XX Conference

Applications of Logic in Philosophy and the Foundations of Mathematics

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XX Conference

Applications of Logic in Philosophy and the Foundations of Mathematics

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Abstracts

Editorial note

(EN) means that the talk is presented in English, (PL)—in Polish.

Dominating Graphs and Gamma Graphs

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Gamma graphs $\gamma.G$, $G(\gamma)$, and k-dominating graphs $D_k(G)$ are graphs, whose vertices correspond to dominating sets of G. Vertices of gamma graphs correspond to γ-sets of G, i.e. dominating sets whose cardinality is minimal. A subset S of the set of vertices $V(G)$ is a dominating set of the graph G iff every vertex of G belongs to S or is adjacent to some vertex, which belongs to S.

Different types of dominating sets have wide applications and have been extensively studied since 1963, when Ore published his book „Theory of Graphs" with first three theorems on dominating sets, (see for example [4]). Kernels of directed graphs, which correspond to dominating sets of undirected graphs have also been studied for the purpose of some logical problems [1].

It is known, that some graphs have exponentially many γ-sets, hence it is worth to ask if a γ-set can be obtained by some transformations from another γ-set. Gamma graphs and k-dominating graphs can be applied in the study of this reconfiguration problem.

In the talk the relation between the gamma graphs introduced in 2010 [5] and 2011 [2], and k-dominating graphs described in 2014 [3] will be discussed. Also gamma graphs of caterpillars with one leg will be presented. Finally the answer to the following question will be given: Is there a graph G such that $\chi(G) = a$ and $\chi(\gamma.G) = b$ for any positive integers a and b, where $\chi(G)$ denotes the chromatic number of the graph G?

References

[1] C. Berge, A. Rachmandra Rao: *A Combinatorial Problem in Logic*, Discrete Math., 17 (1977), 23–26.

[2] G.H. Fricke, S.M. Hedetniemi, S.T. Hedetniemi, K.R.Hutson: *γ-graphs of Graphs*, 31 (2011), 517–531.

[3] R. Haas, K. Seyffarth: *The k-dominating Graph*, Graphs and Combinatorics, 30 (2014), 609-617.

[4] T.W. Haynes, S.T. Hedetniemi, P.J. Slater: *Fundamentals on Domination in Graphs*, CRC Press, New York, 1998.

[5] S.A. Lakshmanan, A. Vijayakumar: *The Gamma Graph of a Graph*, AKCE J. Graphs and Combin., 7 (2010), 53–59.

A Survey of Hypersequent Calculi for S5

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Hypersequent calculi were developed as a generalised form of sequent calculus well-behaved with respect to many non-classical logics. In particular some of the systems were proposed to solve problems with unsuccessful formalisation for **S5** in standard sequent calculi. **S5** is one of the most important modal logic with nice syntactic, semantic and algebraic properties but its standard sequent calculus fails to be cut-free. On the other hand several approaches to formalisation of **S5** in hypersequent calculi were provided. In this survey we present HC for **S5** proposed by Pottinger, Avron, Restall, Poggiolesi, Lahav and Kurokawa. We are particularly interested in examining different methods which were used for proving the eliminability/admissibility of cut in these systems. Finally we present our own variant of a system which admits relatively simple proof of cut elimination.

A Purely Algebraic Proof of the Fundamental Theorem of Algebra

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1. Proofs of the fundamental theorem of algebra (FTA for short) can be divided up into three groups according to the techniques involved: proofs that rely on real or complex analysis, algebraic proofs, and topological proofs. Algebraic proofs make use of the fact that odd-degree real polynomials have real roots. This assumption, however, requires analytic methods, namely, the intermediate value theorem for real continuous functions. We develop the idea of algebraic proof further towards a purely algebraic proof of the intermediate value theorem for real polynomials (IVT).

2. The theory of real closed fields, started by Emil Artin and Otto Schreier in the 1920s, provides a general framework for our development. Taking advantage of theorems of this theory, we show that in order to prove FTA it suffices to prove IVT.

3. In our proof of IVT, we neither use the notion of continuous function nor refer to any theorem of real and complex analysis. Instead, we apply techniques of modern algebra: we extend the field of real numbers to the non-Archimedean field of hyperreals via an ultraproduct construction and explore some relationships between the subring of limited hyperreals, its maximal ideal of infinitesimals, and real numbers.

References

[1] P. Błaszczyk, *A Purely Algebraic Proof of the Fundamental Theorem of Algebra.* See http://arxiv.org/pdf/1504.05609v1.pdf.

Axiomatic Development of Euclid's Elements *Book V*

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1. Euclid's *Elements* includes three theories developed in an axiomatic fashion: plain and solid geometry (books 1–4, 11–13), theory of magnitudes (book 5), and arithmetic (books 7–9). Book 5 sets the basis for the theory of similar figures developed in book 6; it is also used in Euclid's stereometry. Theory of magnitudes played a crucial role in ancient Greek and in early modern mathematics till the end of the 17th century; in modern mathematics, since 70s of the 19th century, it has been replacing by the arithmetic of real numbers (see [3]).

Hilbert (1899) and Tarski (1959), to mention the most famous, provided modern accounts of Euclid's plain geometry. Beckmann, in his 1967 paper [1], presented the first axiomatic account of Euclid's theory of magnitudes. Mueller adopted Beckamnn's system and popularized it in his book [6]. Then, in 1975 Malmendier [5] introduced axioms for Euclid's arithmetic. We present another axiomatic account of Euclid's book 5. It differs from that of Beckamnn's both in the set of axioms and methodology.

2. Greek general notion μέγεϑος is exemplified in *Elements* book 6 by different kinds of geometric objects such as line segments, triangles, concave polygons, angles, and circular arcs. Magnitudes of the same kind can be formalized as an ordered additive semigroup, $\mathfrak{M} = (M, +, <)$, characterized by the following five axioms:

(E1) $(\forall A, B \in M)(\exists n \in \mathbb{N})[nA > B]$,

(E2) $(\forall A, B \in M)(\exists C \in M)[A > B \Rightarrow A = C + B]$,

(E3) $(\forall A, B, C \in M)[A > B \Rightarrow A + C > B + C]$,

(E4) $(\forall A \in M)(\forall n \in \mathbb{N})(\exists B \in M)[nB = A]$,

(E5) $(\forall A, B, C \in M)(\exists D \in M)[A : B :: C : D]$.

3. We provide an interpretation of Euclid's theory of proportion. By proving a proposition: If $(\mathbb{F}, +, \cdot, 0, 1, <)$ is an Archimedean ordered field, then for every $x, y, z, v \in \mathbb{F}_+$ equivalences hold

$$x : y :: z : v \Leftrightarrow x \cdot y^{-1} = z \cdot v^{-1}, \quad x : y \succ z : v \Leftrightarrow x \cdot y^{-1} > z \cdot v^{-1},$$

we show that every structure $\mathfrak{M} = (\mathbb{F}_+, +, <)$ is a model of Euclid's μέγεϑος.

4. Taking advantage of provided interpretation we present some independence results. Moreover, we revise a long-standing thesis:

"Ceratin it is that there is an exact correspondence, almost coincidence, between Euclid's definition of equal ratios and modern theory of irrationals due to Dedekind" (T.L. Heath, 1908),

"Dedekind (and before him the author—thought to be Eudoxos—of the fifth book of Euclid) constructed the real numbers form the rationals" (J. Conway, 2001).

References

[1] Beckmann F., *Neue Gesichtspunkte zum 5. Buch Euklids*, Archive for History of Exact Sciences IV, 1967, 1–144.

[2] Błaszczyk, P., Mrówka, K., *Euklides, Elementy, Księgi V–VI. Tłumaczenie i komentarz*, Copernicus Center Press, Kraków 2013.

[3] Piotr Błaszczyk, Nota o rozprawie Otto Höldera *Die Axiome der Quantität und die Lehre vom Mass*, http://www.up.krakow.pl/mat/annal-dyd/2013/blaszczyk.pdf

[4] Heiberg J.L., *Euclidis Elementa.* Vol. II, Teubneri, Lipsiae, 1884.

[5] Malmendier N., *Eine Axiomatik zum 7. Buch der Elemente von Euklid*, Mathematisch–Physikalische Semesterberichte 22, 1975, 240–54.

[6] Mueller I., *Philosophy of Mathematics and Deductive Structure in Euclid's Elements*, MIT Press, Cambridge, Massachusetts, 1981.

