XVIII Conference

Applications of Logic in Philosohpy and the Foundations of Mathematics

Szklarska Poręba
6–10 May 2013

Game Theory

XVIII Conference

Applications of Logic in Philosophy and the Foundations of Mathematics

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Under the auspices of the Polish Association for Logic and Philosophy of Science

Typesetting:

Marcin Selinger
Krzysztof Siemieńczuk
Bartłomiej Skowron

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Abstracts

Editorial note

(EN) means that the talk is presented in English, (PL) — in Polish.

Social Laws: Logic and Games

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Social laws (or *normative systems*) have emerged as a natural and powerful paradigm for coordinating multi-agent systems. The social laws paradigm exposes the whole spectrum between fully centralised and fully decentralised coordination mechanisms. A social law is, intuitively, a constraint on the behaviour of agents, which ensures that their individual behaviours are compatible. Typically, a social law is imposed off-line, minimising the chances of on-line conflict or the need to negotiate.

The social laws paradigm is based on the use of computational logic to reason about multi-agent systems, but frequently also makes use of game theory.

I will give an overview of the state-of-the-art in the use of social laws for coordination. I discuss questions such as: how can a social law that ensures some particular global behaviour be automatically constructed? If two social laws achieve the same objective, which one should we use? How can we construct a social law that works even if some agents do not comply? Which agents are most important for a social law to achieve its objective? I will furthermore exemplify of how, e.g., formal logic, game theory, voting theory and complexity theory, can be used and combined in multi-agent systems research. Depending on the time, I will focus on the following topics:

1. *Introduction to state transition models for multi-agent systems and the social laws paradigm.* I review state-transition models of multi-agent systems. Coordination is concerned with the behaviour of a system

as a whole, with the global properties of the system. To this end, I introduce the language of Computation Tree Logic (CTL), with its natural branching time semantics to model possible computations of distributed systems. I'll focus on model checking, including computational complexity issues. I'll present a by now standard framework for social laws, introduced by Shoham and Tennenholtz in the early 1990s. A key assumption is that the designer (or analyst) of the multi-agent system has an objective, a property he or she wants the global behaviour of the system to satisfy. Key problems involving social laws are discussed, including: the effectiveness problem - will a given social law ensure the objective? The feasilbility problem - given an objective, does there exist an effective social law? The synthesis problem - given an objective, construct an effective social law. I show how these problems can be framed as model checking problems, allowing standard model checkers from multi-agent systems to be used to solve them, and we discuss the computational complexity of the problems.

2. *Dealing with non-compliance.* In many cases it might be that some agents choose not to comply with a given social law. There are many possible causes of non-compliance; it could be deliberate because the agent does not consider it to be in his best interest to comply, or it could be that a component in the system fails. I discuss how to analyse the properties of a social law under possible non-compliance. In particular, I look at how robust the social law is, and try to identify the agents that are most important for the correct functioning of the system. We say that the social law is robust if the objective is still achieved if only a small number of the agents choose not to comply. Key problems here are: which agents are necessary, in the sense that the objective does not hold unless they comply? Does there exist a social law that is robustly feasible in the sense that compliance of a given group (or number) of agents is sufficient to ensure the objective? I further analyse the relative importance of agents by employing power indices, such as the Banzhaf index, to measure the influence an agent has on satisfaction of the objective in terms of choosing to comply with the social law or not. For example, I discuss how we can ensure that power is distributed evenly amongst the agents in a system, so as to avoid bottlenecks or single points of failure, or to understand where the key risks or vulnerabilities in a social law lie.

   I look more closely at one particular type of possible non-compliance: deliberate non-compliance by rational, self-interested agents. Thus we shift from the perspective of the designer to the perspective of the agent, and assume that also each agent has his own objective. Will an agent with a given objective comply with a given social law? As satisfaction of the objective depends upon whether or not the other agents in the system comply, this is a game-theoretic scenario. Key problems here include: does there exist a social law all agents would be better off complying with (as opposed to not complying)? Does there exist a social law that is a Nash implementation, in the sense that complying forms a Nash equilibrium?

3. *Reasoning about Social Laws.* I look more closely at how we can use formal logic to reason about social laws. In particular I, first, discuss how (variants of) deontic logic can be used to reason about different social laws in the context of a multi-agent system, e.g., allowing us to say that something is permitted in one social law but forbidden in another. Second, I show how standard logics can be extended in order to be able to frame the problems discussed in part (2), including robustness properties, as model checking problems.

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Interrogative Games — The Idea of J. Hintikka

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This speech will focus on using games as a kind of theory of dialogues which has written by Jaakko Hintikka in *A spectrum of logic of questioning* and *Rules, Utilities, and Strategies in Dialogical Games.*

Hintikka proposed to use game as a illustrating of dialogues between two person. The general approach assume that games will continue after that one of players finds an answer, furthermore player who ended the searches, in the further course of the game he may find a new question for which he will find the answer. After same modification such game can be treated as dialogue between Scientist and Nature, of course Scientist have a more opportunities in the game, but it is still interrogative games.

I will describe rules and strategies, my attention will be focused on rules used in game between Scientist and Nature because it is quite interesting game and it is worth considering.

Products of Singular Graphs

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A graph G is singular if its adjacency matrix is singular. The interesting question is what structure do singular graphs have? There are known examples of singular graphs, like complete bipartite graphs, odd paths, cycles C_{4n}, square planar grids i.e. products of two paths of the same lenght. The problem of singularity for planar grids has been solved and published in 2011 in [2]. An explicit formula for the determinant of a planar grid was obtained by D. Pragel [3].

We consider cartesian products of singular graphs. According to results presented in *The generalized hierarchical product of graphs* [1] we conclude, that a product of two singular graphs is singular. However, we can obtain a singular graph as a product of two non-singular graphs. An interesting example of such product is every graph $P_2 \times G$, such that 1 or -1 is one of the eigenvalues of the graph G.

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Freedom and Enforcement in Action. Norms and Action Systems.

