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The Conference is organized by:

Department of Logic and Methodology of Sciences, University of Wrocław

Institute of Mathematics, University of Silesia in Katowice

Department of Mathematics, University of Opole

Department of Logic and Methodology of Sciences, University of Łódź

Department of Logic, Nicolaus Copernicus University in Toruń

Department of Logic and Cognitive Science, Adam Mickiewicz University  
in Poznań

Under the auspices of:

Polish Association for Logic and Philosophy of Science

Edited by:

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First published 2025

Publisher:

Department of Logic and Methodology of Sciences, University of Wrocław,  
Wrocław

ISBN 978-83-940690-9-4

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# Abstracts

## Editorial note

(EN) means that the talk is presented in English, (PL)—in Polish.

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## *Agnostic Hedgehog in the Semantic Forest*

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The presentation will introduce the method of semantic trees, which was to be the focus of the textbook ‘The Method of Semantic Trees in Classical Logical Calculus’, co-authored with Professor Jerzy Pogonowski. It will be illustrated with „examples that avoid rigid gloominess”. Historical comments on the discussed method will also be provided.

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## *Hyperintensionality and Constructive Knowledge*

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Drawing inspiration from Suszko’s non-Fregean logics, we introduce several constructive extensions of the basic non-Fregean systems, SCI. We present Kripke semantics for these systems, along with corresponding natural deduction systems.

This framework is then employed to define various concepts of constructive knowledge and belief. These definitions rely crucially on the notion of propositional identity. However, in contrast to the analysis presented by Suszko [1, 2], and later by Ishii [3], the constructive nature of the logic offers a new way of defining epistemic modalities. We demonstrate how these definitions capture key ideas already explored in the literature on intuitionistic epistemology.

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# *Fregean Definite Descriptions in Non-Fregean Logic*

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Definite descriptions are typically regarded as first-order term-forming expressions that denote uniquely determined objects. Given an individual variable  $x$  and a formula  $\varphi$ , the expression  $ix\varphi$  constitutes a term. Definite descriptions are commonly classified as either proper or improper. Proper descriptions have a unique referent, while improper ones lack such a referent.

A distinctive feature of the Fregean theory of definite descriptions [1, 2] is the assignment of a specially designated object as the referent of all improper descriptions. In contrast to Russell's reductionist approach [4], which seeks to eliminate descriptions by translating them into standard first-order logic with identity, the Fregean perspective treats all descriptions as genuine terms.

We present an approach based on a propositional language with propositional identity, inspired by Suszko's non-Fregean logic [5]. In analogy with propositional quantification, we introduce the concept of *propositional definite descriptions*. Given a propositional variable  $p$  and a formula  $\varphi$ , the expression  $ip\varphi$  is a formula denoting the unique proposition that satisfies  $\varphi$ . We designate a special propositional variable as the referent of improper propositional descriptions instead of specially designated object. Suszko's identity connective  $\equiv$  plays a central role in the development of this framework. We aim to provide a semantic account of this approach and to modify Idrzejczak's [3] cut-free sequent calculus for Fregean theory to accommodate propositional definite descriptions.

**Acknowledgements.** The research of Yaroslav Petrukhin is funded by the European Union (ERC, ExtenDD, project number: 101054714). Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the European Research Council. Neither the European Union nor the granting authority can be held responsible for them.

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# *On the Conflict of Obligatory Norms*

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In the standard formulation, deontic logic is the formal study of the normative concepts of obligation, permission, and prohibition. According to the approach presented in this talk, these deontic concepts are not entirely derived from the notion of action. Actions are deontically loaded — they are permitted, prohibited, or obligatory. The notion of a norm is then secondary — actions are inherent components of norms. We speak about a conflict of obligations of actions when we come to deal with a pair of obligations, e.g., legal and moral, legal and religious, medical and religious, medical and religious, etc., which mutually exclude execution of the given action. In the talk we undertake the task to define the conflict in terms of action systems. Two solutions of the problem are presented. Both are based on action systems as the basic semantic unit. It seems that the approach to conflicts presented in the talk has not been considered in the deontology of actions.

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## *A Study on Logic Naturally Associated to Double Stone Algebras*

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This article examines the 4-element chain's logical structure when seen as a double Stone algebra. It has been established that any element of a double Stone algebra can be identified as monotone ordered triplet of sets. We therefore have the 4-valued semantics for the logic LD of double Stone algebras. Moreover, the rough set semantics of the logic of double Stone algebra is obtained by splitting the boundary region (uncertainty) into two disjoint subregions.