Peculiar Decidability of the Predicate Calculus with Identity

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In memory of Professor Leon Gumański

It is assumed that a theory T is ***peculiarly decidable***, when for a certain logic $\langle S, C_A \rangle$, which is equivalent the reducing predicate calculus with identity [1], A is the set of axiom, $T \subseteq S$ and in this logic exists the decidable theory $T' \subset T$ such that $C_A(T) = C_A(T')$. The theory T' is decidable in logic $\langle S, C_A \rangle$, when exists an effective method of verification of $T' \subseteq C_A(\emptyset)$. Moreover, the formula is ***valid***, when: 1^0 each occurrence of a quantifier associated (connects) with another variable, 2^0 any formula derived from this formula has a number of variables less than or equal to the number of variables in this formula. When formula is not valid, then is ***invalid***. A ***formula*** φ ***is conceived as a valid formula*** ϕ, when ϕ is derived from the formula φ.

Fact [2]
Each formula can be conceived as a valid formula.

Thesis of Gumański: *any effective method can be described by the valid formulas.*

Gumański proves [1] using this thesis

Theorem 1. (*about decidability*)
In the reductive calculation of predicates with identity, the set of formulas which is important as calculation thesis is decidable.

Hence

Theorem 2.
The calculation of predicates with identity is peculiarly decidable.

However

Theorem 3.
Conceiving of formulas as a valid is a method of undecidable in the sense of computability.

Therefore

Theorem 4.
The calculation of predicates with identity is undecidable in the sense of computability

However, worth using is

Theorem 5.
The calculation of predicates with identity is decidable in the sense of conceiving of formulas the calculation as valid formulas

References

[1] L. Gumański, *Introduction to the logic of contemporary* (in Polish: *Wprowadzenie w logikę współczesną*), PWN, Warszawa 1990.
[2] Letters from Professor Gumański.

Abductive Question-Answer System for Classical Propositional Logic

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We define abductive problem as a situation, when it is impossible to derive A form the knowledge base Γ. Therefore, the question (*abductive question*) arises: what we have to add to the knowledge base Γ to be able to derive A from it?

In our presentation we focus on describing Abductive Question-Answer System (AQAS) for Classical Propositional Logic. We use methods developed on the ground of Wiśniewski's Inferential Erotetic Logic (IEL) which enables us to transform an initial abductive question into auxiliary questions [5]. Answers to the auxiliary question create the answer we were seeking at the beginning i.e. answer to the initial question. Through this process we obtain two kinds of abductive hypothesis: analytic and non-analytic. The first one gives us answer that contains information only from our database while the second allow us to introduce a new piece of information. We also introduce rules and restrictions for generating abductive hypotheses which guarantee that those hypotheses are significant (an *explanandum* is not entailed by abductive hypotheses alone) and consistent with a given knowledge base. Introduced rules have questions as their premises and propositions as their conclusions. The effect of introducing such rules with certain restrictions is that the set of possible hypotheses is reduced to the optimal one, i.e. redundant (non significant and inconsistent) cases are impossible to obtain. As a result, Abductive Question-Answer System generates 'good' abductive hypotheses in one step, on the contrary to the more standard approach where this process is divided into two parts: generation of hypotheses and evaluation with qualifying selection (see for example [2]).

Our future work will cover also implementation of AQAS in programming language. This would enable us to test the system on huge datasets and compare it with already existing solutions, like one presented by Komosiński [2].

This stage has begun by now (as a first step a simple theorem prover for Classical Propositional Logic in erotetic setting has been developed) and as the implementation language was chosen Haskell. The reason of this choice was that Haskell is a purely functional language and that enables us to define the AQAS almost in the same manner as we introduce it in the logical formalism.

References

[1] Aliseda, A. (2006). *Abductive reasoning. Logical investigations into discovery and explanation.* Springer, Netherlands.

[2] Komosiński, M., Kupś, A., Leszczyńska-Jasion, D., and Urbański, M. (2014). Identifying efficient abductive hypotheses using multi-criteria dominance relation. *ACM Transactions on Computational Logic (TOCL)*, 15(4):28:1–28:20.

[3] Peirce, C. S. (1931–1958). *Collected Works.* Harvard University Press, Cambridge MA.

[4] Urbański, M. (2009). *Rozumowania abdukcyjne.* Wydawnictwo Naukowe UAM, Poznań.

[5] Wiśniewski, A. (2004). Socratic proofs. *Journal of Philosophical Logic*, 33(3): 299–326.

Beyond Finitarity in Abstract Algebraic Logic I From Motivation to a Theory

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Abstract algebraic logic (AAL) is the branch of mathematical logic that provides a systematic framework to deal with the multiplicity of logical systems according to their relation with corresponding matricial (algebraic) semantics (see [3]). Matrices are pairs $\langle \boldsymbol{A}, F\rangle$, where $\boldsymbol{A}$ is an algebra (with an operation for each connective of the logic) and F is a subset of the domain A of $\boldsymbol{A}$ that gives a notion of truth for the logic by selecting designated truth-values.

This framework has several layers of abstraction and generality. Naturally, the level of abstraction/generality is inversely proportional to the strength of the achieved results and the simplicity of presentation. One of the most common restrictions imposed on logical systems for the sake of simplicity is *finitarity*, i.e. the assumption that whenever a formula is derivable from a set of premises it is already derivable from some of its finite subsets.

The most basic result (and a fundamental one) proved for all finitary logics is the Abstract Lindenbaum Lemma, namely the fact that any theory can be extended into a *completely meet-irreducible* theory (in other words: completely meet-irreducible theories form a basis of the system of all theories). This leads to completeness of any finitary logic w.r.t. (relatively) subdirectly irreducible matrices, e.g. completeness w.r.t. the two-valued Boolean matrix in the case of classical logic.

Interestingly enough, this result is unnecessarily strong. Note that the two-valued Boolean matrix is also the only *finitely* subdirectly irreducible matrix for classical logic. A matrix $\langle \boldsymbol{A}, F\rangle$ is relatively finitely subdirectly irreducible (RFSI) iff (assuming that the logic has disjunction) F is a *prime* filter, i.e. for each $a, b \in A$, if $a \vee b \in F$, then $a \in F$ or $b \in F$. Also, a matrix $\langle \boldsymbol{A}, F\rangle$ is RFSI iff (assuming that it is a semilinear logic [1]) F is a *linear* filter, i.e. for each $a, b \in A$, $a \to b \in F$ or $b \to a \in F$. Moreover, the matrix is RFSI iff its filter is *finitely* meet-irreducible. All these facts show that, for many logics of interest, *finitely* meet-irreducible theories (which are sometimes, for obvious reasons, called *intersection-prime theories*) play a more important role than completely meet irreducible theories, and, due to the abstract Lindenbaum lemma, they also form a basis of the system of all theories.

Therefore, it makes sense to define a bigger class of logics (extending that of finitary logics) in which one still has completeness w.r.t. RFSI matrices, namely logics satisfying the *intersection-prime extension property* (IPEP), i.e. logics where each theory can be extended to an intersection-prime theory [2].

In this talk we will present, together with the necessary background notions, the class of logics with the IPEP and show that it is a proper extension of the class of finitary logics.

References

[1] Cintula, P., Noguera, C.: Implicational (semilinear) logics I: A new hierarchy. *Archive for Mathematical Logic* **49**(4):417–446, 2010.

[2] Cintula, P., Noguera, C.: The proof by cases property and its variants in structural consequence relations. *Studia Logica* **101**(4):713–747, 2013.

[3] Czelakowski, J.: Protoalgebraic Logics, *Trends in Logic*, vol. 10. Kluwer, 2001.

Beyond Finitarity in Abstract Algebraic Logic II From Theory to Applications

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In this talk we will demonstrate the usefulness of IPEP in various areas of AAL.

IPEP in disjunctional logics. AAL has led to fine analysis on the role of the connectives of classical logic, identifying their essential properties, and thus suggesting possible generalizations of these connectives (in non-classical logics) still retaining their essential function(s) in classical logic. Notable examples of this approach are the extensive studies on equivalence connectives (see e.g. [1,5]). Disjunction connectives have also been systematically studied (see e.g. [3,5]), based on their main property (and modifications thereof), the proof by cases property:

PCP If $\Gamma, \varphi \vdash \chi$ and $\Gamma, \psi \vdash \chi$, then $\Gamma, \varphi \vee \psi \vdash \chi$.

Taking inspiration from the study of equivalence connectives, one may allow a lot of freedom on the disjunction connective and allow that, instead of being a primitive connective, it may be definable by an arbitrary (even infinite) set of formulas. We will show that, assuming the IPEP, we can obtain several important consequences of the presence of a suitable disjunction in a given logic. Namely, we will show the role of prime filters, characterize disjunctional logics in terms of distributivity of the lattice of filters, find an axiomatization of the extension of a logic semantically given by a positive universal class of its models, and as a particular case we show how to axiomatize the intersection of any finite set of its axiomatic extensions.

IPEP in semilinear logics. The paper [2] proposes an approach to protoalgebraic logics based on implication connectives (instead of equivalence). It shows, in particular, that implications define an order relation in the reduced matrix models of these logics. This yields a natural definition of *semilinear logics* as those complete w.r.t. *linearly ordered* matrix models. Semilinearity is characterized first in [2] (only for *finitary* logics) in terms of a purely syntactical property: the metarule called *Semilinearity Property*:

SLP If $\Gamma, \varphi \to \psi \vdash \chi$ and $\Gamma, \psi \to \varphi \vdash \chi$, then $\Gamma \vdash \chi$.