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Stit semantics gives an account of action from a certain perspective: actions are seen *not* as operations performed in an action system and yielding new states of affairs, but rather as selections of pre-existent histories (or trajectories) of the system in time. Stit semantics is therefore time oriented, and time, being a situational component of action, plays a special role in it. In the talk an approach to action theory which stems from formal linguistics and dynamic logic is presented. An *elementary action system* is a triple (*) $(W, R, \mathbf{A})$, where W is a set of *states*, R is a binary relation on W called the *transition relation* between states, and $\mathbf{A}$ is a non-empty family of binary relations on W. (*) is thus a multi-modal frame in which the relation R is distinguished. The members of $\mathbf{A}$ are called *atomic actions* of the system (*). A *compound action* (over (*)) is a set of finite sequences of atomic actions from $\mathbf{A}$. Every compound action is thus viewed as a formal language over the alphabet $\mathbf{A}$. In *ordered action systems* the set of states W is additionally partially ordered. A *situational action system* is an extension of an elementary action system which takes into account the situational envelope in which actions are immersed. A comparison with stit semantics is discussed.

In the talk an approach to deontic logic is presented according to which actions (or deeds), and not states of affairs, bear deontic values (as being obligatory, forbidden, permitted). The notion of an *atomic norm* and its relationship with the above interpretation of the deontic operators is discussed. The term *norm* receives a wider meaning than in jurisprudence and encompasses moral norms, linguisitic norms, social norms – providing patterns of behaviour in social communities, conventions, etiquettes etc. In our approach norms play a role similar to that of rules of inference in logic (but the analogy is rather loose) – norms are rules of action; they guide actions and determine circumstances under which some actions are permitted, forbidden or obligatory. To each atomic norm a certain normative proposition is assigned; norms however are not reducible to propositions.

Almost Structural Completeness in Quasivarieties and in Logic

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Key words: almost structural completeness, quasivarieties of algebras, admissible rules, passive rules.

A b s t r a c t.

Almost Structural Completeness and Passive rules in quasivarieties of algebras and in some logics and are described and characterized.

We consider structural consequence operations and structural rules of the form $A\,/B$. A rule $r : A\,/B$ is *admissible* in a logic L, if for every substitution τ, whenever $\vdash_L \tau A$, then $\vdash_L \tau B$. A rule $r : A\ /B$ is *drivable* in a logic L, if A logic L is *structurally complete* iff every (structural) admissible rule in L is derivable in L. A *unifier for* a formula A in a logic L is a substitution σ such that $\vdash_L \sigma(A)$. A rule $r : A/B$ is *passive* in L, if the premise A does not have a unifier in L. A logic L is *almost structurally complete* iff every (structural) admissible rule in L, which is not passive, is derivable in L.

Quasi-identity has the form: $s_1(x) \approx t_1(x) \wedge \cdots \wedge s_k(x) \approx t_k(x) \Rightarrow s(x) \approx t(x)$. A *quasi-variety* Q is a class of algebras axiomatized by quasi-identities. Let $\mathbf{F}$ be a countable free algebra in Q. A quasivariety Q is *structurally complete*, if Q $=$ Q($\mathbf{F}$), i.e., every quasi-identity valid in $\mathbf{F}$ is also valid in Q.

Q is *almost structurally complete* if for every quasi-identity q valid in $\mathbf{F}$ either q is valid in Q or premises of q are not satisfiable in $\mathbf{F}$. Characterization of almost structural completeness in terms of subalgebras of $\mathbf{F}$ is given and several examples are presented.

Some results of this talk are based on a joint research with M. Stronkowski (Warsaw).

Investigation in Combining Intuitionistic and Classical Logics

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Different attempts to create a new logic that contains all advantages of intuitionistic and classical logics have been undertaken in recent years. Best known example of such a refinement is Girard's linear logic. Instead of emphasizing *truth*, as in classical logic, or *proof* as in intuitionistic logic, linear logic emphasizes the role of formulas as *resources*. Linear logic contains a fully involutive negation while maintaining a strong constructive interpretation. In addition it provides new insight into the nature of proofs in both classical logic and intuitionistic logic. Due to its focus on resources, linear logic has found many applications in theoretical foundations of soft computing methods, for example it enables to design new, expressive languages of logic programming. However linear logic has some limitations and its main drawback is undecidability of its propositional fragment.

The main focus of the talk is to present Intuitionistic Control Logic (ICL) introduced lately by Chuck Liang and Dale Miller [3]. This logic adds to intuitionistic logic elements of classical reasoning without collapsing it into classical logic. It is achieved by adding a new logical constant for *falsum*. The idea of using two different constants for *falsum* was derived from Linear Logic. Distinction between two symbols 0, which is a *falsum* in intuitionistic logic, and $\perp$, which is the new constant, allows to define two forms of negation: $\sim A$ and $\neg A$. Neither of them is involutive, as both negations are defined by intuitionistic implication.

The new constant requires a simple but significant modification of intuitionistic logic both proof-theoretically and semantically. Intuitionistic Control Logic has natural deduction proof system *NJC*. In the proof theory of ICL, the new constant $\perp$ indicates points in a proof where contraction and multiplicative disjunction can be used. *NJC* is sound and complete with respect to the Kripke semantics. A Kripke model for ICL, called r-model, is based on a rooted Kripke frame $\langle W, r, \preceq \rangle$, where $\preceq$ is a partial ordering relation on the set of possible worlds W and $r \in W$ is the unique root such that $r \preceq u$ for all $u \in W$. Standard forcing relation $\models$ in a Kripke model maps elements of W to sets of atomic fomulas and is defined as usual. The only differences between forcing rules in r-models and those of regular Kripke models for intuitionistic logic are in regard to $\perp$. All worlds properly above r force $\perp$, but not r itself.

We present an algorithm for deciding if a formula is provable in ICL based on a concept of Ladner's algorithm for modal logic S4 [2]. The algorithm is a contribution in finding answers to basic questions concerning Intuitionistic Control Logic, in particular computational complexity of ICL. Our main conjecture is that adding a new constant to intuitionistic logic should not enlarge the computational complexity beyond PSPACE. The algorithm is also a foundation for future work on other combination of intuitionistic logic and classical logic presented by Liang and Miller [4], namely Polarized Intuitionistic Logic (PIL) which is a system based on a distinction between two dual polarities. The strength of this logic is that it allows connectives that are intuitionistically oriented to mix freely with classically oriented ones and still it is guaranteed that they do not collapse into each other's counterpart.

