XIX Conference

Applications of Logic in Philosohpy and the Foundations of Mathematics

Szklarska Poręba

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XIX Conference

Applications of Logic in Philosophy and the Foundations of Mathematics

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Abstracts

Editorial note

(EN) means that the talk is presented in English, (PL) — in Polish.

Mantiq and Halakha — the "Roads" of Reasoning in Arabic Logic and Rabbinic Logic

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The concepts of Arabic logic and rabbinic logic still remain vague and controversial — there are many problems with determining their essence, content, time-frames, sources, or the criteria of differentiation with respect to the so-called classical logic. Even the selection of a proper name for disputed disciplines is essentially non-transparent in content: instead of Arabic logic is better to talk (sometimes) about the logic of Muslim, Islamic, or Koranic logic; and in the case of rabbinic logic other names are: Judaic logic, Talmudic logic. On the other hand, it is possible to identify the specific nature of the discussed disciplines, for example, by specifying the methods and tools used. In my presentation accent will be put on the "roads" of reasoning (mantiq in Arabic logic and halakha in rabbinic logic) — which will be identified as sets of rules and methods for generating inferences used in the process of interpretation.

The starting point of discussion is to draw attention to the common features of Arabic and rabbinic logic — for example, in both cases, a main purpose is to formulate a set of rules of correct reasoning (inference) as a tool for the interpretation of principles of law and religion. Similarly, characteristic of both logics is to emphasize the role of analogy. Reconstructions using the formal apparatus indicate from the one hand on the specificity and from the other hand on the similarity of the two discussed logics. An important background of considerations will be the problem of the relation of Arabic and rabbinical logic to classical logic — especially Aristotle's syllogistic.

Value Priorities in Logical Reasoning

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The visible structure of an argument, i.e. the reasons and the conclusion, is sometimes rendered ambiguous due to an unstated belief that supports the explicit reasoning. An unstated belief in a logical reasoning is called an assumption. Assumptions are either necessary for the reason(s) to support the conclusions or for a reason to be true. Value assumptions are especially functional in the movement from reason to conclusion in the sense that they direct the reasoning behind a screen. This is why some reasonable people declare that war is evil, while other equally reasonable people see it as patriotism. The different conclusions arise out of the existence of different ethical values. The talk is about the legitimacy of incorporating ethical values into logic. On a commonsensical level, we feel that a logician should not legalize illegal drugs, violence, crimes and euthanasia, etc. But why?! Is this logical or the consequence of our incorporation of our value priorities into our logical thinking? This is what the talk aims to bring an answer to.

Cut Elimination in Hypersequent Calculi for $S5$

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Cut rule is one of the most important rules in sequent calculus but it makes a system nonanalytic which has negative impact on decidability and proof search. Solution for the troubles is usually provided by cut elimination theorem. However, for many important logics their standard sequent formalizations do not admit cut elimination. One of the well known example is modal logic $S5$.

Such problems can be interpreted as a reason to create hypersequent calculi. The differences between this two type of calculi will be discussed, as well as different motivations underlying hypersequents calculi proposed by A. Avron, F. Poggiolesi, G. Pottinger.

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On the Determinant of Hexagonal Grids $H_{n,k}$

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The problem of singularity of hexagonal grids is analysed. We introduce certain methods of reduction of weighted graphs, based on operations which do not change the determinant of the adjacency matrix. The methods are used to calculate the determinant of all graphs which are hexagonal grids $H_{n,k}$. The hexagonal grid $H_{n,k}$ is a graph described in chemical literature as a hexagon-shaped benzenoid system $O(k,1,n)$. As the final result, the explicit formula for the determinant of the adjacency matrix of a hexagonal grid $H_{n,k}$ is presented. The formula also proves that there are no singular $H_{n,k}$ graphs.

The Logic of Conceiving of the Sentential Schemes

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In memory of Professor Andrzej Grzegorczyk

Formalization can be conceived as a representation of logical knowledge [1]. In the pragmatic sense, texts are conceived, i.e. there are procedures of unique usage of data for agents communicating with each other. Identical usage of data types determines the unit of knowledge, and the set of unit of knowledge is (in this sense) knowledge [2]. Conceiving of texts is establishing of knowledge represented by these texts. Texts representing logical knowledge include the following expressions: "*a scheme of a sentence*", "*a scheme of a true sentence*", "*a scheme a sentence that is not true*" and the phrase "*is conceived as ...*". This phrase is denoted as $\equiv$. There are considered schemes of complex sentences: the negation ($\neg$), the conjunction ($\wedge$), the alternative ($\vee$), the implication ($\Rightarrow$) and the equivalence ($\Leftrightarrow$) of sentences. These schemes are not only for the two sentences but also for three, four or more sentences. These sentences represent knowledge, which may consist of several fields of knowledge. For example:

Consider the equivalence of sentences "Kowalski is a minister in the Polish government", "Kowalski works in Warsaw", "Kowalski works in the Ministry". If the last sentence "Kowalski works in the Ministry" is true, then first two sentences are conceived as equivalence. Furthermore, if the last sentence is not true then first and second sentences are not conceived as equivalence.

The **logic of conceiving** is a formal system of conceiving texts representing logical knowledge. The system of conceiving of logical knowledge represented by logical expressions of the propositional calculus is the **logic of conceiving of the sentential schemes**. Moreover, the relation of conceiving of formulas satisfies the following conditions:
<formula> $\equiv$ a scheme of a sentence; 1 $\equiv$ a scheme of a true sentence; 0 $\equiv$ a scheme a sentence that is not true; $\neg$<formula> $\equiv$ a scheme of a sentence negation; < connective > (< finite sequence of formulas >) $\equiv$ schema binding connective finite sequence of sentences.

