Admissible rules and their complexity

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A rule is admissible in a given logic if the set of tautologies of the logic is closed under the rule, or equivalently, if the addition of the rule to the logic does not create any new tautologies. The concept also has a natural generalization to multiple-conclusion rules. Algebraically speaking, admissible rules of a nicely algebraizable logic correspond to quasi-identities (or clauses, in the multiple-conclusion case) valid in free algebras of the corresponding variety.

Admissibility is closely related to unification: equational unification over the equational theory of the corresponding variety can be stated in terms of the logic, namely that a unifier of a formula is a substitution that turns it into a tautology. This makes it a special case of inadmissibility (for rules with inconsistent conclusions). Unification can be generalized to the disunification problem, in which case it encompasses inadmissibility of multiple-conclusion rules.

It is standard in unification literature to work in the expansion of the given equational theory by free constants. We may do this in the logical setting as well, leading to admissibility with a new kind of atoms—variously called parameters, constants, coefficients, or metavariables—that are required to be left intact by substitutions.

In this talk, we are going to investigate admissibility with parameters in transitive modal logics (extensions of **K4**). We will be primarily interested in logics satisfying suitable frame extension properties (cluster-extensible logics), but we will also look at other logics, in particular logics of finite depth and width.

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We shall be interested for example in semantic descriptions of admissible rules, constructions of complete sets of unifiers, and axiomatization of admissible rules by means of bases. We will pay special attention to algorithmic complexity questions, such as what is the computational complexity of admissibility in various logics; as we will see, they are intimately connected to structural properties of the logics.

The talk is mostly based on [1, 2].

References

[1] Emil Jeřábek, *Rules with parameters in modal logic I*, Annals of Pure and Applied Logic 166 (2015), no. 9, pp. 881–933.

[2] ———, *Rules with parameters in modal logic II*, in preparation, 2019.