

Definite Descriptions and Proof Theory

Andrzej Indrzejczak

This talk is concerned with two topics which usually do not come together. Definite descriptions are ubiquitous in natural languages and provide a very popular topic for the philosophy of language and logic. Since the publication of B. Russell famous paper “On Denoting”, many researchers provided deep and detailed studies of this phenomenon. One can mention here for example: Frege, Hilbert, Bernays, Carnap, Quine, Rosser and Hintikka - just a few eminent scholars from the earliest stage of investigation. In the second half of XX century a lot of new proposals were added which were developed on the basis of several nonclassical logics. Yet, despite the long history and variety of proposed solutions we can hardly say that some approaches may be treated as obvious or commonly acceptable. In fact, proper definite descriptions having a unique designatum, are rather not problematic, in contrast to those which fail to designate, called improper (or unfulfilled) definite descriptions. The famous Russellian “the present King of France”, is of this kind but even innocent-looking “the son of Jack” may be problematic in case Jack has no son, or more than one.

In the first part we survey the most important and interesting theories of definite descriptions with focus on their advantages and disadvantages. In the context of classical logic we will focus on the well known reductionist approach of Russell and the chosen object theory of Frege and its formalization provided by Kalish and Montague. The former shows how to get rid with definite descriptions (and individual names in general) and is one of the most popular solution, however at the costs of many drawbacks of different kind. The latter is one of the four approaches sketched by Frege which treat descriptions as genuine names. It is formally convenient but has its own disadvantages. Next, we describe some of the theories developed in the framework of free logic by Lambert, Scott, van Fraassen and others. In general, free logic is much better tool for developing a satisfactory theory of definite descriptions but some of them are too weak. We finish this part of the presentation with three theories developed on the ground of modal logic by Thomason and Garson, Goldblatt, Fitting and Mendelsohn. It seems that relational semantics with varying domains and nonrigid terms offers even better framework for definite descriptions, yet the presented approaches are significantly different in many respects.

The second part will be devoted to presentation of proof theory for definite descriptions. In fact, a modern proof-theoretic apparatus was not applied in this field so far. We hope to show that the application of techniques taken from structural proof theory may shed a new light on the good and bad sides of different approaches to definite descriptions. Sequent calculi for two different theories of definite descriptions will be examined. The first is equivalent to Kalish and Montague version of Fregean theory developed in the setting of classical logic. The second, equivalent to Thomason and Garson’s theory, is for modal system with rigid and nonrigid terms based on free logic. We focus

on proof theoretic features and problems with their application to description-operator as additional constant. For both theories we prove cut elimination theorem, discuss some of its properties, and – in the latter – some extensions by extra rules. We also discuss problems which makes some other theories of definite descriptions more complicated to deal with in proof theory.

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Department of Logic
University of Łódź
Lindleya 3/5
90–131 Łódź
e-mail: indrzej@filozof.uni.lodz.pl