The conceiving of schemes of sentential conjunctions can be determined recursively:

$$\Delta(F_1, F_2) \equiv (F_1 \Delta F_2)$$

$$\Delta(F_1, \ldots, F_{n-1}, F_n) \equiv \begin{cases} \Delta(F_1, \ldots, F_{n-1}), \text{ if } F_n \equiv 1; \\ \neg(\Delta(F_1, \ldots, F_{n-1})), \text{ if } F_n \equiv 0; \end{cases}$$

where Δ is one of the symbols of the sentential conjunctions, and F_i, for $i = 1, 2, ..., n$, are arbitrary formulas. The **tautologies** are the formulas conceived as schemas of true sentences, regardless of conceiving propositional variables (sentential variable) as 1 or 0. For example:

1. $(\Leftrightarrow (p,q,r)) \Leftrightarrow ((p \Leftrightarrow q) \Leftrightarrow r)$; $(\Leftrightarrow (p,q,r)) \Leftrightarrow ((r \Leftrightarrow p) \Leftrightarrow q)$; $(\Leftrightarrow (p,q,r)) \Leftrightarrow ((q \Leftrightarrow r) \Leftrightarrow p)$,
2. $\neg(\wedge(p,q,r)) \Leftrightarrow (\wedge(p,q,\neg r))$; $\neg(\vee(p,q,r)) \Leftrightarrow (\vee(p,q,\neg r))$
3. $\neg(\Rightarrow (p,q,r)) \Leftrightarrow (\Rightarrow (p,q,\neg r))$; $\neg(\Leftrightarrow (p,q,r)) \Leftrightarrow (\Leftrightarrow (p,q,\neg r))$

De Morgan laws

4. $\wedge(p, q, r) \Leftrightarrow \neg(\vee(\neg p, \neg q, r))$; $\quad \vee(p, q, r) \Leftrightarrow \neg(\wedge(\neg p, \neg q, r))$;

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Inference Anchoring Theory

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A series of talks:

- Inference Anchoring Theory: Philosophical Foundations
- Inference Anchoring Theory: Linguistic Applications
- Inference Anchoring Theory: Computational Applications

This series of three talks introduces the links between different communication structures including: inferential structures typical for human and agent communication; dialogue structures determining linguistic behaviour amongst agents; and ethotic structures related to the credibility of speakers and trust management in a multi-agent system and artificial intelligence.

The connection between formal theories of inference on the one hand, and dialogical processes of disagreement and persuasion on the other is surprisingly understudied. Inference Anchoring Theory (IAT) provides, for the first time, a well-grounded account of this connection exploiting the well-known theory of speech acts. IAT allows us to understand, for example, how it is that when A asks why it is that p, and B replies that q, an inference is established from q to p.

IAT tackles a number of challenging theoretical issues, including using the statements of others (such as appeals to expert opinion and authority), locutions that attacks ethos (that is, a speaker's character and credibility), and inference that is established by virtue of its dialogical context. We also study different areas of applications of IAT from linguistic analysis to protocol design. Finally, IAT is the lynchpin in extensions to the argument interchange format, a computational standard for the representation of argument by machines, and those extensions are now supporting a raft of innovative, exciting software applications.

The series will be run in three parts:

* Inference Anchoring Theory, IAT
  - Connecting Inference with Dialogue (Budzynska and Reed 2011)
  - Authorization for Performing a Speech Act (Budzynska 2010)
  - Dialogue Templates (Bex and Reed 2012)
* Application to human communication
  - Non-inferential structures with trust (Budzynska 2012)
  - Using authority (Budzynska and Reed 2011)
  - Analysis of dialogues from the BBC Radio 4 programme The Moral Maze (Budzynska et al. 2014)
* Application to agent communication
  - Attacking the opponent's credibility in dialogue games (Budzynska and Reed 2012)
  - Software tools supporting argument analysis (www.arg.dundee.ac.uk)

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Erotetic Calculi, Cut Rule and the Minimal LFI

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A proof of a formula of a certain propositional language can be considered as a sequence of yes/no questions such that each element of that sequence is obtained from the preceding element by means of an erotetic rule of inference. Both the premise and conclusion of this rule are questions. It can be shown that the affirmative answer to the question which play the role of a premiss entails the affirmative answer to the question, which is the conclusion and *vice versa*. This fact can be described in the framework of Wiśniewski's Inferential Erotetic Logic (**IEL**) as a special case of *erotetic implication* [Wisniewski (2013)].

Loosely speaking a proof of a formula A is a sequence of questions (called *Socratic transformation*) which starts with a question $?(A)$ ('is A valid?') and each question from that sequence is implied by the previous question by means of an erotetic rule of inference (which 'decompose' or 'simplify' a question on which it acts) such that the affirmative answer to the last question is in a sense evident. From the fact that the answer to the last question of a Socratic transformation is affirmative follows that the answer to the initial question $?(A)$ is affirmative [Wisniewski (2004)].

One of the key aspect of Wiśniewski's method is that deductive problems worded in a language of a given logic can be resolved by pure questioning i.e. without help of an external source of information, be it the Nature, fellow inquirers or a database. Such a process of problem solving may be generally called *internal question processing* or *ultimate question processing*. An interesting philosophical consequence of solving problems by pure questioning is the following: if we are in a situation of solving problem D in a logic L using only simple erotetic rules and we do not have to ask queries to some external source of information, then the deductive problem D is in some sense *analytical*.

I give a sound and complete proof method for classical propositional logic (CPL), the propositional part of the logic CLuN and CLuNs [Batens (2005)] and its extension: the mbC system [Carnielli (2003)]. The method has two basic roots. The first one is Wiśniewski's Inferential Erotetic Logic, the second is the proof procedure which may be called 'backward dual resolution' (BD-resolution) [Ligeza (2006)] or 'direct resolution'. I present four structural erotetic calculus $\mathbb{E}^{\mathsf{CPL}}_{cut}$, $\mathbb{E}^{\mathsf{CLuN}}_{cut}$, $\mathbb{E}^{\mathsf{CLuNs}}_{cut}$ and $\mathbb{E}^{\mathsf{mbC}}_{cut}$.

Generally speaking, the proposed calculi differs from the existing ones in at least two facets:

- in structural erotetic calculi we decompose a formula to its disjunctive normal form

- in structural erotetic calculi we use both logical and structural erotetic rules of inference and the structural rules can not be eliminated

A consequence of the work on structural erotetic calculi which is of some importance for logic lays in showing the existence of an intuitive normal forms of formulas in non classical logics CLuN, CLuNs and mbC.

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Abductive Questions and Their Complexity

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It seems natural to consider abduction as an art of solving a special class of problems. One may call these class of problems *abductive problems*, a reasoning which leads to a solution of abductive problem is a *abductive reasoning* and the solution is the set of *abductive hypotheses* [Urbański, 2009]. A lot of work was done for understanding abductive reasoning and an evaluation of abductive hypotheses. Though, we are interested in formal representation of the concept of abductive problem.