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## *Modal Logics of Classes of Finite Structures*

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In a class of models of first-order theory  $T$  a sentence  $\phi$  is said to be *possible* in a model  $M$ , symbolically  $M \models \Diamond\phi$ , if  $\phi$  is true in some extension of  $M$ . In this interpretation of modal logic, we will ask questions concerning only those models of a theory  $T$  that are finite. We investigate logics of theories with only finite models, as well as finite models of an arbitrary first-order theory.



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## *Constructive Neologicism in the Theory of Classes*

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Logicism currently is not a popular approach to the foundations of mathematics. But it is still alive, in the variety of neologicist positions, in particular represented by Tennant's constructive neologicism. We present the general features of this form of neologicism and apply its principles to set theory. Instead of focusing on some specific form of set theory, like ZF or NBG, we will provide a sequent calculus for Quinean virtual theory of classes (VTC). This approach introduces in the safe, non-committal way, the notion of a class represented by set abstracts, and permits for investigation of the domain of classes without existential assumptions. It may be used instead as a neutral platform for the development and comparison of alternative theories of sets. Sequent calculus we present is cut-free and allows us to prove syntactically the consistency of VTC.

**Acknowledgements.** Funded by the European Union (ERC, ExtenDD, project number: 101054714). Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the European Research Council. Neither the European Union nor the granting authority can be held responsible for them.

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## *Discovery or Creation? A Few Words About Jerzy Pogonowski's Essay of the Same Title.*

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The aim of my paper is to recall the arguments given for the thesis that mathematics is discovered, as well as for the opposite thesis that mathematics is created. I am referring to Jerzy Pogonowski's book entitled *Myślenie matematyczne. Drobne eseje przedemerytalne*.

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## *120 Years of Russell's 'On Denoting'*

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Bertrand Russell's 'On Denoting', published 120 years ago (*Mind* Vol. 14, No. 56 (Oct., 1905), pp. 479-493), is undeniably one of the most influential papers in analytic philosophy. Its importance is hard to overestimate. It has been the object of criticism, inspiration and study ever since its publication. It is famous for the Theory of Definite Descriptions, and this is what many philosophers retain from it. It is infamous for the Gray's Elegy argument. But the paper contains many other gems, such as questions regarding the primitives of logic and the distinction between knowledge by acquaintance and by descriptions. It played a pivotal role not only in the development of Russell's mathematical logic, but also in that of his metaphysics and epistemology.

The present talk has two objectives. First, to celebrate this paper and to entice the listener to read this classic again. Second, to present recent work in the formalisation of the theory of definitive descriptions.

I will begin with a brief reminder of the contents and the overall line of argument of 'On Denoting'. This will place the paper in the development of Russell's mathematical logic and his philosophy as a whole and demonstrate why it is so important for analytic philosophy. The bulk of 'On Denoting' concerns the theory of definitive descriptions, which is also the focus of the present paper, and so I will restrict myself to discussing only those other aspects of 'On Denoting' that have an immediate bearing on this theory. These are the choice of logical primitives, the project of logical analysis, which ultimately lead Russell to the philosophy of logical atomism, and the distinction between knowledge by acquaintance and by description. Due to the great difficulties in interpreting it, I'll leave the Gray's Elegy argument aside, apart from pointing out that it is intended to be an argument against Frege's distinction between sense and reference, and plays a role in establishing Russell's view of meaning.

Many agree with Ramsey that the theory of definite descriptions is a paradigm of philosophy. As it is so well known, I can confine myself to a brief outline of the theory as it appears in 'On Denoting'. I'll also say a few words on its formal development in *Principia Mathematica* in relation to the notion of scope.

According to Russell, 'The  $F$  is  $G$ ' means 'There is one and only one  $F$  and it is  $G$ '. To us, who are so familiar with the theory of definite descriptions, it may come as a surprise that Russell felt that his interpretation would be considered 'somewhat incredible'. The reason is that according to Russell, 'the  $F$ ' is not a genuine singular term; in fact in isolation it has no meaning at all. (*Principia* even contains a proof of this.) Comments I have received on my work in this area have shown that even today some consider this to be incredible. But it is

the essence of Russell’s view: denoting phrases have no meaning in themselves, but every sentence in which they occur is given a meaning by Russell’s analyses in terms of the logical primitives.

To round things off, I’ll present a formalisation of Stephen Neale’s reconstruction of Russell’s theory of definite descriptions. Neale stays very close to Russell, indeed, but clarifies a few points in particular in the formal representation of the theory. Neale takes more seriously Russell’s view that definite descriptions are really quantificational phrases than Russell himself did in his symbolism in *Principia*. The present account formalises complete sentences in which definite descriptions occur by a binary quantifier, where  $Ix(Fx, Gx)$  formalises ‘The  $F$  is  $G$ ’. I’ll give rules of inference for  $I$  that are suitable for classical logic, but could be added to other systems, too.

