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(Forty Years of Structural Completeness)

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Nota redakcyjna. Symbol (EN) przy nazwisku referenta znaczy, że (w razie obecności zainteresowanych gości zagranicznych) referat będzie przedstawiony w języku angielskim. Symbol (PL) znaczy, że referat będzie zaprezentowany w języku polskim.
Editorial note. (EN) means that the talk is presented in English, (PL) — in Polish.

Algorithmic structural completeness

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We shall introduce the notion of the algorithmic structural completeness, which is some kind of the notion of structural completeness introduced by W. A. Pogorzelski. At first we shall consider the substitution rule, finitary rule and the notion of structural rules in algorithmic logic. Next we shall study interrelation between all structural, finitary and admissible rules on one hand, and derivable rules on the other hand. We say that the consequence operation C is algorithmically structurally complete iff every structural, finitary and admissible rule is derivable in C. We shall prove that the consequence operation C_R, strengthened by the substitution rule, i.e. C_R^*, is incomplete. The same result can be obtained for the consequence with non–deterministic programs $C_\Box$, $C_\Box^*$, and the consequence of algorithmic logic with quantifiers C_E. Moreover, we shall show that the consequences with identity: $C_R^=$, $C_R^{*=}$, C_E^*, $C_\Box^*$ are incomplete. At last we shall prove that the consequence C_R and $C_R^=$ are not algorithmically structurally complete but for example the consequence C_R^* of algorithmic logic is algorithmically structurally complete.

Methods of reduction of singular graphs

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The definition of a singular graph, which is typically algebraic, is connected with its adjacency matrix which is singular. The aim of introducing methods of reducion of singular graphs is to describe a structure of these graphs in a language of the graph theory.

The presented approach is known from an article of H.M.Rara [3]. She introduces methods which can be applied instead of complicated algebraic calculations. These methods involve removing vertices or edges and they do not change the determinant of an adjacency matrix or change it in a certain way.

I will introduce new methods of reduction, which are applicable to a broader class of graphs. The methods are identifying P_3 paths, contracting P_5 paths, removing planar subgrids and reduction of graphs circumscribed on cycles. The methods are based on the following theorem:

Theorem 1. Let $P_5 = [v_1, v_2, v_3, v_4, v_5]$ be an induced subgraph of G, $deg_G(v_2) = deg_G(v_4) = 2$ and $N_G(v_1) \cap N_G(v_5) = \emptyset$. If G^* is a graphs obtained from G by identifying vertices v_2 and v_4, v_1 and v_5, then

$$detA(G^*) = -detA(G).$$

An interesting proof of this theorem is based on a combinatorial formula of F.Harary [2].

$$detA(G) = \sum_{\Gamma \in S} (-1)^{|V(\Gamma)| - c(\Gamma)} \cdot 2^{|E(\Gamma)| - |V(\Gamma)| + c(\Gamma)},$$

where S denotes a set of all sesquivalent spanning subgraphs of graph G.

The main theorem is applied in the solution of the problem of singularity of planar grids. The method of contracting P_5 path together with Rara's methods suffices to give the complete solution. The algorithm for calculating the determinant of the adjacency matrix of a planar grid is similar to the Euclidean algorithm, hence it is more effective then calculating the determinant in a standard way. The recursive formula is a consequence of a more general fact.

Theorem 2. If $G = H \cup (P_n \times P_{n+1})$ and $H \cap (P_n \times P_n) = \emptyset$, then $detA(G) = (-1)^{\lfloor (n+1)/2 \rfloor} \cdot detA(G - (P_n \times P_{n+1}))$

The theorem can be applied to certain graphs which are not planar grids.

In the last part of my speech I will introduce a definition of a graph circumscribed on a cycle and a method of removing subcycles from this kind of graphs. In this method a relatively big number of vertices and egdes is removed and a degree of any vertex does not become greater. It is essential, because in all presented methods we assume that there are some vertices with small degrees. To find methods which can be applied to graphs with all degrees greater than two is an open problem .

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The Rational Postulates for Dynamic Epistemic Logics

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Introduction

Determination of administering knowledge by a rational agent acting in compliance with certain protocols within a real system of interaction, a system of communicating, requires postulating rationality of acting by the agent, as well as postulating restriction of this rationality appropriately to the real actions.

A description of administering knowledge by the rational agent in compliance with DEL protocols was presented in [1], [2], [3], [4], [5], [6].