This result is too limited because there are many prominent examples of infinitary semilinear logics. In this talk we will show that the characterization can be extended to IPEP logics, showing that semilinearity is actually equivalent to the conjunction of IPEP and SLP [4].

References

[1] Blok, Willem J., and Don L. Pigozzi, *Algebraizable Logics*, vol. 396 of *Memoirs of the American Mathematical Society*, American Mathematical Society, 1989.

[2] Cintula, P., Noguera, C.: Implicational (semilinear) logics I: A new hierarchy. *Archive for Mathematical Logic* **49**(4):417–446, 2010.

[3] Cintula, P., Noguera, C.: The proof by cases property and its variants in structural consequence relations. *Studia Logica* **101**(4):713–747, 2013.

[4] Cintula, P., Noguera, C.: Implicational (semilinear) logics II: additional connectives and characterizations of semilinearity. Submitted.

[5] Czelakowski, J.: Protoalgebraic Logics, *Trends in Logic*, vol. 10. Kluwer, 2001.

The Concept of Meaning in Formalized Languages

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1. Natural language and formalized languages.
2. Real object and intentional object.
3. **The meaning** of primitive and defined terms in axiomatic systems.
4. An analysis of the notion of **equinumerosity**.

Rasiowa-Sikorski Sets and Forcing

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The talk is concerned with the problem of building models for first-order languages from the perspective determined by the classic paper of Rasiowa and

Sikorski [9]. The notion of a Rasiowa-Sikorski set of formulas for an arbitrary language L is introduced. Investigations are confined to countable languages. Each Rasiowa-Sikorski set defines a countable model for L. Conversely, each countable model for L is determined, up to isomorphism, by some Rasiowa-Sikorski set. Consequences of these facts are investigated.

Rasiowa-Sikorski sets enable one to build a substitutional semantics for first-order logic. This is due to the fact that the satisfaction relation in the model $\boldsymbol{A}_\Delta$ corresponding to a Rasiowa-Sikorski set Δ is expressed in a straightforward way in terms of "double" substitutions of variables in the formulas of L. Since each consistent closed set of formulas Σ is the intersection of a family of Rasiowa-Sikorski sets, the Extended Completeness Theorem for first order logic in terms of the above substitutional-semantics immediately follows. This shows that the class of Rasiowa-Sikorski sets (and even a narrower family of model sets) suffices for establishing an adequate substitutional semantics for first-order logic.

The relationship between Rasiowa-Sikorski sets and forcing for first-order languages is also outlined.

Literature

[1] Adamowicz, Zofia and Zbierski, Paweł "Logic of Mathematics: A Modern Course of Classical Logic", John Wiley & Sons, New York 1997.

[2] Bell, John B. "Set Theory. Boolean-Valued Models and Independence Proofs", Third Edition, Oxford Science Publications, Clarendon Press, Oxford 2005.

[3] Dzik, Wojciech *The existence of Lindenbaum's extensions is equivalent to the axiom of choice*, Reports on Mathematical Logic **13** (1981), 29–31.

[4] Feferman, Solomon *Review of the paper: H. Rasiowa and R. Sikorski, "A proof of the completeness theorem of Gödel"*, Journal of Symbolic Logic **19** (1952), 72.

[5] Fitting, Melvin "Intuitionistic Logic, Model Theory and Forcing", North-Holland, Amsterdam 1969.

[6] Goldblatt, Robert *On the role of the Baire Category Theorem and Dependent Choices in the foundations of logic*, Journal of Symbolic Logic **50** (1985), 412–422.

[7] Henkin, Leon *The completeness of the first-order functional calculus*, Journal of Symbolic Logic **14** (1951), 159–166.

[8] Hodges, Wilfrid "Model Theory", Cambridge University Press, Cambridge 2008. (The digitally printed version).

[9] Rasiowa, Helena and Sikorski, Roman *A proof of the completeness theorem of Gödel*, Fundamenta Mathematicae **37** (1950), 193–200.

[10] Rasiowa, Helena and Sikorski, Roman "The Mathematics of Metamathematics", PWN, Warsaw 1963.

[11] Scott, Dana "Boolean-Valued Models for Set Theory", Mimeographed notes for the 1967 American Math. Soc. Symposium on axiomatic set theory.

[12] Scott, Dana, *The Algebraic Interpretations of Quantifiers: Intutionistic and Classical*, in: "Andrzej Mostowski and Foundational Studies" (ed. by A. Ehrenfeucht, V.W. Marek and M. Srebrny), IOS Press 2008, 289–312.

[13] Weaver, Nick "Forcing for Mathematicians", World Scientific 2014.

[14] Zygmunt, Jan *A survey of the methods of proof of the Gödel-Malcev's completeness theorem*, in: "Studies in the History of Mathematical Logic" (ed. by Stanisław J. Surma), Ossolineum, Wrocław 1973, 165–238.

Introduction to Projective Unification in Predicate Logics

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Key words: unifiers, projective unifiers, admissible rules, passive rules, almost structural completeness in predicate logics.

We define and investigate projective unifiers in predicate logic.

Projective unifiers were introduced by S. Ghilardi and successfuly applied in propositional logics, especially in intuitionistic and modal logics.

In propositional logics, a *unifier for* a formula φ in a logic L is a substitution σ such that $\vdash_L \sigma(\varphi)$. A unifier τ for φ is called a *ground unifier* if τ has $\{\top, \bot\}$ as the co-domain, i.e. $\tau : Var(\varphi) \to \{\top, \bot\}$. A unifier σ for φ is a *projective unifier* for φ in L if, for each $x \in \mathrm{Var}(\varphi)$,

$$\varphi \vdash_L \sigma(x) \leftrightarrow x.$$

Using substitutions for atomic formulas (endomorphisms modulo bounded variables) introduced by W.A. Pogorzelski and T. Prucnal we define and investigate projective unifiers in predicate logic. Applications to admissible rules and almost structural completeness follow.

In the first, introductory part, a *method of ground unifiers* for building projective unifiers will be presented for some predicate logics like classical predicate logic, predicate modal logic **QS5** and other predicate logics.

The Status of Modal Propositions in Avicenna: A Comparision with that in Wittgenstein Logic

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Modality sets up the kind of relation that the underlying situation that a proposition points at establishes with the truth. Modality in classical logic denotes in what specific relation the concordance between subject and predicate in a proposition is conditioned on.

In this paper, I am going to evaluate the concepts in Avicenna logic that express modality and their functions in propositions in that logic. I will analyse what kind of originality the Avicenna logic owned in the cultural realm it came out. I, then, will point out at the significant differences as well as the similarities between Avicenna and his predecessor, Farabi, and shed light on the issue by way of giving some examples. Additionally, I will go on conveying the form and implications of modality in the applications of the Avicenna logic with a specific emphasis to bring forth its special linkage to ontological and epistemological conceptions in Islamic thought. Lastly, I will assess the Avicenna logic's similarities to and the differences from the modality in logic as conceived by Wittgenstein, the prominent counterpart in Western thought.

The Concept of "Relation" and its Function in Classical Logic: Case of Ibn Hazm

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Ibn Hazm, grown and educated in Andalus, Spain, was a prominent scholar of Islamic jurisprudence and thinker closely associated with the logical tradition of Averroes and Aristotle.

In this paper, we analyse the fundamental characteristics of the Ibn Hazm logic. We analyse the epistemological status of propositions in the Ibn Hazm logic, and demonstrate the distinctive place and the very special conditions under which they provide basis for knowledge. In this vein, we demonstrate how propositions and syllogisms are formed on a relational base and the explicative power they thus attained in applications. We deal with the issue of place and function of relations in classical logical applications, and demonstrate these based on examples by Ibn Hazm. We show the powerful function relation plays in the Ibn Hazm logical thinking in particular attribution to applications as widely varying as, judicial inference, boundedness, place, quality, quantity, contradiction, etc., and hence draw special attention to its value and richness in logical applications.

Negational Fragment of Intuitionistic Control Logic

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The use of formal logic as a tool for knowledge representation in Computer Science frequently requires an integration of several logic systems into a homogeneous environment. In [1] Ch. Liang and D. Miller introduced a combination of intuitionistic and classical logics called Intuitionistic Control Logic (ICL). This logic arises from Intuitionistic Propositional Logic by extending its language with additional new constant for *falsum* $\perp$ which is distinct from intuitionistic falsum 0.