Acknowledgements. The research in this paper was funded by the European Union (ERC, ExtenDD, project number: 101054714). Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the European Research Council. Neither the European Union nor the granting authority can be held responsible for them.

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## *A Logic of Interrogative Attitudes*

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In recent years, a growing trend in epistemological literature has placed the process of questioning and questions at the center of attention [1–5,8–9]. Some even speak of an “interrogative turn” in epistemology [5]. One might question whether this constitutes a true breakthrough or merely a rediscovery of familiar ideas, but there is no doubt that foregrounding the notion of the interrogative attitude (or questioning attitude) opens up an intriguing research area. Topics explored within this framework include the explication of erotetic rationality (though, in the context of rediscovering the wheel, see [10]). At the same time, some valuable works provide proof-theoretic formalizations of these interrogative epistemological concepts [6-7]. However, systematic logical studies on interrogative attitudes remain lacking.

In my talk, I propose a logic of interrogative attitudes – that is, attitudes of an agent towards questions – treating interrogative attitudes as modal operators. In addition to the interrogative attitude, I introduce an operator for cognitive access to knowledge and analyze these operators in the context with the standard knowledge operator.

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## *Didactics and Publishing Plans of Professor Jerzy Pogonowski in the Years 2015-2024*

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Between 2015 and 2024, Professor Jerzy Pogonowski was a faculty member at the Department of Logic and Cognitive Science at the Faculty of Psychology and Cognitive Science of Adam Mickiewicz University in Poznań. In my talk, I will discuss two key topics: Professor Pogonowski’s invaluable contributions to teaching cognitive science students and the publishing plans he pursued in the final period of his life.

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## *From Questions to Proofs. On the Method of Socratic Proofs*

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The logic of questions is a discipline aimed at formally analyzing issues such as the structure of interrogative sentences, types of questions and answers, presuppositions of questions, question semantics, relations between questions, and, finally, systems of questions. Many of these topics have been present in the literature for over a century (see [1,2,6]) or at least for several decades [3,7,9,10]. One of the relatively newer research areas is the proof theory involving questions [4,11,13,14]. Can a question be proven? While this is certainly possible, one might debate what it actually means. If a traditional proof of  $A$  serves as a certification of the epistemic value of  $A$ , then a proof of the question “Is it  $A$  or  $B$ ?” certifies the epistemic value derived from partitioning the examined (logical) space using the  $A/B$  dichotomy. The logic of questions encompasses well-established paradigms that emphasize different aspects of questioning, for instance: epistemological [8], semantic [5], or inferential [12]. My work focuses on the proof-theoretic formalisms. Drawing on the paradigm of Inferential Erotetic Logic, developed and advanced by Andrzej Wiśniewski [12,14], I define proof systems referred to by Wiśniewski as the Method of Socratic Proofs [13]. In my talk I will present the main results concerning the development of this method for classical, modal, and intuitionistic logic, as detailed in my monograph [11].

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## *On the Difference Operation in Lattices*

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The set-theoretic difference of two sets  $a$  and  $b$ , i.e.,  $\{x \in a : x \notin b\}$ , can be translated into lattice theory in at least three ways:

$$a - b = \max\{x \in L : x \leq a \text{ and } x \wedge b = 0\},$$

$$a \div b = \min\{x \in L : x \vee b \geq a\},$$

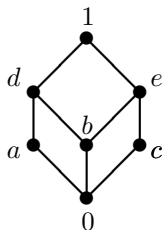
$$a \setminus b = \min\{x \in L : x \vee b = a \vee b\}.$$

(Here,  $\max$  denotes the greatest and  $\min$  the least element satisfying the given conditions.) The first two operations are well known:

$$a - b = a \wedge b^*, \text{ where } * \text{ denotes the operation of } \textit{pseudocomplementation},$$

$$a \div b = b \rightarrow_d a, \text{ where } \rightarrow_d \text{ denotes the operation of } \textit{relative pseudocomplementation} \text{ in dual lattice } L_d.$$

In Boolean lattices, all operations coincide; in finite distributive lattices, the second and the third coincide; in non-distributive lattices, they might not exist. For instance, in the following lattice



$1 - b$  and  $d \div a$  do not exist:

$$1 - b = \max\{a, c, 0\}, \quad d \div a = \min\{b, c, d, e, 1\}.$$

On the other hand, one can verify that the operation  $\setminus$  is well defined here, and in particular,  $1 \setminus b = \min\{1\} = 1$  and  $d \setminus a = \min\{b\} = b$ .

