## Manuel António Martins Modal structures over lattices

In this talk, we will present a modal framework that integrates modal worlds governed by different logical systems, each defined by sublattices within a shared lattice structure.

Dealing with worlds operating in different logic systems poses a challenge for the standard principles in normal modal logic. When evaluating a modal sentence  $\Box A$  in such setting, it is not sufficient to inspect the truth of A in accessed worlds (possibly in different logics). Rather, ways of transferring more subtle semantic information between logical systems must be established.

We will introduce modal structures that accommodate communication between logic systems by fixing a common lattice *L* that contains as sublattices the semantics of each world. Our approach redefines necessity and possibility, based on a comparative analysis of assignment values across accessible worlds and the base lattice. The value of a formula  $\Box A$  in a world with lattice *L*' will be defined in terms of the values of *A* in accessible worlds relativized to *L*' using the common order of *L*. We will explore simple instances where a formula can be said to be necessary/possible even though all the accessible worlds falsify it.

We will also examine frames that characterize dynamic relations between logic systems: classically increasing, classically decreasing, and dialectic frames. To exemplify the kind of issue one should face in this framework, we formalize the semantics of considering worlds operating in classical logic or logic of paradox.