The original impetus for ICL came from the search for a logic that would preserve the crucial connective of intuitionistic implication and at the same time would be able to type programming language control operators such as *call/cc*. So far, the Curry-Howard correspondence between proofs and programs related *call/cc* to Peirce's law, which extends intuitionistic logic to classical logic. ICL is fully capable of typing programming language control constructs while maintaining intuitionistic implication as a genuine connective. This is achieved by adding to the language of intuitionistic logic the additional constant $\perp$.

Having two different falsum constants enables to define two distinct negations: an ordinary intuitionistic negation denoted $\sim A = A \to 0$ and $\neg A = A \to \perp$ which bears some characteristics of classical negation. Combination of these two kinds of negation results in possibility of forming new operators.

In our talk we would like to briefly describe ICL paying special attention to its monadic negational fragment. We analyse the number of distinct operators that can be defined by sequences of both negations and give the complete characterization of the interaction between them by presenting a poset of non equivalent formulae of this fragment of ICL.

References

[1] Liang Ch., Miller D.: *An Intuitionistic Control Logic.* To appear.

[2] Glenszczyk A.: *Negational Fragment of Intuitionistic Control Logic.* To appear in Studia Logica.

Interpolation and Proof Systems

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There are various reasons for developing proof systems for logics. They may, for example, be used to prove that a logic is consistent or decidable, or provide a means to uncover certain structural properties of a logic, such as interpolation.

Interpolation is considered by many to be a "good" logical property because it indicates a certain well-behavedness of the logic, vaguely reminiscent to analycity: if an implication $\phi \to \psi$ holds in the logic, then there is a formula χ in the common language of ϕ and ψ that *interpolates* the given implication, that is, such that $\phi \to \chi$ and $\chi \to \psi$ hold. What the common language is depends on the logic one considers. In propositional logics it typically means that all atoms in χ occur in ϕ as well as in ψ.

As expected, many well-known logics satisfy interpolation, such as classical propositional and predicate logic, which was shown by William Craig in 1957. More than three decades later it turned out that some of the standard logics with interpolation also satisfy the stronger property of *uniform interpolation*, where the interpolant only depends on the premiss or the conclusion of the given implication.

Whereas in the presence of a decent analytic proof system, proofs of interpolation are often relatively straightforward, proofs of uniform interpolation are in general quite complex. In this talk I will describe a method to extract uniform interpolants from sequent calculi and prove, using this method, that logics without uniform interpolation lack certain calculi. Thus having uniform interpolation becomes a property of proof systems rather than of logics. The method applies to many propositional logics, including modal and intermediate logics, and thereby provides a way to prove that several of such logics do not have proof systems of a certain form.

Building Rooted Frames for Some Polimodal Logics

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In the talk we consider two polimodal logics L_1 and L_2 having disjoint sets of modal operators. Logics L_1 and L_2 are characterized by classes of rooted frames $\mathcal{C}_1$ and $\mathcal{C}_2$, respectively. The fusion $L_1 \otimes L_2$ of L_1 and L_2 is the smallest polimodal logic containing $L_1 \cup L_2$.

Let $\mathcal{C} = \{\mathfrak{F}_i; i \in I\}$ be a family of rooted frames, where x_i is a root of $\mathfrak{F}_i$ and let $\mathfrak{F}$ be a rooted frame with a root x_0. The point x_0 is called a $\mathcal{C}$-root if for each $i \in I$ there exists a p-morphizm $f : \mathfrak{F} \to \mathfrak{F}_i$ such that $f(x_0) = x_i$.

Assume that there exist L_1-frame $\mathfrak{F}^1$ with $\mathcal{C}_1$-root and L_2-frame $\mathfrak{F}^2$ with $\mathcal{C}_2$-root. In the talk we present a construction of a rooted frame $\mathfrak{F}$ for the fusion $L_1 \otimes L_2$. Moreover, we obtain the following property

$$\mathfrak{F} \models \varphi \iff \mathfrak{F}, x_0 \models \varphi,$$

where x_0 is a root of $\mathfrak{F}$.

On Halldén Completeness in Brouwer Logics Determined by Nets of Clusters[1]

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We continue research on Brouwerian modal logics which are Halldén complete. We study the Brouwer modal logic **KTB** and its normal extensions, which are determined by a class of Kripke frames with a tolerance relation and having special forms. Each reflexive and symmetric Kripke frame may be divided into blocks of tolerance. Blocks of tolerance are linearly ordered if one of them has non-empty intersection with at most two other blocks. If one block of tolerance sees at most k other blocks then we call such a Kripke frame as k-branching net of clusters. Brouwerian linear logics may be axiomatized by adding the following axiom (see [3]):

$$(3') \ := \Box p \vee \Box(\Box p \to \Box q) \vee \Box((\Box p \wedge \Box q) \to r).$$

It was proved in [3] and [4] that

Theorem 1. *All logics from $NEXT(\mathbf{KTB.3'})$ are Kripke complete and have f.m.p. The cardinality of $NEXT(\mathbf{KTB.3'})$ is uncountably infinite.*

Referring to Kripke frames in which blocks of tolerance are connected with some bounded number of others, we may generalize the axiom (3') as follows:

$$(n') \ := \Box p_1 \vee \Box(\Box p_1 \to \Box p_2) \vee ... \vee \Box((\Box p_1 \wedge ... \wedge \Box p_n) \to \Box p_{n+1})$$

[1]This work was supported by the financial support by the NCN, research grant DEC-2013/09/B/HS1/00701.

and study the logics $NEXT(\mathbf{KTB.n'})$.

Similarly as for linear case we proved that:

Theorem 2. *For the given $n \in \mathbb{N}$ the logics from $NEXT(\mathbf{KTB.n'})$ are Kripke complete and have f.m.p.*

In our talk, we describe Halldén complete logics from $NEXT(\mathbf{KTB.n'})$.

References

[1] J.F.A.K. van Benthem, I.I. Humberstone, *Halldén-completeness by Gluing of Kripke Frames*, Notre Dame Journal of Formal Logic, Vol. 24, (4): 426–430, (1983).

[1] Z. Kostrzycka, *On interpolation and Halldén-completeness in $NEXT(\mathbf{KTB})$*, BSL 41, (1–2): 23–32, (2012).

[1] Kostrzycka, Z. *On linear Brouwerian logics*, Mathematical Logic Quarterly 60 (4–5): 304–313, (2014).

[1] Kostrzycka, Z., Miyazaki, Y. *Normal modal logics determined by aligned clusters.* Submitted.

On Describing Theories of Kripke Models for Intuitionistic Logic

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The notion of logical equivalence still remains one of the most interesting subjects of investigation. In many logical systems the question that arises is how to describe the theory of a considered structure by means of a single formula.

In the case of classical logic this problem is simplified, since the law of excluded middle holds. Namely, given a classical first-order structure $\mathcal{A}$, any formula φ remains true or false in $\mathcal{A}$. Under some additional assumptions, any information concerning $\mathcal{A}$, whether positive or negative, can be encoded in the language. And hence, the theory of $\mathcal{A}$, defined as the set

$$Th(\mathcal{A}) = \{\varphi \colon \mathcal{A} \models \varphi\},$$

can be described by means of a single formula.

In the talk we consider Kripke semantics for intuitionistic first-order logic, and solve the aforementioned problem. Since intuitionistic connectives differ significantly from the classical ones, one might expect a more complex representation.

For an arbitrary node α of a Kripke model $\mathcal{K}$ we construct so-called *Yes/No Formulas* that describe the theory of α. Furthermore, we establish the relationship between Yes/No Formulas and the notion of logical equivalence of Kripke models.

Abstract Banach-Mazur Games

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The Banach-Mazur game is usually played in a topological space, using its nonempty open subsets. The idea is that two players alternatively build a decreasing sequence of sets and the result of the game is its intersection. We develop a much more abstract setting for this game. Namely, we study the Banach-Mazur game in the context of category theory, aiming at an abstract notion of completeness. We focus our attention to categories where the Banach-Mazur game is determined and results in certain universal objects. We shall give several examples coming from algebra, topology and analysis.

Beyond Finitarity in Abstract Algebraic Logic III Hierarchy and Separating Examples

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In this last talk, we perform a finer analysis of the landscape of infinitary logics in AAL. We will consider, besides the IPEP [3], the following three properties: σ-IPEP (namely, the fact that completely meet-irreducible theories form a basis of all theories), completeness w.r.t. relatively subdirectly matrices (RSI matrices), and completeness w.r.t. relatively subdirectly matrices (RFSI matrices).

We show the mutual relations between all these notions and build separating examples. We will also consider the position of the separating examples in the (extended) Leibniz hierarchy [1,2,4].

These results will be presented in a forthcoming paper.

References

[1] Blok, Willem J., and Don L. Pigozzi, *Algebraizable Logics*, vol. 396 of *Memoirs of the American Mathematical Society*, American Mathematical Society, Providence, RI, 1989.

[2] Cintula, P., Noguera, C.: Implicational (semilinear) logics I: A new hierarchy. *Archive for Mathematical Logic* **49**(4):417–446, 2010.

