UNIVERSAL HOMOGENEOUS STRUCTURES

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A structure U is *universal* for a given class of structures $\mathfrak{K}$ if $U \in \mathfrak{K}$ and every structure from $\mathfrak{K}$ embeds into U. Universality is a well-known concept in several areas of mathematics. Another property, often making a universal structure unique up to isomorphism, is *homogeneity*. Namely, given a class $\mathfrak{S}$ whose objects will be called *small*, we say that a structure U is $\mathfrak{S}$-*homogeneous* if every isomorphism between its small substructures (i.e. substructures belonging to $\mathfrak{S}$) extends to an automorphism of U. This notion usually makes sense if U can be "reached" from the class $\mathfrak{S}$, for example, when U is the union of a chain of small substructures. This gives rise to the category-theoretic concept of a *generic sequence*, a functor from $\mathbb{N}$ into $\mathfrak{S}$ which has special properties leading to a universal homogeneous object in the bigger category $\mathfrak{K}$.

We shall describe some aspects of the category-theoretic framework for universal homogeneous structures, emphasizing on the uniqueness problems, solved by a *back-and-forth argument*, which in turn can be viewed as a game between two structures of the same type. We shall also discuss an *approximate back-and-forth method*, in the context of categories enriched over metric spaces.

Historically, the first work on universal homogeneous structures in model theory, exploring the back-and-forth argument, was made by Roland Fraïssé [1], later continued by several authors. We will present some some material from our works [2, 3, 4].

References

[1] Fraïssé, R., *Sur quelques classifications des systèmes de relations*, Publ. Sci. Univ. Alger. Sér. A. **1** (1954) 35–182

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