Discrete dualities for some lattice-based algebras

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Discrete duality is a relationship between classes of algebras and classes of frames (relational systems). If Alg is a class of algebras and Frm is a class of frames, establishing a discrete duality between these two classes requires the following steps:

1. With every algebra L from Alg associate a canonical frame Cf(L) and show that it belongs to Frm.
2. With every frame X from Frm associate a complex algebra Cm(X) and show that it belongs to Alg.
3. Prove two representation theorems:
(3a) For each algebra L in Alg there is an embedding h: L->CmCf(L),
(3b) For each frame X in Frm there is an embedding k: X->CfCm(X).

In case of distributive lattices canonical frames correspond to dual spaces of algebras in the Priestley-style duality, however in case of discrete duality they are not endowed with a topology and hence may be thought of as having a discrete topology. Similarly, in case of distributive lattices complex algebras of canonical frames correspond to canonical extensions in the sense of Jonsson-Tarski.

In my talk discrete dualities for the following two lattice-based classes of algebras will be discussed: a not first order-definable class of algebras based on Boolean algebras and a non-canonical class of algebras based on distributive lattices. Furthermore, it will be shown that, in general, not necessarily distributive lattices do not admit discrete duality (theorem 3b does not necessarily hold).