However one may ask: what is an abductive problem? The common answer would be that it is a situation in which a subject lacks information which is needed for an explanation of a certain data D. Then he/she raises an *abductive question* i.e. a question of the form: *what information should be added to my information set to explain data D?*

The concept of the abductive question can be precisely described on the ground of Wiśniewski's Inferential Erotetic Logic [Wiśniewski, 2004a,b]. We give a formalization of this concept and we propose a certain tools for measuring the degree of complexity of abductive questions. Finally we prove that using rules of erotetic calculi one is able to reduce the complexity of abductive problems in such a way that the initial problem becomes easier to resolve.

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Nonstandard Hierarchy of Infinite Sets

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1) Terminology:
"hyper-real number", "hyper-limes" (conditions of existence), "hyper-measure of the set";
2) The number of infinitely small — (positive) — less than each one of real number;
3) Hyper-number h — the hyper-limes of the sequence $\{1/n\}$, the family of the hyper-small of the h row;
4) The hyper-low of the h_1 row ($h_1 = 2^h$) is less than each one of hyper-low of the h row, hyper-low of the h_{-1} row ($h_{-1} = \log_2 h$) is greater than each one of the hyper-low of the h row.
5) The hyper-low of the h_k row ($h_k = 2^{h_{k-1}}$) is less than each one of the hyper-low of the h_{k-1} row; hyper-low of the h_k row ($h_k = \log_2 h_{-(k-1)}$) is greater than each one of the hyper-low of the $H_{-(k-1)}$ row.
6) The hyper-natural number H ($H = 1/h$) — the hyper-limes of a sequence $\{n\}$, it is the unit of hyper-measure of the numerical set.
7) For the sequence $\{a_n\}$ we define the sequence $\{^*a_n\}$, where $^*a_n$ is the number of elements of the sequence $\{a_n\}$ which are less than or equal to n.
8) Hyper-limes of the sequence $\{^*a_n\}$ is the hyper-measure of the set $\{a_n\}$.
9) The hyper-natural number $f(H)$ — the hyper-limes of a sequence $\{f(n)\}$;
10) $f^{-1}(H)$ is a hyper-measure of the set $\{a_n\}$, when $\bigvee_m$, that $\bigwedge_{n>m} : a_n = f(n)$.
11) The hyper-great of the H_1 ($H_1 = 2^H$) row is greater than each one of the H row, the hyper-great of the H_{-1} ($H_{-1} = \log_2 H$) row is smaller than each one of the H row.
12) The hyper-great of the H_k ($H_k = 2^{H_{k-1}}$) row is greater than each one of the hyper H_{k-1} row, and the hyper-great of the H_{-k} ($H_{-k} = \log_2 H_{-(k-1)}$) row is smaller than each one of the $H_{-(k-1)}$ row.
13) The non-standard definition of zero and ∞:
14) The generalization on any family of homogeneous sets.

Consequence Operations Extending Modal Logic S4.3

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Part 1.
Wojciech Dzik
Finitary Consequence Operations

Generalizing well-known results by R.Bull [1] and K.Fine [5] we proved in [3]

Theorem 1. *Each finitary consequence operation Cn extending* **S4.3** *has a finite basis, over some $L \in \mathrm{NExt}(\mathbf{S4.3})$, consisting of finitary passive rules.*

In the proof we use our characterization of projective unification in modal logic, see [2]. The rule α/β is called *passive* in L, if α is not unifiable in L.

Theorem 2. *Each modal formula unifiable in* **S4** *has a projective unifier. Consequently, each modal consequence operation extending* **S4.3** *is almost structurally complete (for finitary inferential rules) and can be obtained by extending a normal modal logic with a collection of passive rules of the form: $\frac{\Diamond\theta_1 \wedge \cdots \wedge \Diamond\theta_s}{\delta}$, $2 \leq s \leq 2^n$ and $\{p_1, \ldots, p_n\} \cap$ Var $(\delta) = \emptyset$, $\theta_k : p_1^{\sigma(1)} \wedge \cdots \wedge p_n^{\sigma(n)}$*

We also consider some properties of the lattice $\mathrm{EXT}(\mathbf{S4.3})$ of all finitary consequence relations extending **S4.3** and an example of $\mathrm{EXT}(\mathbf{S5})$.

Part 2.
Piotr Wojtylak
Infinitary Consequence Operations

Let us recall that a consequence operation Cn is finitely approximable if $Cn = \overrightarrow{\mathbb{K}}$ for some class $\mathbb{K}$ of finite matrices.

Theorem 3. *Each finitary consequence operation Cn extending* **S4.3** *coincide on finite sets with a finitely approximable modal consequence operation.*

In case of infinitary rules, we have neither projective unification nor (any variant of) structural completeness, for **S4.3**. We prove in [4]

Theorem 4. *Let Cn be a modal consequence operation extending* **S4.3**. *Then Cn is almost structurally complete (with respect to infinitary rules) iff Cn is finitely approximable.*

We also provide an uniform basis, consisting of infinitary rules, for all admissible rules of any $L \in \mathrm{NExt}(\mathbf{S4.3})$. This rule basis is uncountable. It contains, as a sample, the rule of the form:

$$\frac{\{\Box(\alpha_i \leftrightarrow \alpha_j) \to \alpha_0 : 0 < i < j\}}{\alpha_0}$$

It also follows that

Theorem 5. *The lattice of all almost structurally complete extensions of* **S4.3** *is a complete sublattice of the lattice of all consequence operations over* **S4.3**, *which is isomorphic with the lattice of all finitary extensions of* **S4.3**.

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Contraction Contracted

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In sequent calculi the rule of contraction plays an important role since it is often necessary to decrease the number of occurrences of a formula in a sequent. On the other hand, contraction is technically embarassing in Cut elimination proofs. Hence it is desirable to eliminate it, at least from the set of primitive rules.

Several ways of dealing with that problem were proposed: Gentzen [1934] to avoid the complications connected with contraction introduced a special rule Mix (or Multicut) instead of Cut. Curry [1963] provided a proof of Cut elimination where global transformations of proofs are defined. Dragalin [1988] depending on Ketonen's invertible rules provided a system with no structural rules at all and allowing a proof of elimination directly for Cut, but in order to obtain the result he had to show first the admissibility of contraction. Recently Negri and von Plato [2013] provided a system where deletion of new introduced copies of formulas is implicitly introduced into rules formation.

