THE FINITE AS THE NEW INFINITE, Mirna Džamonja, Institut de Recherche en Informatique Fondamentale (CNRS & Université de Paris-Cité, Paris, France)

The infinite has puzzled philosophers from the time of the Ancient Greece to our days. When it started mixing with mathematics, it brought huge controversies, mostly about the difference between the potential and the actual infinite. From its beginning with Cantor in the 19th century, set theory was associated with the revolutionary actual infinite, an infinite that can be named rather than viewed as a limit. This approach has had a lot of success in the sense of really understanding the large infinite sets, in set theory, in model theory and in many other areas where set theory has been applied. This includes topology, analysis and notably, philosophy through the work of Alain Badiou. However, something might have been lost in the process: the connection between the finite and the infinite. It seems that the combinatorial properties of the finite and of the infinite objects are so different, that there is no connection between them. After all, an infinite set can be bijective with a proper subset of itself, so how much worse can this get?

A recent trend in mathematics and in theoretical computer sciences is to bridge this gap by studying 'reasonable infinite objects'. This means the infinite objects which are built out of the finite ones in some precise way: as a Fraïssé limit, a result of some infinite automaton computation, a morass, an ultraproduct, a graphon… There have been several breakouts in making such connections, which we shall review. Then we shall talk about a possible connection with the most abstract of the infinite: abstract elementary classes.

Our thesis is that the study of the 'reasonable infinite' closes the controversy between the potential infinite built as a limiting structure of some finite processes and the actual infinite. It provides a third way.