Although ICL can be viewed as a specific case of PIL, it is a stand-alone logic not based on any notion of polarity or duality that is assumed to exist a priori. Propositional fragments of ICL and PIL are decidable which alongside with simple languages that maintain intuitionistic implication as a genuine connective and elegant traditional semantics (Kripke, algebraic and topological) give them advantages over linear logic.

The main goal would be to create a corresponding algorithm for checking the provability of formula in PIL and proving the conjecture of PSPACE-completeness of Polarized Intuitionistic Logic along with further studies on this logic.

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Abstract Argumentation, Logic & Games

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Abstract argumentation [2] is the theory of graphs of the type $\langle A, \rightarrow \rangle$—called *attack graphs*—where A is a set and $\rightarrow$ a binary relation. These are high-level models of the sort of conflict that occurs in argumentation where arguments (the elements of A) interact by attacking one another (through the binary 'attack' relation $\rightarrow$). The theory has proven to be a prolific abstraction from which to study several aspects of argumentation. In these lectures I aim at providing an introduction to this theory highlighting its relationships with logic and games.

First, I will show how attack graphs can be used to provide mathematical definitions of criteria of the 'rationality' or 'justifiability' of sets of arguments, which I call *solution concepts* for attack graphs. The development of such criteria constitutes the main bulk of the theory of attack graphs as developed in the last two decades within the field of Artificial Intelligence (cf. [1] for a recent overview). In introducing these notions I will draw a parallel with modal logic [3], showing that many solution concepts of abstract argumentation can be naturally formalized in well-known modal languages by interpreting the modal diamond $\Diamond$ as expressing the property "there exists an attacker such that ...". A good example is the formula of the modal μ-calculus:

$$\mu p.\Box\Diamond p \tag{1}$$

which, for a given graph $\mathcal{A}$, expresses the smallest set p of arguments such that $p \leftrightarrow \Box\Diamond p$. That is, the smallest set p which is equal to the set of arguments whose attackers are attacked by some argument in p.

Second, I will move to a more dynamic and interactive view of a argumentation. Solution concepts can be viewed as specifications of abstract standards of proof, i.e., as specifications of the conditions under which an argument is 'satisfactorily' proven within a given graph. Two-players (proponent and opponent), zero-sum games with perfect information can be used as interactive procedures 'implementing' such standards of proof. More concretely, for a given solution concept S—like the one expressed by Formula (1)—one can define a game $\mathcal{G}_S$ satisfying the following property:

> *An argument a belongs to solution S if and only if the proponent has a winning strategy in the game $\mathcal{G}_S$ played starting with argument a.*

Third, I will address the issue of when two arguments, in two attack graphs, can be considered to be 'equivalent' [4]. In abstract argumentation arguments have no internal structure (no premisses, no conclusions), being just points in a network of attacks. So the notion of equivalence I propose is of a structural type and concerns the 'positions' that arguments occupy in their respective graphs. I will look at this intuition from two perspectives: a modal one, whereby two arguments are equivalent (w.r.t a given solution concept) whenever they satisfy the same modal formulae in an appropriate fragment of the basic modal language; a game-theoretic one, whereby two arguments are equivalent whenever a same player has a winning strategy of the same type (according to a precise definition of 'type') in the games for the two arguments. The two perspectives will be shown to be equivalent in the case of the solution concept expressed by Formula (1).

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Methods of Proving Cut Elimination

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Cut elimination/admissibility theorem is commonly treated as one of the most fundamental results in modern proof theory. It is not surprising that during the last 80 years, since the classical paper of Gentzen, a lot of different techniques were introduced to deal with the problem. In the talk we compare some of the proposed methods, due to Curry, Dragalin, Schutte, Tait, Smullyan and Buss, and discuss their merits.

Relative Negations as Binary Functors of Classical Sentential Calculus

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In classical sentential calculus there is only one negation and this makes an analysis of logical relations between certain sentences of a natural language within the classical logic a little difficult. There are various kinds of sentential negations in an ordinary language and we often use at least two – already known in frame of traditional logic – weaker negations than the classical one, i.e. contrary and subcontrary negations. My purpose is to give to the each of mentioned sentential weak negations (as well as the standard one) a formal representation inside of classical calculus as two arguments truth value functions. I obtain new extensional functors forming propositions from two propositions: a negated statement and a proposition giving a context of the negation. Because of the clear reference to a context of denying I have named them: relative negations. The new connectives have got their truth values lattice definitions and, of course, every formula containing them can be equivalently replaced by a formula containing only standard connectives. Thereafter, using just classical logic, one may e.g.: formalize and investigate the relations among contrary, subcontrary or contradictory negations and the other sentences, explain paradoxes like the liar paradox, recognize some expressions called "metalinguistic negations" as examples of one of the weak negations.

Countable Frame for Bimodal Logic $Grz.3 \otimes Grz.3$

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Mono-modal logic has been widely investigated in philosophy and mathematics. It is known that there is a great deal of high powered results: (strong) completeness, decidability or finite model property (see [2]). However, when we turn to the polimodal case, there are a lot of unanswered questions. In some cases of polimodal systems, to determine the completeness or the decidability problems, it suffices to confine our considerations to the independently axiomatizable bimodal logic.

In our talk we focus on independently axiomatizable bimodal system $Grz.3 \otimes Grz.3$ which is a fusion of two $Grz.3$ systems. It means that this is the smallest multimodal system containing the following axioms

K_i	$\Box_i(\varphi \to \psi) \to (\Box_i\varphi \to \Box_i\psi)$
$D1_i$	$\Box_i(\Box_i\varphi \to \psi) \vee \Box_i(\Box_i\psi \to \varphi)$
Grz_i	$\Box_i(\Box_i(\varphi \to \Box_i\varphi) \to \varphi) \to \varphi.$

which is closed under the rule of Modus Ponens (MP) $\frac{\varphi\to\psi,\varphi}{\psi}$ and the rules of Necessitation (RN_i) $\frac{\varphi}{\Box_i\varphi}$, for $i = 1, 2$.

It is already known that completeness is preserved under the formation of fusions (see [3]). Namely, the system $Grz.3 \otimes Grz.3$ is characterized by the class of finite and connected frames $\mathfrak{V} = \langle V, S_1, S_2\rangle$ whose relations are linear order on every S_1 (or S_2) $-$ *connected* component, which is connected with respect to S_1 (or S_2) (see [1]).