[3] Cintula, P., Noguera, C.: The proof by cases property and its variants in structural consequence relations. *Studia Logica* **101**(4):713–747, 2013.

[4] Czelakowski, J.: Protoalgebraic Logics, *Trends in Logic*, vol. 10. Kluwer, Dordrecht, 2001.

The Classification of Properties in CIFOL

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CIFOL (Case-Intensional First Order Logic) is combination of first-order predicate logic with modal logic resulting from modifications of Bressan's *General interpreted modal calculus*. It has some interesting features, which distinguish it from standard predicate logic and standard modal logic. For example, individuals here are not represented simply by elements of a domain, but by function from a set of cases to a domain. It enables to treat predication as intensional, that is, dependent on intensions of singular terms.

One of possible applications of this system is a classification of properties, broadly understood. Authors themselves wanted to use it to distinguish between sortals and qualities. They defined three modal features which every property could have or not, namely Extensionality, Modal Constancy and Modal Separation. Then, they use these features to characterize sortals and qualities. However, these features could be used to create broader classification of properties. Simple combinatorics gives us eight possibilities, because each property could have or have not each of three features.

I will present basics of CIFOL and show some helpful method of representing individuals and properties in this system. It will be used to analyse 8 types of properties mentioned above. All of them could be nonempty, but two turn out to be trivial. The other six are identical with or include some interesting kinds of properties, for example sortals or natural kinds, qualities (which are divided into necessary and accidental), indexicals.

This method of analysis of properties allows us to make distinctions between them that are sharper than in a natural language, but also can grasp some important pre-theoretical intuitions about them.

Distributivity for Upper Continuous and Strongly Atomic Lattices

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A lattice L is said to be *upper continuous* if it is complete and the following condition is satisfied:

$$(UC) \qquad x \wedge \bigvee C = \bigvee\{x \wedge c : c \in C\},$$

for any $x \in L$ and for any chain $C \subseteq L$. A lattice L is *strongly atomic* if:

$$(SA) \qquad (\forall x, y \in L)\big(x < y \Rightarrow (\exists z \in L)(x \prec z \leq y)\big).$$

We consider the following conditions:

$$(D) \qquad (\forall x, y \in L)(x \wedge y \prec x, y \ \Rightarrow\ [x \wedge y, x \vee y] \cong B_2),$$
$$(D^*) \qquad (\forall x, y \in L)(x, y \prec x \vee y \ \Rightarrow\ [x \wedge y, x \vee y] \cong B_2),$$

where B_2 denotes a four-element Boolean lattice. These conditions are strengthenings of well known Birkhoff Conditions. Note that, if a lattice L is finite then the conjunction of Birkhoff Conditions is equivalent to the modularity of L. The main result is:

Theorem. *If L is an upper continuous and strongly atomic lattice then L is distributive iff L satisfies (D) and (D^*).*

In the talk we present the main idea of the proof. Moreover, we discuss some consequences of the theorem:

- **A strengthening of a Dilworth's theorem:** For an upper continuous and strongly atomic lattice upper and lower local distributivity imply distributivity.

- **A strengthening of a Birkhoff's theorem:** If L is an upper continuous, strongly atomic, modular but non-distributive lattice then L contains a covering diamond.

References

[1] R. P. Dilworth, Lattices with Unique Irreducible Decompositions, **Annals of Mathematics**, Vol. 41, No. 4 (1940), pp. 771–777.
[2] R. P. Dilworth, P. Crawley, Decomposition Theory for Lattices without Chain Conditions, **Trans. Amer. Math. Soc.**, 96 (1960), pp. 1–22.
[3] M. Ward, Structure Residuation, **Annals of Mathematics**, Vol. 39, No. 2 (1938), pp. 558–568.

A $\to$-Decomposition Property

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In the talk we discus the notion of $\to$-irreducibility in finite Heyting lattices. An element a is called *$\to$-irreducible* if $a = x \to y$ implies $a = x$ or $a = y$.

Theorem. ([1]) *An element $a \in L$ is $\to$-irreducible iff a is the least element in some maximal Boolean interval in L.*

As an easy consequence we get a new characterization of the skeleton (the least elements of maximal Boolean intervals) of a lattice L:

$$S(L) = \{a \in L : a \text{ is } \to \text{-irreducible}\}.$$

Definition. We say that L has the *$\to$-decomposition property* if each element of L can be presented as a $\to$-combination of elements of $S(L)$.

The natural question arises:

Problem. How to characterize lattices with the $\to$-decomposition property?

We present a partial solution of the above problem for lattices with a skeleton isomorphic to a chain. For any $n, k > 0$ consider the set:

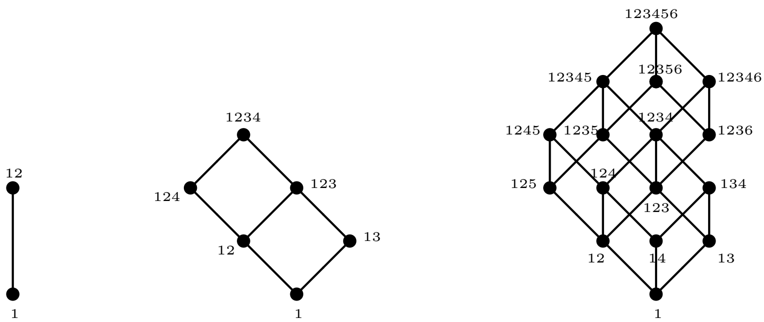
$$k \otimes B_n = \bigcup_{i=1}^{k} [I_i, I_{n+i}],$$

with inclusion order, where $I_m = \{1, \ldots, m\}$ and $[I_i, I_j] = \{A \subseteq \mathbb{N} : I_i \subseteq A \subseteq I_j\}$. Lattices $1 \otimes B_1$, $2 \otimes B_2$ and $3 \otimes B_3$ are presented below.

Our main result is the following:

Theorem. *If $k \geqslant n$, then the lattice $k \otimes B_n$ has the $\to$-decomposition property.*

We present the idea of the proof and discuss selected problems:



Problem. If $k < n$ then $k \otimes B_n$ has no $\rightarrow$-decomposition property.

Definition. A lattice L is said to be *n-regular* if each maximal Boolean lattice of L is isomorphic to B_n.

Problem. To find a criterion of $\rightarrow$-decomposition property for n-regular lattices.

References

[1] M. Łazarz, K. Siemieńczuk, New Characterization of Skeleton of Finite Distributive Lattice, **Bulletin of the Section of Logic** 43/1–2 (2014).

On an Application of Sets with Atoms in Logic

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In our presentation we aim at introducing to the audience the apparatus of *sets with atoms* and show how it can be applied in logic. In particular we demonstrate the results of our own research on the extension of modal $\mu-$calculus to this new setting. Use of sets with atoms in this context allow us to define a quite expressive logic (in particular properly extending traditional modal $\mu-$calculus) with infinitary boolean operations which enjoy some plausible from computational point of view properties.

The machinery of sets with atoms can be traced back to the origins of set theory where they appeared under the name of *Fraenkel-Mostowski sets* or *permutation models* and they were used to show that the Axiom of Choice is not a consequence of a theory called *ZFA*. The main idea was to lay the universe of sets (in the sense of *ZFC*) not on one set, but on a collection of countably many "indistinguishable" elements, called atoms. One can define such a universe in a standard way by a transfinite induction, in each step allowing the sets to contain as elements not only sets previously defined, but also atoms. Not all sets are legal in such a universe: we keep only those which are invariant under the action of a pointwise stabilizer of some finite set with atoms (so called: finitely supported sets).

Nowadays sets with atoms reappeared and found its use in theoretical computer science. In the context of our presentation the most important idea of using sets with atoms is the following: let us observe that some sets with atoms can be covered by (i.e. are subsets of a sum of) finitely many orbits of the action of the permutation group of atoms. Such sets will be called *orbit-finite*. The most important observation of the above cited paper is that finitely supported, orbit-finite sets with atoms, whose all elements are finitely-supported (such sets are called *nominal*), can be represented in a uniform way by *finite sets*. This representation gives rise to the possibility of performing algorithms on infinite, but orbit-finite sets.

In our presentation we introduce all the definitions required to understand what sets with atoms are and demonstrate the important steps in the proof of the representation theorem. As an important and interesting example of application we show how to define an extension of modal μ−calculus which admits infinite conjunctions and disjunctions and which formulae can nevertheless be represented by finite sets. Moreover we can show that the model-checking problem for this logic is decidable, where the class of models for this logic consists of orbit-finite, nominal and finitely-supported Kripke structures.