The solution proposed in this talk allows for eliminating contraction in some types of sequent calculi. It is based on the simple change in the way of reading composition of contexts in sequents. Instead of additive sum, which is common solution, we apply multiset union of contexts which greatly simplifies matters. For simplicity sake we consider only the case of propositional classical logic but the proposed solution may be applied also to sequent formalizations of extensions of classical logic. Resulting system:

1. does not need contraction as primitive rule and does not require proving its admissibility by complicated induction;
2. does not require also other preliminary results like height-preserving admissibility of weakening and invertibility of rules;
3. allows for simple cut elimination proof;
4. may be applied also to stronger logics where invertible sequent rules for constants are not known (e.g. modal logics).

Software Tool for Lorenzen Natural Dialogue

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This talk is a continuation of the work [4] where the description of formal dialogues in terms of speech act theory was discussed. In particular, the dialogical logic DL introduced by Lorenzen [2] was mapped into a general language for natural dialogue systems. The result is LND system. In this talk we show a software tool which implements the protocol for LND given in [5] and present the rules for embedding LND into PND system [1].

Participants in dialogue games perform a variety of actions, some of which can be recognized as justifications of a player's standpoint. Some of these justifications may use deductive arguments based on propositional tautologies. The LND (*Lorenzen Natural Dialogue*) game tests propositional formulas and decides whether the corresponding inference is correct. We extended this system to include a new protocol enabling the reconstruction of natural dialogues in which parties can commit formal fallacies. In [1] we introduced PND (*Prakken Natural Dialogue*) system in which players are allowed to commit formal fallacies, i.e. fallacies that use schemes which are not equivalent to valid formulas of the underlying logic. PND allows for modelling of dialogues in which inference rules used by players are publicly declared and can be challenged. In this approach we limit ourselves to propositional calculus and use as a departing point the general framework for dialogues for argumentation proposed by Prakken [3]. Prakken's system was extended to include specific locutions allowing players to use incorrect arguments, to directly show the inferences on which these arguments are based, and to challenge them. We also defined rules determining how LND can be nested in PND. The main advantage of the unification of these two systems is that during the course of a dialogue the participants can verify their sets of rules and create new arguments. This idea makes it possible to study argumentation systems in which participants have the ability to learn. The dynamic nature of dialogues can be reflected not only in players' revision of their beliefs and commitments but also in changes in the way they argue and reason. The dialogue systems LND and PND can be used both as a simulation of natural dialogues conducted in artificial intelligence systems and as a tool for argumentation and persuasion communication in multi-agent systems.

The implementation of LND was written in Java language, which will facilitate further development of the application, as well as software portability. This choice helps us to avoid the limitations of other protocols than the one proposed by us. The participants in the dialogue and the game manager are implemented as separate classes distributed over the network. The aim of this implementation is a dialogue game simulation and decision-making support during such a game. It also allows dialogue games to be recorded for later analysis. Subsequent versions of the application will reflect our progress in combining numerous formal systems for modelling natural dialogues as games and analysing the properties of such dialogue games.

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Building Countable Frames for some Bimodal Logics

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There are many results on completeness in monomodal logic (see [1]). In some cases of polimodal systems, to solve completeness problem, it suffices to consider independently axiomatizable bimodal logics.

As we already known, completeness is preserved under the formation of fusions (see [2] and [3]). Usually every monomodal logic is characterised by a class of frames

$\mathcal{C}$. Moreover, in some cases it is possible to replace the class $\mathcal{C}$ with only one countable frame.

In the talk we focus on fusions of two monomodal logics. The additional modality makes the problem of finding one countable frame more complex.

Given a monomodal system, let $\mathcal{C} = \{\mathfrak{F}_i; i \in I\}$ be the family of connected frames and $\mathfrak{F}$ be a connected frame. A point x_0 from $\mathfrak{F}$ is a $\mathcal{C}$-starting point if every mapping $f : \{x_0\} \to \mathfrak{F}_i$ can be extend to a p-morphism $f : \mathfrak{F} \to \mathfrak{F}_i$, for each $i \in I$.

We present a method of constructing a countable connected frame for fusion of two monomodal systems, which are characterised by the frames with $\mathcal{C}$-starting points. We are able to construct countable frames for fusions of monomodal logics L_1 and L_2, where $L_1, L_2 \in \{S5, Grz.3, GrzB_2, S4.3B_2M, \ldots\}$.

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On Modal Logics Determined by Homogenous Kripke Frames†

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In this talk we consider modal logics determined by classes of homogenous Kripke frames. We study nontransitive frames; we allow they are symmetric or reflexive but actually, they do not have to be. First, we pay attention to the Halldén completeness of the determined logics. Then we look for logics with the Craig interpolation property (CIP). Let us remind two definitions.

Definition 1. *A logic L has the Craig interpolation property (CIP) if for every implication $\alpha \to \beta$ in L, there exists a formula γ (interpolant for $\alpha \to \beta$ in L) such that $\alpha \to \gamma \in L$ and $\gamma \to \beta \in L$ and $Var(\gamma) \subseteq Var(\alpha) \cap Var(\beta)$.*

Definition 2. *A logic L is Halldén complete if*

$$\varphi \vee \psi \in L \ \text{ implies } \ \varphi \in L \ \text{ or } \ \psi \in L$$

for all φ and ψ containing no common variables.

There is an important connection between the Craig interpolation property and Halldén completeness of modal logics. It is presented in the following lemma due to G. F. Schumm [7]:

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Lemma 1. *If L has only one Post-complete extension and is Halldén-incomplete, then interpolation fails in L.*

It is known that many modal logics **T**, **S4**, **KTB** and **S5** are Halldén complete. It is also known that **K**, **T**, **K4** and **S4** have (CIP), see Gabbay [3]. Also the logics from $NEXT(\mathbf{S4})$ are well characterized as regards interpolation (see [6]). For example, it is known that **S5** has (CIP). The last fact can be proven by applying a very general method of construction of inseparable tableaux (see i.e. [2], p. 446). The same method can be applied in the case of **KTB** and $\mathbf{KTB} \oplus \Box^n p \to \Box^{n+1} p$, $n \geq 2$. Therefore, without getting into details, we get:

Lemma 2. *The logics* **KTB** *and* $\mathbf{KTB} \oplus \Box^n p \to \Box^{n+1} p$*,* $n \geq 2$ *have (CIP).*

It is also proven that there are infinitely many Halldén incomplete logics in $NEXT(\mathbf{S4})$ (see [7]) as well as in $NEXT(\mathbf{KTB} \oplus \Box^2 p \to \Box^3 p)$ (see [4]).

It will be shown how to construct Halldén complete normal extensions for many modal logics. Our approach to this problem is purely semantic. The main key-tool will be a lemma due to van Benthem and Humberstone [1]. It is a conclusion of more general theorem (Theorem 1 from [1]).