In mono-modal case, it is easy to show that $Grz.3$ is complete with respect to $\langle\omega, \geq\rangle$. Additional modality make the problem more complicated. Thus, our aim is to distinguish only one countable frame with that property for the system $Grz.3 \otimes Grz.3$. To that end, first we have to consider notions such as p-morphism or finite model property. Then, we obtain our main result which states that the system $Grz.3 \otimes Grz.3$ is complete with respect to the frame $\mathfrak{D} = \langle U, R, B\rangle$ in which $U = \{(c_1p_1, \ldots, c_{n-1}p_{n-1}, c_n0); n \in \mathbb{N}, c_k \in \{r, b\}, c_k \neq c_{k+1}, p_k \in \{-\frac{n}{n+1}; n \in \mathbb{N}\} \cup \{\frac{1}{n}; n \in \mathbb{N}\} \cup \{-1\}\}$ and R, B are some particular relations.

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On the Lattice of Logics in NEXT(KTB.3′)

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We examine normal extensions of the Brouwer modal logic which are determined by a class of Kripke frames equipped with a tolerance relation and having special linear forms. Each reflexive and symmetric frame $\langle W, R\rangle$ can be divided into blocks of tolerance. Blocks of tolerance are linearly ordered if one of them has non-empty intersection with at most two other blocks.

The motivation for our research has two sources **S4.3** and $\mathbf{KTB} \oplus alt_3$. For both these logics the appropriate Kripke frames have linear shape. Referring to brouwerian linear logics, they can be axiomatized by adding the following axiom (see [1]):

$$(3') := \Box p \vee \Box(\Box p \to \Box q) \vee \Box((\Box p \wedge \Box q) \to r).$$

It was proved in [1] that

Theorem 1. *All logics from $NEXT(\mathbf{KTB.3'})$ are Kripke complete and have f.m.p.*

In contrast to logics from $NEXT(\mathbf{S4.3})$ and $NEXT(\mathbf{KTB} \oplus alt_3)$, it occurred (see [2]) that:

Theorem 2. *The cardinality of $NEXT(\mathbf{KTB.3'})$ is uncountably infinite.*

We describe the lattice of $NEXT(\mathbf{KTB.3'})$ using the method of splitting. The general splitting theorem due to Kracht (see [3]) will be applied for this purpose.

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Bisimulations of Kripke Models for Intuitionistic First-Order Logic with Strong Negation

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The problem of logical equivalence has been widely investigated in many logical systems. As we know the query whether two classical first-order structures validate the same formulae can be reduced to a structural description for the mentioned notion.

In classical model theory the problem, stated by Alfred Tarski, was to find a structural description for the notion of elementary equivalence of two classical first-order structures. It was solved by Fraïssé and then by Ehrenfeucht, and refered to the notion of Ehrenfeucht–Fraïssé Game.

Subsequently, the issue of structural description of logical equivalence has been transferred to intuitionistic logic. In the case of Kripke semantics for intuitionistic first-order theories, the aim is to find a structural description for the notion of logical equivalence of two Kripke models.

As it turned out, the notion of bounded bisimulation of two Kripke models provides such a description. Let us consider Kripke models $\mathcal{K}$ and $\mathcal{M}$ and two arbitrary nodes α and β of those models respectively. Then, by a bisimulation we mean any relation between α and β which fulfills particular 'zig' and 'zag' conditions.

Since quantifiers $\forall$, $\exists$ and propositional connectives are not mutually definable, as a measure of formula's complexity we consider the *characteristic* of a formula. We say that the characteristic of a formula φ, in symbols $char(\varphi)$, equals $(^{\rightarrow}p,^{\forall}q,^{\exists}r)$ whenever there are p nested implications, q nested universal quantifiers and r nested existential quantifiers in φ.

It was already known (see [5]) that, in the case of Kripke models for intuitionistic first-order logic, the existence of (p,q,r)-bisimulation between nodes α and β, in symbols $\alpha \sim_{p,q,r} \beta$, implies their logical equivalence with respect to all formulae of characteristic not greater than $(^{\rightarrow}p,^{\forall}q,^{\exists}r)$, denoted by $\alpha \equiv_{p,q,r} \beta$.

Then, it has been shown in [2] that the inverse implication holds for so-called *strongly finite* and *finitely saturated* Kripke models.

Afterwards, our attention was drawn to intuitionistic first-order logic with strong negation, that was first introduced in [4] by Nelson and independently by Markov (see [3]). In intuitionistic logic, the negation operator $\neg$, which we will call *Heyting's negation*, lacks of some properties. For example, the derivability of $\neg(\varphi \wedge \psi)$ is not equivalent to the derivability of at least one formula of $\neg\varphi$ or $\neg\psi$. But, if we substitute the Heyting's negation with a strong negation, that equivalence will be obtained.

We consider an operator of *strong negation* $\sim$ and an extention of intuitionistic first-order logic with it. In this system, not only are we able to verify statements, but also falsify them. The negative information is as primitive as the positive one, and both of them are equally important. According to the Kripke semantics, at a stage α we can determine whether an element a has property P, in symbols $\alpha \Vdash P(a)$, or, on the other hand, we can establish if a does not have property P, $\alpha \Vdash\sim P(a)$. But, let us also bare in mind that no atomic formula can be accepted and simultaneously refuted at any stage α. Moreover, we may also encounter a case where neither $\alpha \Vdash P(a)$, nor $\alpha \Vdash\sim P(a)$.

Consequently, in Kripke semantics for intuitionistic first-order logic with strong negation, we deal with positive and negative forcing, and investigate logical equivalence of Kripke models, in both positive and negative sense.

Moreover, in this case quantifiers $\forall$ and $\exists$ are mutually definable. Hence, as a measure of formula's complexity we consider a *strong characteristic* of a formula. We will say that the strong characteristic of a formula φ equals $(^{\sim}s,^{\rightarrow}t,^{\forall}w)$ whenever φ consists of s nested strong negations, q nested implications and r nested universal quantifiers.

