Model Theory, a survey with particular emphasis on topological methods

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Model theory was founded by Abraham Robinson and Alfred Tarski in mid-twentieth century. Over the years it grew in volume and depth. Now it is an established part of mathematics and mathematical logic. In the talk I will survey the development of model theory and will present some recent ideas in it, related to topological dynamics.

In the development of model theory there are some major stages. The turning point was the Morley categoricity theorem (1964), answering the conjecture of Jerzy Łoś. Gradually model theory incorporated more and more methods from various areas of mathematics and integrated with them. Also the nature of model theory was changing. So Keisler and Chang around 1970 gave a succint definition of model theory in the form of equation:

Model Theory = Universal Algebra + Logic

25 years later Wilfrid Hodges changed it into:

Model Theory = Algebraic Geometry – Fields

The change of definition reflects the change in model theory over those years, due to the contributions of several leading researchers, most notably Saharon Shelah, Boris Zilber, Anand Pillay, Ehud Hrushovski and many others. In the talk I will explain these definitions in greater detail.

Topological methods were present in model theory already in its early stages. In fact, they played a prominent role in the proof of the Morley categoricity theorem. The major idea there was to measure definable sets in models by means of Morley rank that is a variant of Cantor-Bendixson rank in the space of types. Later Shelah invetigated variants of Morley rank by combinatorial methods that led to his discovery of forking and development of geometric model theory.

In the years 2000 I suggested applying in model theory some stronger and more precise topological tools, coming from topological dynamics. These tools turned out to be useful to investigate deep nature of theories, like strong types, Galois groups and Borel complexity of various model-theoretic equivalence relations. I will survey this development.