Lemma 3. *If a modal logic logic L is determined by one Kripke frame, which is homogeneous, then L is Halldén complete.*

In the construction of Halldén complete logics, we are however bounded by theorem due to Lemmon [5]. We say that two logics L_1, $L_2 \in NEXT(L)$ are incomparable, there exist two formulas φ and ψ such that $\varphi \in L_1$ but $\varphi \notin L_2$ and $\psi \in L_2$ but $\psi \notin L_1$.

Theorem 1. *Let L_1, $L_2 \in NEXT(L)$ be two incomparable logics. Then the logic $L_0 = L_1 \cap L_2$ is Halldén incomplete.*

In our talk we take advantage of the above lemma and theorem, and define countable many normal extensions of the given logic, which are Halldén complete, as well as uncountably many normal extensions, which are not.

Then we take a closer look to the Halldén complete logics and find among them the logics with (CIP).

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Games for Kripke Models for Intuitionistic First-Order Logic with Strong Negation

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Kripke models are a powerful and comprehensive tool in the semantical investigations of constructive first-order theories. Using an attractive possible-world interpretation, many important problems can be solved. Nevertheless, in contrast to classical model theory, the general theory of Kripke models still remains not well developed.

In the talk we consider Kripke models for intuitionistic logic $\mathcal{S}$ with strong negation. Logic $\mathcal{S}$, first introduced by Nelson in [1], is an extention of intuitionistic logic with a constructive negation operator. In this system, not only are we able to verify statements, but also falsify them. The negative information is as primitive as the positive one.

We recall the notion of bounded bisimulation as a structural description of logical equivalence between two Kripke models. Subsequently, we tackle a more model-theoretical approach and introduce the concept of a game for Kripke models (in [2]). Given two Kripke models $\mathcal{K}$ and $\mathcal{M}$, the game is played between two players, $\forall$ and $\exists$, who compare the models in question. In terms of games we give a condition for two Kripke models to be bisimilar. Namely, we show that there exists a bisimulation between two Kripke models if and only if the $\exists$ player has a winning strategy in the game on those models. Moreover, we also establish links between the notion of a game for Kripke models and logical equivalence.

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A Characterization of Distributive Lattices of Finite Length

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Let L be a lattice of finite length. It is well known fact that L is modular if it satisfies the following two conditions:

$(SM1)$ $(\forall x, y \in L)(x \wedge y \prec x, y \Rightarrow x, y \preceq x \vee y)$,

$(SM2)$ $(\forall x, y \in L)(x, y \prec x \vee y \Rightarrow x \wedge y \preceq x, y)$.

Our aim is to give an analogous characterization of distributivity. Using certain known facts, we prove that the distributivity of L can be characterized by the conjunction of the following conditions:

$(B1)$ $(\forall x, y \in L)(x \wedge y \prec x, y \Rightarrow [x \wedge y, x \vee y] \cong \mathbf{4})$,

$(B2)$ $(\forall x, y \in L)(x, y \prec x \vee y \Rightarrow [x \wedge y, x \vee y] \cong \mathbf{4})$,

where $\mathbf{4}$ denotes a four-element Boolean lattice.

An advantage of this characterization is a simplicity—it is expressed in a language of two- and four-element Boolean intervals. Moreover, it provides an effective and economical algorithm establishing that property.

A Logical Theory of Actions

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To act is intentionally to bring about or to prevent a change in the world. So the two kinds of actions could be distinguished, the productive one and the preventive one. A change in the world is a transformation of states of affairs. It takes places when a state of affairs ceases or comes to be. The lecture provides a tentative formal logical study of concepts of action and the interconnection between the notions of action, state of affairs and change. It provides an axiomatic characterization of these concepts within the framework of a multi-modal propositional logic and then, presents a semantic analysis of these concepts. The semantics is a slight modification to the standard relational semantics for normal modal propositional logic.

A Few Notes on Internal and External Alternative-Indicating Connectives

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The relationships between sentences containing an external connective and sentences with an internal connective are the subject of a linguistic investigation. The aim of my paper is to show how the results obtained by linguists correspond to the logical approach to structures which contain an external alternative-indicating connective and those containing an internal connective.

A New Proof of Knaster-Tarski's Fixed Point Theorem

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Two examples of Galois connections and their dual forms are presented. First one, (f, g), is responsible for the dual isomorphism between a complete lattice $(A, \leq)$ and a closure system of subsets of a meet-generating subset of the lattice. Given any subset B of A, the mappings $f : A \longrightarrow \wp(B),\ g : \wp(B) \longrightarrow A$ are defined for any $a \in A$ and $X \subseteq B$ by $f(a) = \{x \in B : a \leq x\},\ g(X) = \inf_A X$. The dual residuated function with respect to f is then of the form: $f_d(a) = \{x \in B : x \leq a\}$ and its residual is defined by $g_d(X) = \sup_A X$. Due to the induced closure and interior operations on the lattice $(A, \leq) :\ C_B(a) = \inf_A\{x \in B : a \leq x\},\ I_B(a) = \sup_A\{x \in B : x \leq a\}$, the following criterion of being a complete lattice can be formulated.

Lemma. *Let $D, O \subseteq A$ be closure and interior systems of a complete lattice $(A, \leq)$, respectively. Then the following conditions are equivalent:*
- *(i) for each $a \in O,\ C_D(a) \in O$,*
- *(ii) for each $a \in A,\ C_D(I_O(a)) \in O$,*
- *(iii) for each $a \in A,\ I_O(C_D(a)) \in D$,*
- *(iv) for each $a \in D,\ I_O(a) \in D$,*

Moreover, any of these conditions implies that the poset $(D \cap O, \leq)$ is a complete lattice in which for any $X \subseteq D \cap O,\ \sup X = C_D(\sup_A X)$ and $\inf X = I_O(\inf_A X)$. The inverse implication in general does not hold.

Next the criterion is applied to prove in a simple short way the Knaster-Tarski's fixed point theorem (A. Tarski, *A lattice-theoretical fixpoint theorem and its applications*, Pacific Journal of Mathematics 5(1955), pp. 285–309).

THEOREM. *Given a complete lattice* $(A, \leq)$ *and a monotone function* $\alpha : A \longrightarrow A$, *the poset* $(\{x \in A : x = \alpha(x)\}, \leq)$ *is a complete lattice in which for any its subset* X, $\sup X = C_{f(\alpha)}(\sup_A X)$ *and* $\inf X = I_{f_d(\alpha)}(\inf_A X)$.