In the talk we present the results of our research on the issue of logical equivalence of Kripke models for aforementioned logic. First, we define a notion of bounded bisimulation of two models. Then, we prove a theorem which states that the existence of (s, t, w)-bisimulation between nodes α and β implies their both positive and negative logical equivalence with respect to all formulae of strong characteristic not greater than $({}^{\sim}s, {}^{\rightarrow}t, {}^{\forall}w)$. Finally, we show that the inverse implication holds for strongly finite and finitely saturated Kripke models.

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Universal Homogeneous Structures

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A structure U is *universal* for a given class of structures $\mathfrak{K}$ if $U \in \mathfrak{K}$ and every structure from $\mathfrak{K}$ embeds into U. Universality is a well-known concept in several areas of mathematics. Another property, often making a universal structure unique up to isomorphism, is *homogeneity*. Namely, given a class $\mathfrak{S}$ whose objects will be called *small*, we say that a structure U is $\mathfrak{S}$*-homogeneous* if every isomorphism between its small substructures (i.e. substructures belonging to $\mathfrak{S}$) extends to an automorphism of U. This notion usually makes sense if U can be "reached" from the class $\mathfrak{S}$, for example, when U is the union of a chain of small substructures. This gives rise to the category-theoretic concept of a *generic sequence*, a functor from $\mathbb{N}$ into $\mathfrak{S}$ which has special properties leading to a universal homogeneous object in the bigger category $\mathfrak{K}$.

We shall describe some aspects of the category-theoretic framework for universal homogeneous structures, emphasizing on the uniqueness problems, solved by a *back-and-forth argument*, which in turn can be viewed as a game between two structures of the same type. We shall also discuss an *approximate back-and-forth method*, in the context of categories enriched over metric spaces.

Historically, the first work on universal homogeneous structures in model theory, exploring the back-and-forth argument, was made by Roland Fraïssé [1], later continued by several authors. We will present some some material from our works [2, 3, 4].

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On some Modification of the Nash Arbitration Scheme in Two-person Matrix Games

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In the speech we analyze the Nash Arbitration Scheme in two-person non-zero-sum games. We recall basic concepts of cooperation related to Nash's theorem on existence of fair solution of a game. Next, we consider some assumptions that have to be fulfilled to apply Nash Arbitration Scheme, and we introduce the notions of weak cooperation and the solution on the basis of weak cooperation. We outline connections between the payoff polygon of a game and the set of outcomes obtainable by mixed strategies, and we propose axioms of theory of fair solution on the basis of weak cooperation. Finally, we remark a couple of open problems.

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A Logical Analysis of the Ontological Concepts of Form and Matter

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The lecture provides a tentative formal logical study of ontological concepts of form and matter within the framework of modal propositional logic. It discusses the interconnection between the notion of form and the notion of well-foundation and the interconnection between the notion of matter and the notion of dependence.

The Connective "chyba że"

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This paper will discuss the problem of finding in extensional logic the nearest equivalent of the "chyba że" connective existing in natural language. The connective "chyba że" is the Polish equivalent to "unless" in English.

First-Order Logic with Imperfect Information

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Game-theoretic semantics defines truth and satisfaction in terms of (semantic) games. Although the fundamental intuition that quantifiers can be interpreted as moves in a game appears as early as Peirce's second Cambridge Conferences lecture [8] and Henkin's seminal paper on branching quantifiers [2], game-theoretic semantics was first popularized by Hintikka [3, 4] (see also [5, 7]).

The semantic game associated with a first-order sentence is a contest between two opponents. One player tries to verify the sentence by choosing the values of existentially quantified variables, while the other tries to falsify it by picking the values of universally quantified variables. Disjunctions prompt the existential player to choose which disjunct to verify; conjunctions prompt the universal player to pick which conjunct to falsify. Negation tells the players to switch roles. A first-order sentence is true (false) in a suitable structure if and only if the existential (universal) player has a winning strategy.

In order to define the semantic game for an open first-order formula, one must specify the values of its free variables. Usually, this is done using an assignment. If the open formula in question is a subformula of some first-order sentence, then we can think of the assignment as encoding the previous moves of the players in the semantic game for the sentence.

In the semantic game for a first-order formula, the players take turns making their moves, and at each decision point the active player is aware of every move leading up to the current position. Thus semantic games can be modeled as extensive games with perfect information.

First-order logic with imperfect information is an extension of first-order logic obtained by considering semantic games with imperfect information. In a game with imperfect information, the active player may not be aware of every move leading up to the current position. To specify such games, we must extend the syntax of first-order logic to be able to indicate what information is available to the active player. We briefly describe two approaches found in the literature.

Independence-friendly (IF) logic, introduced by Hintikka and Sandu [6], adds a slash set to each quantifier that indicates which variables the active player is not allowed to access when choosing the value of the quantified variable. For example, in the independence-friendly sentence

$$\forall x \big(\exists y/\{x\}\big) Rxy,$$

the existential player must choose the value of y without knowing the value of x.

Dependence logic, introduced by Väänänen [9], utilizes new atomic formulas of the form

$$=(t_1, \ldots, t_n)$$

whose intuitive meaning is that the value of the term t_n is determined by the values of the terms $t_1, \ldots, t_{n-1}$. The atomic formula $=(t)$ asserts that the value of t is constant. Thus, when playing the semantic game for the dependence logic formula

$$\forall x \exists y \big(=(y) \wedge Rxy\big),$$

the existential player knows the value of x when choosing the value of y, but if the game is repeated she must choose the same value for y as before, regardless of the new value of x.

Both IF logic and dependence logic have the same expressive power as existential second-order logic, a result first proved independently by Enderton [1] and Walkoe [10] in the context of first-order logic with branching quantifiers.

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Proving Conservativity by Means of Kripke Models

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Given a set of sentences S of a first-order language, we can consider the closure of S with respect to derivability in classical or intuitionistic first-order logic. In this way we get the classical or intuitionistic counterpart of the theory axiomatised by S, respectively. Now, if Γ is a class of formulae, we say that the classical theory axiomatised by S is Γ-conservative over its intuitionistic counterpart, if every sentence of Γ which is provable classically from S is also provable from S intuitionistically. For example, a well-known result of this kind is that of Π_2-conservativity of Peano Arithmetic over its intuitionistic counterpart, Heyting Arithmetic.