In the talk, we describe the scope of all these operations, and give necessary and sufficient conditions for:

$$(\forall a, b \in L)(a - b = a \setminus b), \quad \text{and} \quad (\forall a, b \in L)(a \div b = a \setminus b).$$

Then, we discuss the basic properties of  $\setminus$  and characterize Boolean, distributive, complete, and completely normal lattices in terms of  $\setminus$ .

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## *On Oughtness, Goodness and Badness*

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One can say that an object is valuable exactly when it is as it ought to be. On the other hand, one can also say that an object has a certain value when it is good that it is as it is. Similarly, an object can be said to be counter-valuable or negatively valuable when it is bad that it is as it is. That an object is as it ought to be means that the object is such-and-such, while at the same time it ought to be that it is such-and-such. That it is good that the object is such-and-such means that the object is such-and-such and at the same time it is good when it is such-and-such. That it is bad that the object is such-and-such means that the object is such-and-such that and at the same time it is bad when it is such-and-such. The concepts of the value and counter-value of an object are thus closely related to the concepts of the oughtness of a certain situation, the goodness of a certain situation, and the badness of a certain situation, and to the corresponding de dicto modalities. However, the logical relationships that occur between expressions of the form ought to be so-and-so, good when it is so-and-so, and bad when it is so-and-so are not entirely clear. The reading will present a logical analysis of such relationships. Some multimodal propositional logic and some version of standard relational semantics will be used.

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## *A Little Remark About a Certain Proof*

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When working on a proof, we focus on its correctness. After we become familiar with a new rule of elaborating given proof, we pay attention to certain technical details. We want our proof to be beautiful in some way. However, I think that our thoughts rarely concern something so basic. And yet I want to dwell on such a trifle.

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## *Logics Genealogy*

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Since 1997, the Mathematics Genealogy project, run by the University of Minnesota and then the University of North Dakota, has been operating with great success. Logicians are well represented there, including logicians from our country. Unfortunately, many people important to logic are missing there. I would like to present this project in its current form and then propose a way to expand or supplement it in such a way that logic, and especially the Polish school of logic, are properly represented.



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## *Kripke-Style Semantics for Weak (Modal) Logics*

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We present Kripke-style semantics for modal and non-modal propositional languages. We consider several kinds of structures and definitions of truth which we then apply to characterize some weak systems of logic, including certain intermediate subnormal modal logics. Among others, completeness theorems are provided together with results about modal disjunctive property.

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## *Copernicus' Distinguished Work as a Pre-Genesis of Paraconsistency in Jaśkowski's Style*

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The essential aspect of the so-called Copernican revolution consisted in changing the frame of reference: no more the earth at the center of the universe, but the sun. In this paper, we focus on the content of Book III of *De revolutionibus*, where Copernicus presented the set of geometrically equivalent models that, despite differences, gave the same explanatory effect from the point of view of the observer. While reconstructing his reasoning on these astronomical models, we approach their equivalence by recalling S. Jaśkowski's logical approach. The latter was aimed at proposing a model in which systems with contradictions can be presented in a non-contradictory manner. We think that Copernicus, in his research, adopted a non-contradictory point of view, that is, the observer's point of view, by which he disconfirmed contradictory models. Thus, we hypothesize that the Polish (Torunian) approach to paraconsistency can be noted as early as Copernicus. Moreover, we argue that the Copernican approach to the inconsistencies present in scientific explanation should not be treated as the decisive argument for a realistic or antirealistic interpretation of his theory.

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## *A Systematic Overview of Paraconsistent Logics, with an Emphasis on Jaśkowski*

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Paraconsistent logics are characterized by the failure of *ex contradictione quodlibet* (ECQ hereafter). Since the modern birth of paraconsistency, infinitely many systems of paraconsistent logic have been devised and studied based on various motivations. After all, it seems that paraconsistent logics are only loosely connected to each other by a rather general requirement that ECQ should be invalid. But is it really impossible to unify paraconsistent logics? The purpose of the talk is to address this question from a semantic perspective.

The overall project aims at the following claim: there are only two kinds of paraconsistent negation, one being negation as representing falsity, and the other being negation as negative modality. Most systems of paraconsistent logic will be classified into these two categories, and further differences can be seen in terms of the additional connectives and their semantics. The aim of the talk is to spell out the details for the first half of the claim. It turns out that Jaśkowski's insight is to be found beyond his own system, namely discussive logic.