Here $f : (Mon, \leq) \longrightarrow (\wp(A), \subseteq)$ is a Galois function defined by $f(\alpha) = \{x \in A : \alpha(x) \leq x\}$, where $(Mon, \leq)$ is the complete lattice of all monotone mappings from A into A. The second component of the Galois connection just now considered, is the map $g : (\wp(A), \subseteq) \longrightarrow (Mon, \leq)$, defined for any $B \subseteq A$ as $g(B) = C_B$. In turn, $f_d : (Mon, \leq) \longrightarrow (\wp(A), \subseteq)$, is the residuated function dual to f and it is defined by $f_d(\alpha) = \{x \in A : x \leq \alpha(x)\}$. The residual $g_d : (\wp(A), \subseteq) \longrightarrow (Mon, \leq)$ is defined for any $B \subseteq A$ as $g_d(B) = I_B$. The induced closure Cl and interior Int operations on the lattice $(Mon, \leq)$ are such that for any monotone α : $Cl(\alpha) = C_{f(\alpha)}$ and $Int(\alpha) = I_{f_d(\alpha)}$. $C_{f(\alpha)}$ is the least closure operation c defined on $(A, \leq)$ such that $\alpha \leq c$, and $I_{f_d(\alpha)}$ is the greatest interior operation I defined on $(A, \leq)$ such that $I \leq \alpha$.

Some Philosophical Remarks on Completeness in the Sense of Halldén

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A logic L is complete in Halldén's sense (in short: H-complete) if the following condition is satisfied: if L contains a disjunction of formulas α, β with no variable in common, then L contains at least one of the formulas: α, β.

The notion of H-completeness entered the logical scene in the context of modal logics in the early 1950s. Some logicians discretely suggested that the lack of the property is 'highly undesirable' [McKinsey, 1953], or 'disquieting' [Hughes and Cresswell, 1968]. H-completeness is now regarded as a technical property of logics and is intensively studied by modal logicians (see e.g. [Chagrov, Zakcharyaschev, 1993]). G.F. Schumm was the first who has made an attempt to clarify philosophical issues connected to H-completeness [Schumm, 1993].

In my talk I will describe the historical background of H-completeness and survey the most important results concerning H-completeness. Secondly, Schumm's ideas and his argumentation will be presented and critically examined. Finally, I will indicate how H-completeness is related to other basic properties of logics, e.g. to the interpolation property.

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Conservatibility of Arithmetic and its Subtheories

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The well-know result on Π_2-conservativity of Peano Arithmetic over Heyting Arithmetic states that every Π_2 sentence provable in the former is also provable in the latter. This result can be generalized in the form of the following way: For every intuitionistic theory T^i closed under the Friedman and the negative translation and such that all atomic formulas are decidable in T^i, its classical counterpart T^c is $\forall\exists$-conservative over T^i.

In our talk we consider possible generalizations of this result. However, instead of using syntactic methods, we exploit semantic methods and present some new conservativity results proven by means of Kripke models for first-order theories. We focus on first-order arithmetic and some its subtheories.

Conductive Arguments

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The term "conduction" is not widespread in the literature on arguments. It was first introduced by Wellman (1971), but contemporarily is not used in the original meaning by many authors (Blair, Johnson 2011). Currently, conductive arguments are often understood as *pro* and *contra* arguments, which consist not only of normal *pro*-premises supporting a conclusion, but also of *contra*-premises (exceptions) denying it (Walton, Gordon 2013). We explain why such an approach seems to be attractive to the theory of argumentation. We also propose a formal method of representing conductive arguments and calculating the acceptability of their conclusions. The method is based on the model of structure and evaluation of arguments presented in (Selinger 2014). Our formalization recognizes internal structure of conductive arguments, allows infinitely many degrees of acceptability, reflects the cumulative nature of convergent reasoning, and it enables to interpret the attack relation.

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Mathematics as a Network. Saunders Mac Lane's Philosophy of Mathematics.

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The aim of this paper is to present the philosophy of mathematics as it was perceived by Saunders Mac Lane (1909–2005).

Mac Lane saw mathematics as a connected network of formal rules, axiom systems, concepts, and connections. Mathematics is not a single formal system such as ZFC or others. One can perceive the real complexity of mathematics thanks to the category theory. But the category theory does not have to be regarded as the "true" foundation of mathematics, after all mathematics might not need foundations in the sense in which the set theory is one. In his book titled *Mathematics: form and function* Saunders Mac Lane has singled out six possible positions in the philosophy of mathematics: *Logicism*, *Set Theory*, *Platonism*, *Formalism*, *Intuitionism*, and *Empiricism*. He claimed that they are not sufficient since they do not fully explain Mathematics (Mac Lane uses the capital 'M'). He called his own point of view *Formal Functionalism*. In his own words: "Instead, our study has revealed Mathematics as an array of forms, codifying ideas extracted from human activities and scientific problems and deployed in a network of formal rules, formal definitions, formal axiom systems, explicit theorems with their careful proof and the manifold interconnections of these forms. More briefly, Mathematics aims to understand, to manipulate, to develop, and to apply those aspects of the universe which are formal." (*Mathematics: form and function*, p. 456) Mac Lane's formal functionalism can be viewed as supplementation of the missing essential aspects of mathematics in terms of the six listed above. Mac Lane complained that philosophers, while they cultivate the philosophy of mathematics, limit themselves only to the simplest structures (e.g., numbers and geometric figures), not taking into account the whole of modern mathematics, thus they simplify and impoverish it.

The thesis of this paper is that Mac Lane, despite the fact that he deprecated Platonism, was to a large extent a Platonist. In particular, he used the concept of form in the sense close to Plato.

Logic, Knowledge and Time

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In my talk I am going to consider the problem of implementing formal logic apparatus to represent and investigate knowledge undergoing changes in time. By 'formal logic apparatus' we understand systems which are a combination of epistemic modal logic systems applied in the description of knowledge and temporal logic systems which enable the expression of the temporal context.

In the first part of my talk we will discuss basic systems of modal epistemic logic. We supply a basic notional apparatus of modal epistemic logic concerning knowledge of a singular cognitive subject and the notional apparatus connected with issues of knowledge of groups of cognitive subjects. Formal languages considered here are appropriate for the description of static knowledge which is not subject to changes.