Usually conservativity results are proven by means of syntactical methods. One of the most important among them is a combination of the Friedman translation and the so-called negative translation. We will show that some conservativity results can be also proven semantically by means of semantical methods, in particular Kripke models. In particular, in some cases we can replace the assumption that the theory in question is closed under the negative translation with that of completeness with respect to conversely well-founded Kripke models with constant domains. Moreover, we find a purely model-theoretic conditions which allow to prove conservativity results for the class of formulae that strictly contains all $\forall\exists$-formulae. The conditions in question are satisfied by Heyting Arithmetic, so we get a result which improves the mentioned result that Peano Arithmetic is Π_2-conservative over Heyting Arithmetic.

Ontological Dimension

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In many ontological universes of various ontologies, the basic objects that, arranged in appropriate groups, form the objects of higher class, are distinguished. J. Perzanowski's combination ontology, inspired by Leibniz's *Monadology* and Wittgenstein's *Tractatus*, can be the example of such ontological procedure. In combination ontology, J. Perzanowski distinguished, among other things, superelements, elements, complexes, situations and possible worlds. In a sense, one can say that they are successive levels or ontological dimensions. Yet, how to define the notion of ontological dimension, and whether it is needful? In my speech I will show how one can join this ontological intuition of dimension to the dimension in topological sense. I will point out the simple interpretation of the parthood relation in Hilbert cube. Using this interpretation I will demonstrate that ontological dimension of elements amounts to 1, the dimension of situation amounts at least to 1, and the possible world is an infinitely-multidimensional object. What's more, modeling the ontological universe using Hilbert's cube, i.e. the kind of topologization of universe, refines ontological discussion, for it allows, for example, to examine and express the fact that the set of possible worlds is dense (in topological sense) in ontological universe.

"Linguistic Turn": from Frege to Cognitive Science

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The term "linguistic turn" denotes the paradigm in philosophy biased toward an important role of the language in the epistemological foundations of science. This perspective has resulted in the development of various branches of knowledge associated with language, from logic to linguistics, theory of argumentation, philosophy of mind, studies connected with computer science, artificial intelligence and, eventually, modern cognitive science discussions. The idea of the paper is the claim that, in spite of an attempt to undermine the role of logic in contemporary philosophical trends, its role is, in more or less explicit way, unquestionable.

A Formal Description of Changeability Enriched by Modalities and Quantification

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In frame of philosophical logic the notion of changeability is usually described refering to certain temporal notions. A big pallete of many different definitions of changes in certain temporal logics was presented by J. Wajszczyk in [3]. However there are also some philosophical reasons to proceed in opposite direction and to base temporal notions on a primitive notion of change. This point of view with explicit connection with Aristotelian ontology and some intentions of Leibniz was originally presented in [1] and extracted in [2]. In result it has been axiomatized as sentential logic LCG which describes two sorts of chaneability symbolized by operators C and G. C-changes are dichotomic in this sense that sentences describing carriers of such changes, change their values from truth to falsehood or conversely. G-changes consist in the occurence of new objects in the universe of all considered elementary situations. (The original interpretation of both C and G operators is ontological. However there are also possible epistemic ones, e.g. changes of convictions of some agent.) To mention some characteristic features of C-changes let us say that - according to LCG axiomatics - $CA \to C\neg A$ (if A changes its truth value then not-A also changes it), and from A it may be inferred $\neg CA$ (theorems do not change). G-changes are connected with the idea of growing languages (and not of encreasing the set of truths). Logic LCG is interpreted in semantics of so called histories which are understood as sequences of elementary situations which transform facts to fictions or fictions to facts. LCG is complete in respect to this semantics.

In the presented lecture we are going to enrich LCG description of changeability in two respects. At first we supplement LCG description by considering a counterpart of changeability - constancy expressed by $\Box$ operator. This kind of unchangeability is not defined by C operator but is introduced via axioms - we assume e.g. that: $\Box A \to \neg C \Box A$ (if A is constant, then it does not change) and we connect C with $\Box$ introducing some kind of ω-rule. Secondly we want to speak about changes consisting in losing or acquiring certain properties by individuals. For this reason we extend our language to first order by introducing predicates, individual constants of different levels and indexed quantifiers. (Levels and indexes are connected with the idea of G-changes and growing languages). The relation between quantifiers and C operator is expressed by C-versions of Barcan formula, e.g. $\forall_x Ax \to (C\forall_x Ax \to \exists_x CAx)$. We extend the semantics of LCG to the modal first order case: we assign to every level n a domain of individuals D_n (where $D_n \subset D_{n+1}$) and for every n we chose a set of elementary sentences which are considered to be true about individuals from D_n. Now we call a history any sequence of such sets. We interpret our extension of LCG in this semantics (we use past-possibilistic interpretation of quantifires) and we give a completenss proof.

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Structural Completeness of the Relevant Logic R

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Theorem. There exists exactly one structural complete extension of the relevant logic **R**.

Iconic Sign: Definition and Other Fundamental Questions

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The notion of iconic sign is analysed and a definition of presented. Other questions which are analysed are: the world of iconic signs, schemas and pleromas, iconic signs and the essence of objects, the informational value of iconic signs and the suggestive power of iconic signs.

Friendship as a Game

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Friendship is defined as a binary relation F on a set of people. We assume that F is non-reflexive to avoid the tedious discussion whether everybody is a friend of himself. Evidently F is non-symmetric and non-transitive too. Friendship is characterized as an alternating sequence of gifts. The gifts can be material but most frequently they are representing certain amount of time devoted to a friend. Therefore a measure of the value of the gift is a number of time units. The value of material gifts can be converted in terms of time. In this way we obtain the alternating sequence of numbers adequate to the values of gifts. This sequence can be represented as a matrix of non-zero-sum game. The type of friendship — altruistic or egoistic — depends on strategy chosen. According to other assumptions the friendship can be considered to be a certain form of Prisoner's Dilemma. The final moment of the friendship depends on the payoff matrix and it can be fixed and analyzed by means of game theory.

Logic and Artificial Intelligence

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One of the tasks facing designers of artificial intelligence is to build a machine that could achieve the level of competence beyond human possibilities. In many fields, certainly managed to achieve success. Computers count faster, better able to remember and analyze extremely complex models. Still, they cannot explain the language well and recognize emotions of people shown in the photographs.