It Almost Certainly…

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The lecture provides a tentative formal logical study of contexts like *prima facie* **A** (and *prima facie not* **A**) or *it almost certainly* **A** *obtains* (and *it almost certainly* **A** *doesn't obtain*) where **A** stands for a proposition or for a state of affairs. Such contexts play important role in moral reasoning and in theoretical reasoning like reasoning from probabilistic evidence. The concepts of *prima facie*(or *it almost certainly*) are treated as modal operators. The lecture provides an axiomatic characterization of these expressions within the framework of a modal propositional logic and then, presents a semantic analysis of these concepts. The semantics is a slight modification to the standard relational semantics for normal modal propositional logic.

The Connective "względnie" in Polish

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The Polish word "względnie" is most frequently used as an uninflected adverb (equivalent to "relatively" in English).

It is also possible to use this word correctly as a connective (equivalent to "or" in English).

In this paper I will discuss the word "względnie" when used as a connective and then I will focus on its extensional description.

Philosophical Remarks on Non-Fregean Logic

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In my lectures I will introduce the fundaments of the non-Fregean logic and its philosophical aspects. Particularly I will consider philosophical consequences of extensionality of non-Fregean logic. Logical arguments of extra-logical assumptions held in non-Fregean theories will be discussed, particularly assumptions with regard to sentence equality and quantifiers binding sentential variables.

Later I will discuss Barcan's formulae in the language of the non-Freagan logic, and different kinds of definitions which occurs in non-Fregean theories.

The problem of reification of situations will be discussed as well as its relation to the existence of abstract objects. The aim of my lectures is to present general observations concerning the non-Fregean logic, including both the foundations upon which it rests and its possible applications.

Different Approaches to Modality—Hermann Weyl's Philosophy of Logic

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Hermann Weyl is well known for his contributions in the field of mathematics and physics. The aim of my talk is to present his philosophical remarks about the modal words: possibility and necessity in the context of different modal systems.

The notion of modality entered the logical scene in 1930s, when CI Lewis introduced his systems of strict "implication". In this system necessity may be interpreted as deducibility, and therefore is related to a set of axioms. I will focus on Łukasiewicz n-value logic. Here the notion of probability occurs, and one may say that a is possible if probability of a is greater than 0, and similarly, that a is necessary if the probability of a is equal to 1. Then a system based on the set-theoretic topology will be considered. On the ground of this system, notions of necessity and possibility conduct to the properties of being the inner and the limit points of a certain set.

The properties of possibility and necessity between the above systems vary significantly, and these differences will be briefly discussed.

Mathematical Therapy (for Adults)

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It is sometimes claimed that kids enter the school being mathematically highly creative and then their creativity dramatically decreases, most likely because of the boring schematic teaching process. The present author does not have any contact with teaching mathematics at the school level, but he has some experience in teaching mathematical logic and mathematics at the university level, for the students of humanities. After about four decades of such an activity he realized that his role is comparable to that of a therapeutist. Thus, it is not of first importance how much new material you present to the students—much more important is the fact that you should change their attitude towards mathematics. They should forget about being frightened by mathematics and start to like (or even admire) the subject. This is by no means an easy task and it can be achieved only when you choose a proper (accurate and at the same time attractive) way to present mathematical notions, theorems, methods, proofs, etc.

We share with the audience a few reflections concerning our *Mathematical Puzzles* course, offered mainly to students of cognitive science at the Adam Mickiewicz University in Poznań, Poland. These reflections have been already presented at a few conferences in Poland in the years 2013–2015 and we try to summarize them in this talk.

Contrary to the usual mathematical exercises, mathematical puzzles are often connected with that which is unexpected, which contradicts our every-day experience. Thus, such puzzles are instructive, as far as a critical attitude towards informal intuitions is concerned. They teach us that we should be cautious in relying on intuitions, which are sometimes very illusory.

The puzzles are divided into thematic groups, including such topics as: the Infinite, numbers and magnitudes, movement and change, shape and space, orderings, patterns and structures, algorithms and computation, probability, logic. Many of them are connected with paradoxes, i.e. results which seem counterintuitive but are nevertheless true, which can be shown by resolving the paradox in question. We have collected several dozens of such puzzles, accompanied by solutions and commentaries and we hope to publish this material under the title *The Odyssey of the Mathematical Mind*.

Observing the students' activity during our course, we have noticed that it is much more easier for them to acquire small, concise chunks of dissipated knowledge rather than to listen to lengthy expositions of entire theories seldom illustrated with examples.

We claim that *paradox resolution* is very instructive as far as the development of correct mathematical intuitions is concerned. Obviously, one should use several standard (normal, typical, natural) exercises in teaching mathematics—they doubtlessly serve as proper tools for stabilization of intuitions. However, to see clearly the limitations of our mathematical intuitions, we should also investigate the objects which—for several reasons—are called *pathological* in mathematics. Such objects eventually become domesticated, thus leading to new mathematical domains.

Looking for Classical Counter-Models in Intuitionistic Kripke Structures

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Let Γ be a class of formulae of some first-order language. We say that a classical theory is conservative over its intuitionistic counter-part with respect to Γ if both theories prove exactly the same formulae of this class. A typical example of a conservativity result states that Peano Arithmetic (PA) is Π_2-conservative over Heyting Arithmetic (HA). It can be proven in several ways. For example, the so-called (Gödel-Gentzen) negative translation together with the Gödel functional interpretation of HA or proof theoretic analysis of HA can be used or the negative translation and the so-called Friedman translation.

The aim of the talk is to describe conservativity of classical first-order theories over their intuitionistic counterparts from a semantic perspective. In particular, we will consider properties of a class of Kripke models for a given intuitionistic theory that are sufficient to prove conservativity. We also describe semantically a class of formulae for which such results can be proven.

In order to prove classically that a theory T^{c} is Γ-conservative over its intuitionistic counterpart T^{i}, we may show that any formula from Γ which is not derivable intuitionistically in T^{i} is also not derivable classically in T^{c}. So, assume that $A \in \Gamma$ and $\mathsf{T}^{\mathsf{i}} \nvdash A$. Then, by the strong completeness theorem for Kripke semantics, we can find a Kripke model $\mathcal{M}$ of T^{i} such that $\mathcal{M}$ refutes A. Now we need to find a classical structure $\mathbf{M}$ which is a counter-model for A and a model of the theory T^{c}. The most natural idea is to look for such a counter-model among the worlds of the Kripke model $\mathcal{M}$. Thus, since $\mathcal{M} \nVdash A$, we can find a node w of $\mathcal{M}$, such that $w \nVdash A$. In general, the world $\mathbf{M}_w$ corresponding to the node w in $\mathcal{M}$ need not be a counter-model for A nor a model of T^{c}. However, it is enough to find *some* node u such that $\mathbf{M}_u \not\models A$ and $\mathbf{M}_u \models \mathsf{T}^{\mathsf{c}}$, for the world $\mathbf{M}_u$ corresponding to u in $\mathcal{M}$. We show that under suitable assumptions concerning models of the theory T^{i} (and some assumptions on T^{i} itself) this can be done.

On the Mints Hierarchy in First-Order Intuitionistic Logic[2]

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While the prenex normal form is useful for classification of formulas, which was demonstrated in full strength by Börger, Grädel, and Gurevich in their influential book [1], it applies to *classical* logic only. Things become quite different for constructive logic (aka intuitionistic logic), because the prenex fragment of intuitionistic logic is decidable [3]. This contrasts with the undecidability of the general case (see e.g., [5]) and that makes this form of stratification unsuitable in the constructive context.

We can replace the prenex classification by something adequate for intuitionistic logic. As observed by Grigori Mints [2], the principal issue is the alternation of positive and negative occurrences of quantifiers in a formula. This yields the *Mints hierarchy* of formulas:

Π_1 – All quantifiers at positive positions.

Σ_1 – All quantifiers at negative positions.

Π_2 – Up to one alternation: no positive quantifier in scope of a negative one.

Σ_2 – Up to one alternation: no negative quantifier in scope of a positive one.

And so on. In this work we present a systematic study of the decision problem in Mints hierarchy. We restrict attention to the fragment where only the implication and the universal quantifier may occur. Our main results are as follows:

A. The decision problems for classes Σ_2 and Π_2 are undecidable;

B. The decision problem for the class Σ_1 is Expspace-complete.

[2]Project supported through NCN grant DEC-2012/07/B/ST6/01532.

These results are supplemented by the 2-co-NExptime lower bound for Π_1 obtained in [4]. Observe that, because of conservativity, part A applies directly to the full intuitionistic logic, and the same holds for the lower bound in B. The upper bound in B also extends to the general case at the cost of some additional complication.

The undecidabilities in A are shown for the monadic fragment of minimal logic (i.e., the language with only unary predicate symbols). It is slightly different with B, where we conjecture that the monadic case is co-NExptime complete.

References

[1] Egon Börger, Erich Grädel, and Yuri Gurevich. *The Classical Decision Problem*. Perspectives in Mathematical Logic. Springer, 1997.