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## *Some Remarks on the Factivity of Knowledge*

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The factivity of knowledge — i.e., the thesis that knowing that  $p$  entails the truth of  $p$  — is one of the pillars of modern theories of knowledge and an axiom of epistemic logic. Nowadays, however, this property of knowledge is often contested. Discussions take place when we juxtapose the factivity of knowledge with, *inter alia*, (i) epistemic contextualism, (ii) everyday knowledge attributions (including X-Phi surveys), (iii) implicatures or presuppositions of knowledge attributions, (iv) epistemic involuntarism, (v) knowledge as a mental state (in the so-called knowledge-first epistemology), (vi) false-but-approximately-true propositions, (vii) epistemic paradoxes, e.g. the Church-Fitch knowability paradox, the Knower paradox. In my talk, I will briefly discuss selected issues from these discussions, showing that the factivity of knowledge should be preserved.

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## *Levitin's Collection of Algorithmic Puzzles*

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Anany Levitin (1947–2021) was the author of Introduction to the Design and Analysis of Algorithms, used as a textbook in hundreds of universities worldwide. He also co-authored Algorithmic Puzzles with his wife Maria, a collection of puzzles that enriched the pedagogy of Levitin's textbook and subsequently inspired many educators – such as Jerzy Pogonowski – to use the puzzle-like problems as mathematical recreations.

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## *Hypersequent Many-Valued Versions of Essence and Accident Modalities in FDE-Based Logics*

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Modal logic typically studies the notions of necessity ( $\Box$ ) and possibility ( $\Diamond$ ). However, the literature also explores other philosophically significant modalities. Marcos [4, 5], for example, investigates the modalities of *essence* and *accidence*, which he defines respectively as

$$\circ A = A \rightarrow \Box A \quad \text{and} \quad \bullet A = A \wedge \Diamond \neg A.$$

As he explains, ‘one could read  $\bullet A$  as saying that ‘ $A$  is the case, but could have been otherwise’: It works as a kind of (local) connective for ‘accidental truth’. Similarly,  $\circ$  could be read as expressing a (local) notion of ‘essential truth’. [5, pp. 202–203, notation adjusted]. A proposition  $A$  is essentially true if its truth entails its necessity. Analogous *falsity counterparts* of these modalities can also be defined:

$$\tilde{\circ} A = \neg A \rightarrow \Box \neg A \quad \text{and} \quad \tilde{\bullet} A = \neg A \wedge \Diamond A.$$

A proposition  $A$  is *essentially false* if its falsity implies that it is necessarily false. Marcos notes that  $\tilde{\bullet}$  can be interpreted ‘as as a kind of (local) connective for ‘counterfactual truth’, a statement  $A$  ‘is not the case, but it could have been’ [5, p. 191].

This leads us to a many-valued generalisation of the aforementioned modalities. A statement  $A$  *essentially takes* the truth value  $t$  if the fact that it takes the value  $t$  implies that it necessarily takes the value  $t$ . Similarly,  $A$  *accidentally takes* the truth value  $t$  if it takes the value  $t$ , but could have taken a different one. Negative counterparts of these notions are also possible:  $A$  is *essentially not  $t$*  if it does not take the value  $t$ , and necessarily never does;  $A$  is *accidentally not  $t$*  if it does not take the value  $t$ , but could have. These definitions can be formally expressed, provided that the language is expressive enough to characterize statements such as “ $A$  takes (or does not take) the value  $t$ ,” and they can be applied to arbitrary many-valued logics.

In this paper, we explore this idea in the setting of Belnap-Dunn’s logic FDE [1, 2], extended with Boolean negation (in addition to its original De Morgan negation) and implication. We motivate and distinguish 12 essence and 12 accident four-valued modalities, formulate S5-style Kripke semantics for them, and propose a sound, complete, and cut-free hypersequent calculus.

Finally, we incorporate this approach with the modal definite descriptions introduced by Wałęga and Zawidzki [6], where expressions of the form  $@_{\varphi}\psi$  state that “ $\psi$  holds in the modal world where  $\varphi$  holds.” We introduce the following

refined operators:  $@_{\varphi}^{ot}\psi$  expresses that “ $\psi$  essentially takes the value  $t$  in the world where  $\varphi$  essentially takes the value  $t$ ,” and  $@_{\varphi}^{\bullet t}\psi$  expresses that “ $\psi$  accidentally takes the value  $t$  in the world where  $\varphi$  accidentally takes the value  $t$ .”