In the next part of my talk we will consider the problem of methods of temporalizing logical systems. Two basic methods of temporalization will be discussed: *internal temporalization* and *external temporalization*. Due to its possibilities for implementation we will concentrate on the method of external temporalization. We will describe the fusion method and the Finger-Gabbay method [1] and indicate conditions which should be met so that when joining two logical systems using the mentioned methods, the newly created logical system retains the metaproperties of the components systems. We have in mind here properties such as consistency, completeness and decidability. We will also discuss some systems of temporal-epistemic logic created by use Finger-Gabbay method of temporalization of logic systems [3].

In the last part of my talk we will discuss alternating time temporal epistemic logic $ATEL$ [2]. The language of $ATEL$ is the language of ATL extended with knowledge modalities. Combining knowledge modalities with ATL it becomes possible to express some interesting properties of multiagent systems. The $ATEL$ logic is defined with respect of finite set of Π atomic propositions and finite set of Σ $(= \{1, ..., k\})$ agents (players). There are introduced two additional specific operators: $\langle\langle\rangle\rangle$ and $[\![\;]\!]$. The intended interpretation of a formula $\langle\langle\Gamma\rangle\rangle\varphi$ is that the agents Γ can cooperate to ensure that φ holds (or equivalently, that Γ have a winning strategy for φ). The intended interpretation of a formula $[\![\Gamma]\!]\varphi$ is that the agents Γ cannot avoid φ. Formulae of $ATEL$ are interpreted with respect to the alternating epistemic transition systems.

We will discuss an axiomatization of $ATEL$ (it inherits the $S5$ axioms of normal modal logic for knowledge modalities and the associated axioms for common and group knowledge). Moreover, we will discuss some applications of $ATEL$ in communication and game theory (to describe and analyze of extensive games and formulate backward induction method in the language of $ATEL$ [3]).

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*Towards a Deontic Logic of Actions and States**

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Among general norms there are those that concern obligatory, permitted or prohibited *actions* and those that concern desired, acceptable or forbidden *states*. We shall call the norms of the two types shortly a-norms (for action norms) and r-norms (for result norms) respectively. Usually, in deontic logic the two types of norms are not studied together. Moreover, they are often regarded in deontic literature as linguistic variants of the same normative reality.

In our presentation we shall argue that there is a need for deontic logic in which we can reason about a-norms and r-norms together. There are some works that tackle the problem such as [5] and recently [1], but we are not fully pleased by the solutions present there mostly due to the fact that they do not really connect deontic properties of actions with the ones of states. Thus we shall present and discuss our formal solution, being a conceptual extension of works presented in [4,3,2].

We believe, following natural language and legal practice, that the approach to prohibition and obligation should be different. In the case of prohibition if we prohibit the execution of every action denoted by a general action name or if we prohibit bringing about a state described by a proposition we prohibit all their concrete realisations. In contrast, the obligation concerning an action name or proposition is fulfilled if any action token or state fulfilling the specification is realised. However, obligations should not be overgeneralised, i.e., the fact that a set of action tokens or states is obligatory does not entail that its supersets are also obligatory. Otherwise, we would lose information about obligations. We would obtain norms that are less useful then the original ones.

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Moreover, we are interested in two kinds of reasoning about norms. One of them is a derivation of new general norms from already accepted general norms. In the case of norms expressing obligations, a derived norm is always more specific (referring to a smaller set of action tokens or states) than the norms from which it is derived. That guarantees that a derived norm points to new information about the normative system. We want to be able to combine two a-norms together and two r-norms together, but also a-norms with r-norms.

The other kind of normative reasoning we are interested in is discovering individualised norms for a particular agent and situation in a normative environment. Ignoring a sophisticated ontological distinction between general and individualised norms we attempt to obtain that result by finding the most specific general norm derived from all norms applicable to the situation. That allows us to discuss both kinds of norms (general and individualised) in one formal system.

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Applications of Fuzzy Logic in Artificial Intelligence

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One of the main goals of Artificial Intelligence is to create a system which would be able to imitate human thinking. In practice, current researches focus on the development of systems that do not so much think on their own, but rather help humans solving problems. Many of these difficulties cannot be solved using classical logic. In human speech and in the way of thinking vagueness is an inevitable and even a desired effect. Nevertheless, it may be an obstacle for the machines. Sentences like "traffic in the street is heavy" or "this company is doing well" or "the turnover of our company was satisfactorily high, at a fairly low cost" cannot be expressed by means of classical logic. Even relatively clear expressions such as: "tall", "young", "close", are vague. In solving problems relating to the vagueness of terms, the fuzzy logic is used.

The concept of fuzzy logic was introduced in 1965 by Zadeh. (Some his ideas had been studied earlier by Łukasiewicz and Tarski as infinite-valued logics). Fuzzy logic can handle the concept of partial truth where the truth value may range between absolutely true and absolutely false. So, fuzzy logic variables may have a truth value that ranges in degree between 0 and 1.

In my speech I intend to present the motivation for the introduction of fuzzy logic into Artificial Intelligence, I am going to display examples of systems where this approach was applied, and I will discuss their mode of action. I will also show the limitations of such systems and the reasons behind these limitations.

Complexity of Qualitative Spatial Reasoning[‡]

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Although, there are numerous well–known quantitative methods for space representation, e.g., Euclidean or Cartesian approaches, it is believed that they are not adequate for our commonsense representation mechanisms. Therefore, a multi-disciplinary field of Qualitative Spatial Reasoning (QSR) has been established. QSR methods are based on logical formalisms and are mainly focused on representing topology and orientation – the most important aspects of the space. We will present several QSR methods, namely two topological methods, i.e., Region Connection Calculus with 8 basic relations ($RCC8$) ([5]) and Interval Algebra (IA) ([1]), two orientation methods, i.e., Cardinal Direction Calculus (CDC) ([3]) and Rectangle Algebra (RA) ([2]), followed by two combinations of topological and orientation methods, i.e., $RCC8+CDC$ and $RCC8+RA$ ([4]). Assuming that $P \neq NP$, the Economy Principle: *'Our minds are only equipped with such mechanisms which carry out only practically computable operations'* and Edmond's Thesis: *'The class of practically computable problems is the same as the P class'*, we conclude, that our minds are not equipped with any mechanism which carries out NP–complete problems. Therefore, any QSR method which is believed to be adequate for human–like reasoning needs to be P. The aim of our presentation is to introduce various QSR formalisms, specify their complexity and expressiveness. Referring to our conclusion, only P problems may adequately represent human–like reasoning mechanisms, therefore such methods are of our main interest.