[2] G.E. Mints. Solvability of the problem of deducibility in LJ for a class of formulas not containing negative occurrences of quantifiers. *Steklov Inst.*, 98:135–145, 1968.

[3] H. Rasiowa and R. Sikorski. On existential theorems in non-classical functional calculi. *Fundamenta Mathematicae*, 41:21–28, 1954.

[4] Aleksy Schubert, Paweł Urzyczyn, and Daria Walukiewicz-Chrząszcz. How hard is positive quantification? Available at `http://www.mimuw.edu.pl/~alx/positive2conexp.pdf`, 2014.

[5] M.H. Sørensen and P. Urzyczyn. *Lectures on the Curry-Howard Isomorphism*, volume 149. Elsevier, 2006.

The Nature of Mathematics in Terms of Saunders Mac Lane

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Mathematics in Saunders Mac Lane's terms explores forms hidden under facts. Mathematics finds forms in the world and it pulls them out of it; then the forms become independent and mathematicians no longer pay attention to their origin. Mathematics is not a science, because it does not concern the world, it deals only with abstract forms of the world. Examples of these forms are: natural numbers, orders, groups, topological spaces, vector spaces, algebras, etc. These forms can be implemented in many different fields of study. For this reason, Mac Lane claimed that mathematics has a protean character. Mac Lane sees Mathematics (used here with the original upper-case letter provided by Mac Lane) as a formal network of interconnected concepts, definitions and systems. Mathematics is diverse and dynamic. This diversity, however, contains some universals; structures which appear at several points of the network. The thesis of this paper is that an adequate description of the protean character of Mathematics leads to category theory. Forms are in fact structures, structures in essence are categories, i.e. a classes of objects with suitably defined arrows on them. Just as mathematical forms are fished out of the world they can also return to the world in different ways (realizations) so the same universal structures of category theory are manifested in different places of the mathematical network. A simple example of a universal structure in category theory is the product. A product can be realized as a product of topological space, the product of a given group, as the greatest lower bound—depending on which category is given. The advantage of category theory over other representations of Mathematics, according to Mac Lane, is that it does not mean that category theory formulates further foundations of mathematics, only that it reflects both the richness, protean character and diversity of Mathematics, as well as its unity.

Extending the Blok-Esakia Theorem

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The Blok-Esakia theorem states that there is an isomorphism from the lattice $\mathsf{Ext}\,\mathbf{Int}$ of extensions of intuitionistic logic onto the lattice $\mathsf{NExt}\,\mathbf{Grz}$ of normal extensions of Grzegorczyk modal logic [1,2].

There are many extensions of the Blok-Esakia theorem. Just see a recent survey [5] and the references therein. Let us recall two of them.

Let $\mathbf{Int}_\Box$ be the intuitionistic logic augmented by the normal necessity operator $\Box$. Let $\mathbf{Grz}_\Box$ be the bimodal logic equipped with the normal necessity operators $\Box$ and $\boxtimes$, the latter working as the necessity operator in $\mathbf{Grz}$, and possessing the formula

$$\boxtimes\Box\boxtimes p \leftrightarrow p.$$

In [4] Wolter and Zakharyaschev proved that there is an isomorphism from the lattice $\mathsf{NExt}\,\mathbf{Int}_\Box$ of normal extensions of $\mathbf{Int}_\Box$ onto the lattice of $\mathsf{NExt}\,\mathbf{Grz}_\Box$ of normal extensions of $\mathbf{Grz}_\Box$.

In [3] Jeřábek extended the Blok-Esakia theorem to (multi-conclusion) deductive systems. It means that he allows not only axiomatic extensions, but also extensions obtained by adding new rules (with possibly many conclusions).

We present a generalization of these two results.

Theorem. *There is an isomorphism*

$$\sigma\colon \mathsf{DExt}\,\mathbf{Int}_\Box \to \mathsf{DExt}\,\mathbf{Grz}_\Box.$$

From the lattice of (multi-conclusion) deductive systems extending $\mathbf{Int}_\Box$ *onto the lattice of (multi-conclusion) deductive systems extending* $\mathbf{Grz}_\Box$.

We also study the preservation of various properties like admitting (parametrize, local) deduction theorem, (almost) structural completeness and finite axiomatizability.

References

[1] Willem J. Blok. *Varieties of interior algebras.* PhD thesis, University of Amsterdam, 1976.

[2] Leo L. Esakia. On modal companions of superintuitionistic logics. pages 135–136. VII Soviet Symposium on Logic, Kiev, 1976.

[3] Emil Jeřábek. Canonical rules. *J. Symbolic Logic*, 74(4):1171–1205, 2009.

[4] Frank Wolter and Michael Zakharyaschev. On the relation between intuitionistic and classical modal logics. *Algebra i Logika*, 36(2):121–155, 240, 1997.

[5] Frank Wolter and Michael Zakharyaschev. On the Blok-Esakia theorem. In Guram Bezhanishvili, editor, *Leo Esakia on Duality in Modal and Intuitionistic Logics*, volume 4 of *Outstanding Contributions to Logic*, pages 99–118. Springer Netherlands, 2014.

Formal Conditions of Understanding the Concept of Common Good

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The name "common good" is idiomatic i.e. its meaning is different from the meanings of the individual words "common" and "a good". Not every common property can be considered a common good. The author tries to give some logical and epistemic condition under which an everyman understands this concept. There are certain formal conditions which have to be fulfilled to recognize a given object as an example of the common good. According to real approach the common good is an inheritance of objective of objective and irrefutable values and its immaterial components are particularly valuable. The formal approach allows to analyse the idiomatic nature of the common good as well as the process of creation of this concept in everyman's mind.

Default Rules in Spatial Reasoning

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Default rules (*defaults*), are rules of thumb that enable to sanction reasonable but not necessarily true conclusions [1], e.g., "normally birds fly". Given an information that "Tweety is a bird" the above-mentioned *default* may be used to conclude that "Tweety flies". However, when the knowledge is extended with additional information, namely that "Tweety is a penguin" and a rule that "penguins do not fly" it is concluded that "Tweety does not fly". Since the growing set of beliefs may invalidate conclusions that were previously drawn, *defaults* make the reasoning non-monotonic. Additionally, when *defaults* occur, there may be more then one sets of consistent beliefs (*extensions*).

In order to model human-like spatial reasoning, incomplete knowledge and a possibility to jump to conclusions need to be represented. Therefore, we argue that there are strong reasons to involve *defaults* in commonsense spatial reasoning. As an example, consider a situation in which location of a bathroom is known. Although, there is no information about the shower's position, it is reasonably to conclude that it is inside the bathroom (by a spatial *default*).

A number of general formalisms for non-monotonic reasoning have been introduced and studied in depth [6,4,3] but it is not obvious how *defaults* may be obtained in spatial formalisms—notice that in case of a spatial system an additional base of spatial knowledge is needed. One of the most prominent ways to introduce a *default* is presented in [6] as a rule of a form:

$$\frac{\phi : \psi_1, \ldots, \psi_n}{\chi} \tag{1}$$

which has the following intuitive meaning: if ϕ is true and it is consistent to believe that $\psi_1 \wedge \cdots \wedge \psi_n$, then conclude χ. In case of spatial reasoning, formulae in *default* may represent spatial relations between objects, e.g., χ may mean that "the shower is inside the bathroom" but then, in order to find out if a set of beliefs is consistent, semantics of spatial relations need to be known. The reasoning in FOL with mentioned non-monotonic approaches [6,4,3] is not even semi-decidable. In case of propositional form reasoning is decidable, more precisely, brave reasoning, i.e., deciding whether a given formula belongs to some extension is Σ_2^P-complete, while cautious reasoning, i.e., deciding whether a given formula belongs to all extension is Π_2^P-complete [2]. As a result, in order to obtain a decidable spatial default reasoning method, the base of spatial knowledge has to be encoded in propositional logic.

The aim of our presentation is to introduce a concept of a *default* in spatial reasoning systems, discuss its importance and describe its formal properties.

Additionally, we show how a decidable default spatial reasoning system may be obtained for topological relations of Region Connection Calculus (RCC) [5].

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References

[1] G. Antoniou, Nonmonotonic reasoning, Mit Press, 1997.

[2] G. Gottlob, Complexity results for nonmonotonic logics, Journal of Logic and Computation 2 (3) (1992) 397–425.

[3] J. McCarthy, Circumscriptiona form of non-monotonic reasoning, Artificial intelligence 13 (1) (1980) 27–39.

[4] R. C. Moore, Possible-world semantics for autoepistemic logic, Tech. rep., DTIC Document (1984).

[5] D. A. Randell, Z. Cui, A. Cohn, A spatial logic based on regions and connection, in: Principles of Knowledge Representation and Reasoning: Proceedings of the Third International Conference, 1992, pp. 165–176.

[6] R. Reiter, A logic for default reasoning, Artificial intelligence 13 (1) (1980) 81–132.