**Acknowledgements.** This research is funded by the European Union (ERC, ExtenDD, project number: 101054714). Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the European Research Council. Neither the European Union nor the granting authority can be held responsible for them.

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# *The Relationships Between Relevance Logic and Projective Geometry*

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The relevance logic RK arises through the extension of the system  $R$  by the addition of the Duns Scotus’ law. Importantly, the incorporation of this principle as an additional axiom does not result in a collapse into classical logic, a fact established by Robert Meyer. Within the framework of projective geometry, one encounters rich algebraic structures—most notably projective planes—which facilitate the construction of models for relevance logics in a manner that faithfully preserves their intended semantics.

The relationship between relevance logic and projective geometry is both profound and indisputable. This paper aims to elucidate the deep structural connections between projective planes and the ternary relation R. It will be

demonstrated how projective planes can serve as intuitive and geometrically grounded visualizations of abstract logical structures. Consequently, several significant interrelations between these mathematical constructs will be explored and clarified.

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## *Operations on Arguments: An Approach to Defining Argument Structure*

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Based on some ideas proposed by Jerzy Pogonowski (2011), a definition of the structure of a wide class of arguments can be developed. The arguments in question are usually represented in informal logic by the so-called standard diagrams, which have a tree-like form. In this approach, the diagrams will be understood as a kind of bipartite graphs generated from elementary structures of the form  $\langle \text{premises}; \text{conclusion} \rangle$ , i.e., sequents (c.f. Selinger 2011), by means of two operations distinguished by Pogonowski: co-terminal (or alternatively convergent) composition and lengthening (or serial) composition (in Polish “złożenie współkońcowe” and “kompozycja” (or “przedłużenie”), respectively, in the original terminology of Pogonowski). Both operations are important in real-life argumentation and correspond to giving additional reasons for a defended thesis and justifying a challenged premise. Pasting (“wklejenie”), also introduced by Pogonowski in the cited paper, is another operation on arguments that plays a notable role in practice, for example when revealing enthymematic premises.

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## *A Few Remarks About ‘The Odyssey of the Mathematical Mind’ Project*

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In the book *Mathematical Thinking* by prof. Jerzy Pogonowski (2020), you can find a chapter titled *Mathematical puzzles in teaching*. There you can find information about the history of mathematical puzzles and their resources. In addition, there are 36 puzzles of various types, divided thematically into 12 subsections, among others such as: *The Infinite, Numbers and Magnitudes, Change and Motion, Shape and Space, Patterns and Structures, Orderings, Algorithms and Computation*. The puzzles were chosen in order to achieve a better understanding of mathematical concepts involved in their solutions. Contrary to the usual mathematical exercises, mathematical (logical) puzzles are often connected with that which is unexpected, that which contradicts our everyday experience. Selected puzzles are instructive as far as a critical attitude towards intuitions is concerned. The first answer to the puzzle, given without reflection, is simply wrong from a mathematical point of view and only after a deeper analysis is the corrected answer obtained. The book *Odyssey of the Mathematical Mind* is to be a continuation of Professor Pogonowski’s idea and will contain a much more extensive collection of puzzles that develop critical thinking.

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# *Modal Logic with Definite Descriptions, Tableaux, and Python*

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Modal logic with definite descriptions,  $\mathcal{ML}(\mathcal{DD})$  for short, is standard modal logic augmented with expressions of the form:

$$@_{\varphi}\psi,$$

which read ‘in the unique world in which  $\varphi$  is true,  $\psi$  is true, too.’ At first glance,  $\mathcal{ML}(\mathcal{DD})$  looks like a generalisation of the well-known hybrid logic, in which propositional atoms of a special sort, nominals, occur that hold true in a single world. There are considerable differences that arise from the fact that in the case of  $\mathcal{ML}(\mathcal{DD})$ , arbitrary formulas can play the role of worlds’ labels. The first lies in the fact that the negation of  $@_{\varphi}\psi$ , where  $\varphi$  can be any formula, behaves differently than the negation of a formula  $@_i\psi$ , where  $i$  is a nominal. Secondly, the computational complexity of the satisfiability problem increases from PSPACE-complete to EXPTIME-complete, which poses new challenges for proof systems designed to deal with  $\mathcal{ML}(\mathcal{DD})$ -formulas.

In the first part of the talk, we will briefly introduce the logic mentioned above and propose a decision procedure for  $\mathcal{ML}(\mathcal{DD})$  based on analytic tableaux. We will show that two difficulties need to be overcome along the way.