In my presentation I would like to introduce one of the practical applications of logic, namely: how the logical limitation theorems can be used in practice, in this case, in the work of the construction of artificial intelligence. It is being considered a matter of mechanization of the human mind. I intend to present a general idea of artificial intelligence, how does it works, its limitations and possibilities. I'm going to, based on claims Gödel and Church's thesis, show that a certain type of equipment, build on the computational theory of mind, would never solve certain mathematical problems. At the end of my address I'll present alternative to this type of machine, and I will discuss briefly the most important models of artificial intelligence.

Maximal Extension of a Logic of Values of A.A. Iwin

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A Iwin's logic is a bimodal logic. Modal logic with one modality have two or three maximal extension. Iwin's logic of values has infinity many extension. The proof of this fact is purely algebraic.

Quantification of Predicates, Venn's Syllogistic and a Certain Notational Convention

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In his *Formal Logic* (1881) John Venn constructed a certain system of syllogistic, which is one of implementations of the idea of the *quantification of predicates*. An interesting reconstruction of this system was proposed by V. I. Markin (2011). Markin makes use of five primary functors $\{aa, ai, ia, ii, e\}$. The elementary expressions $SaaP$, $SaiP$, $SiaP$, $SiiP$ and SeP are respectively read as: *all S is all P*, *all S is some P*, *some S is all P*, *some S is some P* and *no S is any P*.

Markin gives the axiomacity for the system. He also proposes the rules of translation of its formulas into the language of classical syllogistic of Łukasiewicz's axiomacity $\{SaS, SiS, MaP\&SaM \rightarrow SaP, MaP\&MiS \rightarrow SiP\}$ and the rules of reverse translation.

This formulation of Venn's syllogistic can be simplified by adopting the following notational convention:

$$S\phi\psi P / S\phi P\&P\psi S \quad S\phi P\&P\psi S / S\phi\psi P \quad \text{for } \phi, \psi \in \{a, i\}.$$

A new axiomacity for Venn's syllogistic is proposed here, including the strong understanding of particular-affirmative sentences ($S\iota P$). The logical relations between these two systems are examined.

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Robert Cowens's Hypothesis Concerning Minimally Unsatisfiable CNF's

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Robert Cowen has developed in [1] a method for generating minimally unsatisfiable CNF's that are difficult for automated theorem provers. "Minimal unsatisfiability" means that removing any one clause from the CNF results in a satisfiable CNF. A literal is a propositional variable or its negation. A clause is a disjunction of literals and a conjunctive normal form CNF is a conjunction of clauses. A k-CNF is a CNF where all it clauses have exactly k literals.

Let $g, k \geq 2$ and let be given propositional variables $p_1, p_2, \ldots, p_{(2k-2)g+1}$. They are partitioned, in order, into $g-1$ sets of size $2k-2$ and one "big set" of size $2k-1$. For each cell of the partition form all k-clauses from the variables in that cell and let $C[k,g]$ be the conjunction of all these k-clauses. Next let σ be a permutation on $\{1, \ldots, (2k-2)g+1\}$. Put $q_i = p_{\sigma(i)}$ and order the variables q, again, into $g-1$ sets of size $2k-2$ and one set of size $2k-1$. This time, for each cell of the partition, form all k-clauses from the negated variables in that cell and let $C_\sigma[k,g]$ be the conjunction of all these k clauses.

Theorem. $C[k,g] \wedge C_\sigma[k,g]$ is unsatisfiable.

Computer experiments showed that most of the formulas constructed above are minimally unsatisfiable (MU). Hence, Cowen conjectured that the MU percentages approach 100% for each k, as $g \to \infty$.

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On Non-unifiable Formulae in Chosen Boolean Algebras with Operators

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The notion of *unification* - making two symbolic expressions in a given system equal via substitution of their variables with other symbolic expressions - plays an important role in automated reasoning and logic programming. In case of logic, it means finding a substitution that transforms a formula into a tautology or theorem.

Among many questions regarding unification there are some that help establish more general properties of the given logic, one of them being the question of existence of formulae that cannot be unified, while not being a contradiction. If such formulae exist, they determine the form of passive rules of the given logic, helping to establish if the given logic is *structurally complete* (all admissible rules of the given system are derivable) or *almost structurally complete* (all non-passive admissible rules of the given system are derivable).

The form of non-unifiable formulae in $S4.3$ and its extensions was described by W. Dzik and P. Wojtylak in the following lemma:

Lemma. *If α is not unifiable, then $\alpha \vdash_{S4.3} \Diamond\beta \wedge \Diamond\neg\beta$ for some formula β.*

The purpose of the speech is to present the form of non-unifiable formulae in other systems, like the multi-modal $S5^2$ logic (product of two $S5$ systems, with two discernable operators - $\Diamond_0$, $\Diamond_1$, complete with respect to the variety of Boolean Algebra with Operators with two Henle-like operators) and relation algebras (also a Boolean Algebra with Operators, relevant in scope of arrow logic), namely:

Lemma. *Let α be a non-unifiable formula of $S5^2$, $Var(\alpha) \subseteq \{p_1, \ldots, p_n\}$. Then*

$$\alpha \vdash \bigvee_{i=1,\ldots,n} [(\Diamond_0 p_i \wedge \Diamond_0\neg p_i) \vee (\Diamond_0 p_i \wedge \Diamond_1\neg p_i) \vee (\Diamond_1 p_i \wedge \Diamond_1\neg p_i) \vee$$

$$\vee(\Diamond_1 p_i \wedge \Diamond_0\neg p_i) \vee (\Diamond_1\Diamond_0 p_i \wedge \Diamond_0\neg p_i) \vee (\Diamond_1\Diamond_0 p_i \wedge \Diamond_1\neg p_i) \vee$$

$$\vee(\Diamond_0 p_i \wedge \Diamond_1\Diamond_0\neg p_i) \vee (\Diamond_1 p_i \wedge \Diamond_1\Diamond_0\neg p_i) \vee (\Diamond_1\Diamond_0 p_i \wedge \Diamond_1\Diamond_0\neg p_i)];$$

Lemma. *The following formula is not unifiable and not contradictory in theory of relation algebras:*

$$(x \circ y) \wedge (-x \circ y) \wedge (x \wedge -y) \wedge (-x \wedge -y).$$

Lorenzen's Dialogue System in Natural Communication

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The goal of this talk is to propose the description of formal dialogues in terms of speech act theory. This is a part of the studies on a communication model according to which players can not only perform a dialogue without fallacies, but also discuss about the formal means of reasoning. To this aim two traditions in studying dialogue were brought together. First tradition was initiated by the dialogical logic DL introduced by Lorenzen [3], which allows the representation of formal dialogues in which the validity of argument is the topic discussed. Persuasion dialogue systems as specified by Prakken [4] represent second tradition which focuses on natural dialogues and examines processes typical for real-life communication such as e.g. informal fallacies [1].