On Δ_0-induction for the Compositional Truth Predicate

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In our talk we would like to present some results on proof-theoretic strength of the compositional theory of truth with induction for Δ_0-formulae and some of its variants. In particular we shall present a solution to problem by Albert Visser concerning conservativity of the theory over Peano Arithmetic.

By the **compositional theory of truth** CT^- we mean Peano arithmetic PA with additional predicate $T(x)$ called **the truth predicate** and the following axioms governing this new predicate:

1. $\forall \ulcorner s \urcorner, \ulcorner t \urcorner \;\; T\ulcorner s = t\urcorner \equiv (\mathrm{val}(s) = \mathrm{val}(t))$
2. $\forall \ulcorner \phi \urcorner, \ulcorner \psi \urcorner \;\; T\ulcorner \phi \odot \psi \urcorner \equiv (T\ulcorner \phi\urcorner \odot T\ulcorner \psi \urcorner)$
3. $\forall \ulcorner \phi \urcorner \;\; T\ulcorner Qx\ \phi(x) \urcorner \equiv (Qt\ T\ulcorner \phi(t)\urcorner)$
4. $\forall \ulcorner \phi \urcorner \;\; T\ulcorner \neg\phi \urcorner \equiv (\neg T\ulcorner \phi\urcorner)$,

   where $\odot \in \{\wedge, \vee\}$, $Q \in \{\forall, \exists\}$ the variables ϕ, ψ quantifies over (Gödel codes of) arithmetical formulae, s, t quantifies over (codes of) arithmetical terms and val() represents the valuation of terms.

By a classical and somewhat surprising result of Krajewski, Kotlarski and Lachlan the theory of CT^- is conservative over PA. On the other hand, the theory CT obtained by enriching CT^- with the full induction scheme for truth predicate $T(x)$ is obviously non-conservative over PA, since one may prove e.g. the consistency of PA by induction on length of proofs that from Peano's axioms only true conclusions may be obtained.

Trying to weaken the assumptions needed to obtain nonconservativeness of truth theory over arithmetics, Kotlarski produced a proof that the theory CT_0, i.e. CT^- with Δ_0-induction for truth predicate is not conservative over PA. However, as has been pointed out by Albert Visser, the proof contains an essential gap. At certain point one apparently has to assume Π_1-induction for the truth predicate. It turned out completely unclear what is the expected answer for the conservativeness problem of CT_0.

In our talk, we would like to present a solution to this question. We show that the **global reflection principle**

$$\forall\phi \;\; \mathrm{Pr}\ulcorner\phi\urcorner \rightarrow T\ulcorner\phi\urcorner,$$

where $\mathrm{Pr}(x)$ is a provability predicate for PA is arithmetically conservative over CT_0. Thus we prove that CT_0 is not conservative over PA. Additionally, we show that a slightly modified version of CT_0 actually proves this principle.

Sequential Predication

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V.I. Markin proposed a certain construction—a generalisation of syllogistic—in which he uses the constant @ with indefinite arity. The atomic formulas are of the following sort:

$$S_1S_2\ldots S_m@P_1P_2\ldots P_n, \quad \text{where} \quad m+n>0$$

The construction is accompanied by intended interpretation (*) in a monadic calculus of predicates:

$(S_1S_2\ldots S_m@P_1P_2\ldots P_n)^* = \neg\Sigma x(S_1x \wedge S_2x \wedge \cdots \wedge S_mx \wedge \neg P_1x \wedge \neg P_2x \wedge \ldots \wedge \neg P_nx)$
$(\neg A)^* = \neg A^*$
$(A\nabla B)^* = A^*\nabla B^*$ where ∇ is any logical conjunctive.

The standard syllogistic functors are here interpreted as follows:

$$SaP =: S@P \qquad SeP =: SP@ \qquad SiP =: \neg SP@ \qquad SoP =: \neg S@P$$

Markin constructs a system of *fundamental syllogistic* (**FS**) with constant @ in an axiomatic way. Based on Markin's idea, we propose two constructions, which are formulations of the system of sequential predication built upon the quantifier-less calculus of names. The first one includes the **FS** system. The second one is enriched with individual variables and, among other things, allows to include sequences of individual names in which one has to do with enumerative functors. The proposed instruments can be helpful in the analysis of natural language.

References

[1] Borkowski L.: Bezkwantyfikatorowy założeniowy system rachunku nazw (The quantifier-less presuppositional system of the calculus of names), *Roczniki Filozoficzne* 28 (1980), vol. 1, pp. 133–148 (Part 1) and 41 (1993), vol. 1, pp. 11–21 (Part 2).

[2] Markin V.I.: Fundamentalnaja silogistika s nieopriedielennomiestnoj konstantoj, [in:] *Logicieskije isliedovanija*, 11(2004), pp. 189–201.

[3] Wojciechowski E.: Pewien bezkwantyfikatorowy rachunek nazw (A certain quantifier-less calculus of names) [in:] *Logika & Filozofia Logiczna.* FLFL 1996–1998 (J. Perzanowski and A. Pietruszczak, ed.), Toruń: Wydawnictwo UMK 2000, pp. 109–126.

[4] Wojciechowski E.: Rachunek nazw z listami (Calculus of names with lists), *Roczniki Filozoficzne*, 59 (2011), No. 1, pp. 35–50.

Projective Unification in Modal Predicate Logics

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Let be given a first-order modal language without function symbols. If ε is a substitution for predicate variables, it is usual (see [3]) to assume $vf(\varepsilon(A)) \subseteq vf(A)$ for each formula A, where $vf(\varepsilon(B))$ denotes the set of free variables occurring in B. For our approach, however, such approach would be too restrictive. We need a more general concept where $\varepsilon(P(a_1, \ldots, a_n))$ may contain—in addition to $a_1, \cdots, a_n$—other free variables. These additional variables are regarded as parameters of the substitution.

If L is a propositional logic, then Q-L denotes the corresponding predicate logic. Any predicate logic, in addition to many specific conditions (see [1]), must be also closed under the (above mentioned extended concept of) substitution for predicate variables.

Similarly as in propositional logic, a *unifier* for a formula A in a predicate logic L is a substitution ε (for predicate variables) such that $\varepsilon(A)$ is derivable in L, i.e. $\vdash_L \varepsilon(A)$. A formula A is said to be unifiable in L if it has a unifier. A unifier ε for A in L is *projective* if $A \vdash_L \varepsilon(P_i(a_1,\dots,a_n)) \leftrightarrow P_i(a_1,\dots,a_n)$ for each predicate variable P_i. Clearly, if ε is projective for A in L, then $A \vdash_L \varepsilon(B) \leftrightarrow B$ for each formula B. We say that a logic L enjoys *projective unification* if each unifiable formula has a projective unifier in L.

As it is known, see [2], a propositional modal logic enjoys projective unification iff it extends $S4.3$. However, even at the propositional level, projective unifiers cannot be received in a uniform way (using any form of ground unifier method). Nor one should expect that, in predicate logic, unifiers would satisfy the condition $vf(\varepsilon(B)) \subseteq vf(B)$. Moreover, it is required

$$(\Box IP) \qquad \Box(A \to \exists_x \Box B(x)) \to \exists_x \Box(A \to B(x))$$

and we were only able to prove

Theorem 1. *Any modal predicate logic $L_=$ with equality enjoys projective unification iff $L_=$ extends $IP.Q\text{-}S4.3_=$.*

References

[1] Braüner, T., Ghilardi, S. *First-order Modal Logic*, Chapter 9 in Blackburn, P., **Handbook of Modal Logic** 2007 Elsevier, 549–620.

[2] Dzik, W., Wojtylak, P., *Projective Unification in Modal Logic*, **Logic Journal of the IGPL** 20(2012) No.1, 121–153.

[3] Pogorzelski, W.A., Prucnal, T. *Structural Completeness of the First-order Predicate Calculus*, **Zeitschrift für Mathematische Logik und Grundlagen der Mathematik**, 21 (1975), 315–320.

Gensler's Star Test and its Application in Islamic Syllogistic Logic

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"Star test" is a method of checking the validity of syllogistic arguments in logic devised and first introduced by Gensler in 1973. In his paper called "A simplified decision procedure for categorical syllogism", Gensler contrasts "star test" with the more traditional set of rules traditionally used in checking the validity of syllogistic arguments. Gensler attempts to show that his method is more advantageous and functional insofar as syllogistic and deductive arguments are concerned. The aim of this paper is twofold: to evaluate philosophically Gensler's "star test" in deductive conclusions in general and to check its eventual significance and functionality in the logical enterprises carried out in the tradition of Islamic philosophy. While syllogisms were frequently used in the past by the logicians brought up in Islamic tradition, and while the traditional procedures of checking the validity of syllogistic arguments are painstaking, "star test" might shed light on assessing the validity of syllogistic conclusions extensively put forth in Islamic philosophy.

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