1. Since in some cases, due to the interplay between the  $\neg\Box$  and  $@$  operators, the procedure directly implementing the tableau calculus does not naturally terminate, introducing an additional *blocking mechanism* becomes necessary that prevents producing infinite branches.

2. Even if blocking is applied, it only guarantees that the length of subsequent branches in a tableau is at most exponential in the size of the input formula, which means that the size of the entire tableau can be doubly exponential and this exceeds the complexity of the satisfiability problem in  $\mathcal{ML}(\mathcal{DD})$ . In order to decrease the size of the produced derivation tree by one exponent—and ensure that the procedure is complexity-optimal—a special form of *global caching* must be implemented, thanks to which some branches of the constructed tableau will not need to be checked as repetitive.



In the remainder of the talk, we will briefly describe our ongoing work on building an automated prover that implements the described decision procedure using the Python programming language. First, we will discuss how Python has been used and can be used in automated theorem proving. Secondly, we will focus on the main elements we wish the implementation to contain: automation of the described tableau-based algorithm for testing the satisfiability of  $\mathcal{ML}(\mathcal{DD})$ -formulas; an option to inspect the generated tableau by the user; testing benchmarks and comparing our results to other automated provers; a mechanism for random generation of  $\mathcal{ML}(\mathcal{DD})$ -formulas and for purposes of data exploration, testing, and searching for new benchmarks.

**Acknowledgements.** This research is funded by the European Union (ERC, ExtenDD, project number: 101054714). Views and opinions expressed are however those of the authors only and do not necessarily reflect those of the European Union or the European Research Council. Neither the European Union nor the granting authority can be held responsible for them.

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## *On Certain Undesirable Effects of Counterfactual Assumptions in Physics*

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Counterfactual assumptions in physics generally do not take form of D.K. Lewis's counterfactuals. Usually it is a thesis (a declarative sentence) contradictory to the

description of a certain fragment of reality. Such assumptions are usually made in order to simplify the calculations or lessening the complexity of the researched fragment of the real world. Certain assumptions are made without knowledge of their counterfactual nature, e.g. if the pool of knowledge at certain level of progress is insufficient. Trusting in a counterfactual assumption may be signified by an incorrect result of the prognosis based on such assumption, however, in most cases we cannot unambiguously indicate which out of the numerous assumptions determined the error, especially in case of long-term prognoses. The proposed paper is an attempt to determine the formal conditions of counterfactual assumptions in which such assumptions cease serving the intended function of representing an ideal reality, e.g. by disturbing the essential qualities of given object or creating a model (possible world) too distant from reality and thus contradictory to the essential components of the commonly accepted scientific paradigm.

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## *Graph Neural Networks, Modal Logics and Definite Descriptions*

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The aim of this talk is to present a landscape of surprising relationships between properties expressible by graph neural networks (GNNs) and modal logics, and to determine where do the properties related to definite descriptions lie within this landscape. We will begin by reviewing the seminal results [1, 6, 3, 2] that connect the expressive power of GNNs with graded modal logic  $\mathcal{GM}\mathcal{L}$ —basic modal logic extended with operators that allow for counting accessible modal worlds—and to the fragment of first-order logic  $C_2$ —which allows for using only two variables and counting quantifiers. Next, we will present our new results linking specific classes of GNNs with basic modal logic  $\mathcal{ML}$ , modal logic with universal modality  $\mathcal{ML}(A)$ , and first-order logic with two variables  $\text{FO}_2$ . To determine the role of definite descriptions in this landscape, we will build on our previous results concerning the expressive power of the modal logic with definite descriptions  $\mathcal{ML}(\text{DD})$  [4, 5].

**Acknowledgments.** This research is funded by the European Union (ERC, Ex-tenDD, project number: 101054714). Views and opinions expressed are however those of the authors only and do not necessarily reflect those of the European Union or the European Research Council. Neither the European Union nor the granting authority can be held responsible for them.

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## *Abelian Logic on the Bochum Plan (and the American Plan as Well)*

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In this paper, we introduce two new semantic presentations of Abelian logic, the non-trivial negation inconsistent logic of Abelian lattice-ordered groups, which was independently developed by Ettore Casari, and by Robert Meyer and John Slaney. Abelian logic is presented through a methodology that combines elements of what is sometimes referred to as the “Bochum Plan” and the “American Plan.” While the Bochum Plan is an approach to defining contra-classical logics, the American Plan—developed by Nuel Belnap and Michael Dunn—in particular offers a conception of negation that invites an application of the Bochum Plan. The first semantics is a ternary frame Kripke semantics, and the second is based on ideas from Edwin Mares’ work. Thereby emerges a condition for the falsity of

Abelian implication to be supported, which we analyse further in the separate context of the first-degree entailment logic. The perspectives are united in the end to provide a defence against the scepticism concerning the status of the Abelian negation as a negation.