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An Object-Language Analysis of Speech Acts

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Speech acts are acts. That is, we need to *do* something to perform them and there are norms guiding our speech-actions. Interestingly, the norms are intricately connected to the modal status of the proposition expressed by the sentence used in a speech act. In short, what we should do as a result of a performance of a speech act depends on whether what was said in the act is settled true, settled false, or open to be both ways. The role of the modal element was first recognized by Nuel Belnap who provided an exquisite analysis of speech acts in the setting of *branching-time* semantics (Belnap et al., 2001; Belnap, 2002).

Belnap developed a formal machinery of *double-time references* to grasp the following idea: When we reflect on a previously performed speech act and try to assess whether it was successful, we need to establish if the sentence used in the speech act expressed, *at the moment of performance* of the act, the proposition which is settled true *at the moment of assessment* of the speech act. (The idea was later fruitfully adapted by John MacFarlane, 2003, 2008).

To explicate his idea, Belnap used a three-place metalinguistic predicate $Sett(m_1, m_2, A)$ defined as follows:

Definition 1. $Sett(m_1, m_2, A)$ *iff* $\forall_h (m_2 \in h \Rightarrow m_1, m_1/h \vDash A)$

Belnap uses the definition to provide the truth conditions for phrases like "Agents α_1 and α_2 use the sentence 'A' to ψ (e.g. promise, order, assert, bet, etc.)":

Definition 2 (ψ-ing)**.** $m_c, m/h \vDash \alpha_1$ *ψ-es 'A' to α_2 iff all three of the following hold:*

1. $\forall_{m'>m}$ *if* $Sett(m, m', A)$, *then* α_1 *and* α_2 *should* Π_1 *at* m'
2. $\forall_{m'>m}$ *if* $Sett(m, m', \neg A)$, *then* α_1 *and* α_2 *should* Π_2 *at* m'
3. $\forall_{m'>m}$ *if* $\neg Sett(m, m', A) \wedge \neg Sett(m, m', \neg A)$, *then* α_1 *and* α_2 *should* Π_3 *at* m'

Where Π_1, Π_2, and Π_3 are the obligations relevant to the speech act of ψ-ing.

My aim here is to provide an *object language* definition of speech acts. I think that in the simple cases, to explicate the practice of speech-acting, it is sufficient to appeal to the basic concepts of temporal and modal logic (G, H, $\Diamond$), the truth operator, and the indexical operator Now. However, to give an account of embedded speech-act reports such as "Betty promised to help" or "Betty could have ordered him to come," we need to enrich the object language with additional operators: $Then$ and Ref. Operators of this sort where introduced and studied by Max Cresswell in his *Entities and Indices* (1990).

I will prove that using these operators, we can extract a modal formula whose truth conditions coincide with those proposed by Belnap. Besides a formal one, it serves a philosophical goal. It shows that the inhabitants of the indeterministic world are able to describe what they do when they perform speech acts.

In the presentation, I am going to introduce the formal setting of branching time and the concept of double-time reference in more details. I intend to discuss my ideas with reference to particular genres of speech acts, especially to an act of promise.

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Names, Relations and the Rule of Categorial Shift

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In the analysis of natural language the calculi of names, and especially the so-called *quantifier-less calculus of names* (**BRN**) is the most preferred tool.

The study proposes an enrichment of this calculus with relations (**BRNR**). Within the category of names two subcategories can be discerned, namely that of general names and singular names. To make this tool more flexible in the analysis of natural language, *the rule of categorial shift is introduced.* It allows to go from more elementary formulas (e.g. formulas with individual names) to formulas with general names (SG) and the other way round — from formulas with general names to more elementary formulas (GS). The equivalents of definite pronoun (δ) and indefinite pronoun (σ or η) are proposed here as well.

The study puts new perspective on some issues of classical logic: a certain expression of syllogistic (Nieznański), hamiltonian formulas (*every S is every P*, *every S is some P*, *some S is every P* and *some S is some P*) and the four classical laws — *the law of identity* (I), *the law of double negation* (DN), *the law of non-contradiction* (NC) and *the law of excluded middle* (EM).

Evidence and Philosophizing in Logic

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The evaluation of arguments in philosophy is mostly done by an appeal to the propositional evidence provided in favor of them. However, what counts as evidence for some does not count as evidence for others. Therefore philosophers sometimes tend to charge themselves with not having sufficient evidence for their philosophical beliefs. Nevertheless, it is not the case that philosophers come up with a convincing explanation about how a body of evidence turns out to be sufficient in the final analysis. Therefore, what constitutes a body of evidence sufficient in philosophy seems to be an intriguing and complicated question in its own right. On the other hand, providing evidence in logic seems to be associated with proving the validity and soundness of logical arguments. The talk deals with two questions: a) to what extent a body of propositional evidence reduced to proving the validity and soundness of arguments can be seen sufficiently philosophical in logic, and b) to what extent a body of evidence independent of its logical validity and soundness can be seen logical in philosophy. The aim of the talk is to compare and contrast the function of logical reasoning in philosophy with the function of philosophizing in logic.

Adolf Lindenbaum: His Logical and Mathematical Work

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Adolf Lindenbaum (1904–1941), in the words of A. Hinkis* a "delicate genius", was a member of the Warsaw Schools of Logic and of Mathematics who also collaborated closely with the Lvov School of Mathematics.

In our talk we shall briefly comment on the following:

1. Some aspects of Lindenbaum's private and academic life.
2. A bibliography of his work: papers, abstracts and short notes, reviews, and public lectures.
3. His contributions to general set theory: cardinal and ordinal arithmetic, the axiom of choice, the continuum hypothesis.
4. His contributions to metalogic, and their impact on the subsequent development of logical research.
5. His mathematical work: metric spaces, real analysis, group theory, foundations of geometry.

Some of Lindenbaum's more important papers were written jointly with other authors: two were co-authored with A. Tarski, one with A. Koźniewski and one with A. Mostowski. At the same time, a significant number of his results and ideas were never published at all — or at least not by him — but only appeared as acknowledgments and/or attributions to him in papers written by others. In our discussion of points 3–5 above we will focus on the magnitude and extent of this side of his work, and the way it helped shape the course of developments in several fields.

*A. Hinkis, *Proofs of the Cantor-Bernstein Theorem. A Mathematical Excursion*, Birkhauser 2013; see p. v (the dedication).