Abstract Argumentation, Logic & Games *

Davide Grossi
University of Liverpool

Abstract argumentation [2] is the theory of graphs of the type $\langle A, \rightarrow\rangle$—called *attack graphs*—where A is a set and $\rightarrow$ a binary relation. These are high-level models of the sort of conflict that occurs in argumentation where arguments (the elements of A) interact by attacking one another (through the binary 'attack' relation $\rightarrow$). The theory has proven to be a prolific abstraction from which to study several aspects of argumentation. In these lectures I aim at providing an introduction to this theory highlighting its relationships with logic and games.

First, I will show how attack graphs can be used to provide mathematical definitions of criteria of the 'rationality' or 'justifiability' of sets of arguments, which I call *solution concepts* for attack graphs. The development of such criteria constitutes the main bulk of the theory of attack graphs as developed in the last two decades within the field of Artificial Intelligence (cf. [1] for a recent overview). In introducing these notions I will draw a parallel with modal logic [3], showing that many solution concepts of abstract argumentation can be naturally formalized in well-known modal languages by interpreting the modal diamond $\Diamond$ as expressing the property "there exists an attacker such that ...". A good example is the formula of the modal μ-calculus:

$$\mu p.\Box\Diamond p \tag{1}$$

which, for a given graph $\mathcal{A}$, expresses the smallest set p of arguments such that $p \leftrightarrow \Box\Diamond p$. That is, the smallest set p which is equal to the set of arguments whose attackers are attacked by some argument in p.

Second, I will move to a more dynamic and interactive view of a argumentation. Solution concepts can be viewed as specifications of abstract standards of proof, i.e., as specifications of the conditions under which an argument is 'satisfactorily' proven within a given graph. Two-players (proponent and opponent), zero-sum games with perfect information can be used as interactive procedures 'implementing' such standards of proof. More concretely, for a given solution concept S—like the one expressed by Formula (1)—one can define a game $\mathcal{G}_S$ satisfying the following property:

> *An argument a belongs to solution S if and only if the proponent has a winning strategy in the game $\mathcal{G}_S$ played starting with argument a.*

Third, I will address the issue of when two arguments, in two attack graphs, can be considered to be 'equivalent' [4]. In abstract argumentation arguments have no internal structure (no premisses, no conclusions), being just points in

*`abstractGrossi.tex`, *Friday 22nd March, 2013, 17:49.*

a network of attacks. So the notion of equivalence I propose is of a structural type and concerns the 'positions' that arguments occupy in their respective graphs. I will look at this intuition from two perspectives: a modal one, whereby two arguments are equivalent (w.r.t a given solution concept) whenever they satisfy the same modal formulae in an appropriate fragment of the basic modal language; a game-theoretic one, whereby two arguments are equivalent whenever a same player has a winning strategy of the same type (according to a precise definition of 'type') in the games for the two arguments. The two perspectives will be shown to be equivalent in the case of the solution concept expressed by Formula (1).

References

[1] P. Baroni and M. Giacomin. Semantics of abstract argument systems. In I. Rahwan and G. R. Simari, editors, *Argumentation in Artifical Intelligence*. Springer, 2009.

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[3] D. Grossi. On the logic of argumentation theory. In W. van der Hoek, G. Kaminka, Y. Lespérance, and S. Sen, editors, *Proceedings of the 9th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2010)*, pages 409–416. IFAAMAS, 2010.

[4] D. Grossi and D. Gabbay. When are two arguments the same? equivalence in abstract argumentation. Technical report, University of Liverpool, 2013.