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## *On Deliberate Speech*

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Heinrich Scholz in his article *Sprechen und Denken* initiated the problem of *deliberate speech*. It boils down to the question: what conditions must be fulfilled to consider something one says deliberate. His reflections on this issue provide an inspiration for the studies presented here.

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## *Probability of Conjoined Conditionals: Evaluating the McGee Formula and its Alternatives*

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The analysis of conditional statements and their probabilities remains a topic of ongoing debate, often characterized by a tension between intuition and formal

results. A central issue is how to assign probability to the conjunction of two conditionals,  $(A \rightarrow B) \wedge (C \rightarrow D)$ . Several probabilistic models – such as the Bernoulli-Stalnaker space, fair-bet analysis, and Markov graph space – have been employed to derive the probability formula known as the McGee formula (MG):

$$P((A \rightarrow B) \wedge (C \rightarrow D)) = \frac{[P(ABCD) + P(A^C CD)P(B|A) + P(ABC^C)P(D|C)]}{P(A \vee C)}$$

However, alternative interpretations challenge the validity of MG, proposing different probability calculations. Three main perspectives emerge:

1. The debate is superficial, arising from differences in translating natural language statements into formal logic.
2. MG is correct, and its critics make reasoning errors.
3. MG is incorrect, revealing fundamental flaws in the underlying models.

In this presentation, we reconstruct the key assumptions necessary for the debate to be meaningful and examine whether MG or an alternative approach provides a more accurate calculation. We argue that, in existing examples, an implicit and often unacknowledged interpretation of conditionals leads to incorrect probability computations.

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## *Probabilities of Conditionals of Arbitrary Complexity*

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The aim is to describe a general model for computing probabilities of conditionals. The mathematical environment is the theory of Markov chains (we need only rudimentary facts). The general idea is to represent the conditional as a game and the probability of the conditional is the probability of winning the game – i.e. the absorption probability in the special winning state in the graph. We use the term “Markov graph”, as the presented construction offers a natural and convenient graphical illustration of how the game evolves. We restrict our attention to conditionals where the antecedents have a positive probability.

The game metaphor is clear when considering simple conditionals of the form  $A \rightarrow B$  (like “*If it is Even, it is a Six*” when considering a die: we simply toss the die until an even number appears and check whether it indeed is a six – in which case the game is won). However, it is not obvious how it should be defined in the case of conjoined conditionals, like  $(A \rightarrow B) \wedge (C \rightarrow D)$ , or nested conditionals,

like  $A \rightarrow (C \rightarrow D)$  (not to mention the left-nested  $(C \rightarrow D) \rightarrow A$ ) – or even more complex constructions, like  $D \rightarrow [(E \rightarrow F) \rightarrow (A \rightarrow B) \wedge (C \rightarrow D)]$ .

We present an inductive construction of a family of graphs  $G(\alpha)$  – such that the graph  $G(\alpha)$  is designed to model a particular conditional  $\alpha$ . Each graph  $G(\alpha)$  generates a canonical probability space  $S(\alpha) = (\Omega_\alpha, \Sigma_\alpha, P_\alpha)$ , where  $\alpha$  is given an interpretation as an event  $[\alpha] \subseteq \Omega_\alpha$  so that the probability of  $\alpha$  can be computed as  $P_\alpha([\alpha])$ . Elementary events in  $\Omega_\alpha$  can represent the game scenarios. The graph allows to compute this probability by solving a system of linear equations (being the standard system for absorption probabilities for a Markov chain). In order to give a general inductive definition of the graph  $G(\alpha)$  we define three operations on graphs which correspond to the negation, conjunction and the conditional  $\rightarrow$ .

The definition enables the construction of graphs and the corresponding systems of equations for arbitrarily complex conditionals in an algorithmic and efficient manner.

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## *Jerzy Pogonowski's Contribution to Linguistics*

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The presentation will focus on the figure and linguistic achievements of the recently deceased Professor Jerzy Pogonowski. In addition to discussing the Professor's more significant achievements, the (meta)linguistic background of his scientific achievements will be presented. The point of reference will be the perspective of axiomatic linguistics, which was dynamically developed at the Poznań School of Structural Linguistics. Finally, an attempt will be made to answer why Jerzy Pogonowski abandoned linguistics at the turn of the 20<sup>th</sup> and 21<sup>st</sup> centuries.

ISBN 978-83-940690-8-7