Lorenzen-style dialogue games allow players to check the validity of the formulas in a dialogical way. Yet, those games are not designed for modelling real life communication, e.g. in the systems for natural dialogues players can use different speech acts for making their locutions (e.g. *claim*, *question*), while in Lonrenzen's game they can only attack and defend formulas (e.g. defend $A \wedge B$). This does not allow players to verify the validity of the formulas in the style of natural dialogues.

The solution proposed in the talk is to map the DL into a general language for natural dialogue systems [4]. To this end, dialogical logic was reconstructed by specifying three types of rules. The first type of rules, called locution rules, determines speech acts which players are allowed to perform during a dialogue game (e.g. an attack on the conjunction can be made by performing a speech act *question*); the second type of rules, called protocol, describes the interaction of the speech acts during the dialogue (after *question* φ (attack on conjunction), the player can perform *claim* φ) (defence of conjunction); the third type of rules, called effect rules, specifies effects of performing speech acts during the dialogue (e.g. after the player performs *claim* φ, the formula φ is added to his commitment store, i.e. to the set of proposition that he publicly declared as his beliefs).

Proposed description of dialogical logic in the general language of dialogue systems can be used for studying and systematising structures of sound informal communication as was suggested by Hodges [2]. In particular, it allows to embed Lorenzen's system in a protocol of dialogue systems designed for natural communication and, as a result, to detect formal fallacies committed during a natural dialogue. For example, reconstruction of dialogical logic was extended to include branching rules, and the protocol of LND (*Lorenzen Natural Dialogue*) system was introduced in [5]. During the LND dialogue, the players can decide to verify the validity of an inference scheme they used in argumentation by shifting from the natural dialogue to a formal (Lorenzen-style) dialogue both expressed in the same language.

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Tableau Algorithm with Inequality Solver for the Logic $\mathcal{M}_{\boldsymbol{K}}(\boldsymbol{\mathsf{E}}_n)$

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The algorithm presented in the talk attempts to externalise the algebraic part of the satisfiability checking for the modal logic K with global counting operators, here denoted by $\mathcal{M}_{\mathrm{K}}(\mathsf{E}_n)$ (for a thorough presentation of $\mathcal{M}_{\mathrm{K}}(\mathsf{E}_n)$ see [4]). Arithmetical reasoning involved by counting formulas $\mathsf{E}_{>n}\varphi$, $\mathsf{E}_{<n}\varphi$ is transferred to the **inequality solver** with the aim of improving the performance of the algorithm and **disposing of blocking mechanisms** which are normally necessary to ensure termination of the whole procedure. The solver exploits the notion of **atomic decomposition** defined as follows:

Let $\mathfrak{M} = \langle W, \{R_i\}_{i=1}^n, V\rangle$ be a model of a logic, $w \in W$, $R_{i_1}, \ldots, R_{i_m}$ be accessibility relations such that $\exists v(\langle w, v\rangle \in R_{i_m})$, $i = 1, \ldots, m$, and $\mathcal{P} = \mathcal{P}(\{R_{i_m}\}_{i=1}^m)$ Then by atomic decomposition of the set of all R_i-fillers for w we denote the set:

$$\{\bigcap_{R_i \in S} R_i(w) \setminus \bigcup_{R_j \notin S} R_j(w) \mid S \in \mathcal{P}\}.$$

Primarily, similar hybrid algorithm was established for the description logic $\mathcal{ALCQ}$ in [1] and further extended for the logic $\mathcal{SHOQ}$ in [2]. However, the logic $\mathcal{M}_{\mathrm{K}}(\mathsf{E}_n)$ involves global counting operators which $\mathcal{ALCQ}$ and $\mathcal{SHOQ}$ lack. This makes the calculus presented in the talk significantly differ from the original version and, to our best knowledge, novel in the literature.

Also, the following two theorems hold for the algorithm:

Theorem 1 (Soundness and Completeness) *The expansion rules from of the algorithm can be applied to a formula φ in such a way that they yield a complete and clash-free graph if, and only if there exists a tableau for φ.*

Theorem 2 (Termination) *The algorithm with the inequality solver for the logic $\mathcal{M}_K(\mathsf{E}_n)$ terminates.*

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The Very Beginnings of Set Theory in Poland. (A Preliminary Report)

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Set theory begun in 1874 with Georg Cantor's paper „Über eine Eigenschaft des Inbegriffs aller reellen algebraischen Zahlen" in which Cantor had established the uncountability of the set of reals by using their completeness under limits. It was over thirty years later when several Poles started their interest in *Mengenlehre*. Among them were Jan Łukasiewicz (1878–1956), Wacław Sierpiński (1882–1969), Zygmunt Janiszewski (1888–1920) and Stefan Mazurkiewicz (1888–1945).

My lecture will be devoted to a brief discussion of the historical context and the contents of the following first Polish contributions to set theory:

[1905/1907] „Co począć z pojęciem nieskończoności?" [What can we do with the notion of infinity] by J. Łukasiewicz;

[1908] "O pewnem twierdzeniu Cantora" [Sur un théorème de Cantor] by W. Sierpiński;

[1909] „Pojęcie odpowiedniości w matematyce" [Sur la notion de correspondence en mathématique] by W. Sierpiński;

[1910] „Nowy kierunek w Geometryi" [A new direction in geometry] by Z. Janiszewski;

[1910] "Sur la théorie des ensembles" by S. Mazurkiewicz;

[1912] *Zarys teorji mnogości* [Précies de la théorie des ensembles] by W. Sierpiński;

[1913] *Teorya mnogości. Część druga.* [Set theory. Part two] by W. Sierpiński.