Relating Logics:
Theory and Applications

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When examining reasoning in logic, we usually consider affirming a logical relationship between the premises and the conclusion so that any situation which assigns a meaning of true to the premises, must assign a meaning of true to the conclusion. However, in many cases, there are non-logical relationships that can greatly contribute to the recognition of reasoning.

Such relationships can influence the logical value of a sentence (the meaning of a logical constant) but are different from that determined by only using the logical values of its components. Consider the following standard example:

$$(\diamondsuit)\qquad \frac{\begin{array}{c}\text{If the thief tries to rob your house, you call the police}\\ \text{If you call the police, the thief starts to run away}\end{array}}{\text{If the thief tries to rob your house, the thief starts to run away}}$$

The inference ($\diamondsuit$) can be seen as an instance of the transitivity of classical material implication if we read "if... then..." as the material implication. Thus, in a classical setting, in which the truth values of the implications are determined only by the truth values of the subformuls, the inference ($\diamondsuit$) is correct. But clearly, the conclusion of ($\diamondsuit$) is bizarre as there is no direct logical connection between the thief trying to rob your house and the thief's running away! Thus the classical material implication is unable to account for the causal relationship between the thief's actions and your actions which leads the thief to run away. Our challenge is to accommodate extra, non-logical, relationships such as "causation", to block ($\diamondsuit$). But "causation" is just one example of a non-logical relationship.

In many inferences, similar relationships of a non-logical nature also appear. These include not only causal relationship but also temporal, analytical, content-based, preferential, structural relationships etc. These are intensional relationships, because they are irreducible to the properties of their elements. To express additional intensional relationships, new connectives can be added to the language, which, besides finding dependencies between logical values, also allow for stating the existence of other non-logical relationships. Technically, in interpreting such connectives in the model, in addition to the logical value of individual sentences, we include the valuation of a pair of sentences. The connectives of this kind are called *relating connectives*. The basic idea behind relating connectives is that the logical value of a given complex proposition is dependent on two factors:

(i) the logical values of the main components of the compound proposition

(ii) a valuation of the relation between these components.

The latter element is a formal representation of an underlying intensional relationship that exists between the main components of the proposition, as in (i), but which may not depend upon their logical values. Thus (i) and (ii) together give rise to a connective which is non-extensional. Including such non-extensional relating connectives in the language allows us to represent non-logical relationships, such as causation, in the syntax of our logic. We can then use the traditional connectives to form extensional combinations of these non-extensional relationships. That is, if we define a logic with relating connectives, we are in fact able to cover some relationships that are not extensional; however, the logic itself is extensional.

Relating Logic is a logic of relating connectives (just as Modal Logic is a logic of modal connectives). The basic approach to Relating Logic is two-valued and with one relation in a model to interpret relationships between sentences. However, more complicated implementations are also possible.

In the presentation, we would like to discuss the following problems related to relating logic (with some selected references):

1. motivations
2. the outline of history ([4], [10], [7], [5], [6], [30], [31], [36], [37], [14], [29])
3. the proper definition ([21])
4. possible semantic structures ([21], [15])
5. the fundamentals of proof-theory ([2], [19], [3], [34], [20], [33], [1], [9], [16], [35], [18], [28], [27], [26])
6. applications of relating logic ([32], [8], [23], [22], [13], [12], [24], [11], [17], [19])
7. the process of institutionalization ([25]).

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