The notion of bounded rationality was introduced by H.A. Simon in the 20^{th} century [8], who proposed to distinguish: (1) a set of agents, (2) a set of behaviour alternatives, (3) a set of outcomes of choice among the behaviour alternatives, and (4) a set of order of preferences for making choices of behaviours. According to him, an agent who is invested with “perfect rationality” possesses a full knowledge of distinguished sets, whereas an agent with bounded rationality, in contrast, might not know all alternatives, nor need it know the exact outcome of each; what is more, such an agent might lack a complete preference ordering which is indispensable to obtain the outcomes. We assume that establishing a proper protocol for the agent with bounded rationality leads to linking the real system of interactions with relevant types of communicating. Thanks to fixing a type of communicating, it becomes possible to assign a suitable class of Kripke models for dynamic epistemic logic (*DEL*) to this type. In a real system of interactions, a set of rational agents is limited to a set of subjects of such actions of communicating as: production, rendering available and possession or allocating the objects distinguished by agents. The objects are products of the action of communicating. The products are divided into resources, goods, services and values arising in consequence of actions realized within the real system of interactions. The order of preferences for making choices of actions necessary to obtain certain products expected by agents as a result of a given action, is determined by real conditions that establish the beginning and the end of this action.

In this paper we will present rational postulates which allow executing a certain typology of real systems of interactions. The first group of the postulates allows identifying the dynamics of knowledge within information systems. The other group of postulates serves to identify real systems of interaction.

Postulates concerning information networks

P1. ***Information about an object O*** *(in short:* ***information****) is a set of data about the object* ***O****, or more precisely — a set of data* ***identifying*** *the object* ***O*** *or any object being part of the object* ***O****.*
Pieces of information are ***indiscernible*** *when they identify the same object. Identification of an object* ***O*** *groups information about the object* ***O****, thus it groups indiscernible pieces of information.*

P2. ***Reference of information about objects*** *is an ordered set of information about objects. The first piece of information in the given reference identifies the object which the last piece of information is about in this reference.*

P3. ***Information transmission and processing****. References on elements determining the same object* ***transmit information*** *on this object. The first element of this reference is a piece of* ***input*** *information, while the last one —* ***output*** *information. References not only transmit information,* ***process information****: the first piece of information — the* ***input*** *one — into the last piece of reference information — the* ***output*** *one. Information transmission is a particular case of information processing. We call the object which assigns ordered systems of objects to references* ***information channel****. The first object of the system determined by the information channel is the* ***input of the channel****, while the last object of this system — the* ***output of the channel****. The information channel processes information if each n-th piece of information of reference determines the n-th object of the system of objects ordered by this channel system of objects. We call the collection of information channels an* ***information network****.*

Postulates of the real interactivity system

P4. ***System of communicating*** *is a system of human activity and — at the same time — an information network defined for sets of objects that are subjects or objects of production, rendering available and possession or allocation of resources, goods, services and values being effects of people's activity within the system. Nevertheless, each input and output of this information network is a subject of production, rendering available, possession or allocation.* ***Knowledge*** *is a piece of information processed in a system of communicating. A set of data on the subject, relating to the kind of knowledge that the subject possesses, is understood to be information about the subject.* ***Communicating*** *is processing information within the system of communicating. The following types of communicating are differentiated as determined by their dominance at the input and at the output of the system of such* ***attributes*** *as production, rendering available, possession* *or allocation:*

Interactive — *production and rendering available;*
Verbal — *possession and allocation;*
Public — *allocation;*
Private — *possession;*
Static — *rendering available and allocation;*
Dynamic — *production and possession;*
Decision-making — *production;*
Discursive, based of convictions — *rendering available;*
Intelligent — *production and allocation;*
Behavioural — *rendering available and possession.*

Table 1. Types of communicating determined by input/output attributes

input \ output	***Production***	***Rendering available***	***Possession***	***Allocation***
Production	*Decision-ma*	*Interactive*	*Dynamic*	*Intelligent*
Rendering available	*Interactive*	*Discursive*	*Behavioural*	*Static*
Possession	*Dynamic*	*Behavioural*	*Private*	*Verbal*
Allocation	*Inteligent*	*Static*	*Verbal*	*Public*

The opposition of the types is represented by means of the following juxtaposition of opposing colours:

(■,■), (■,■), (■,■), (■,■), (■,■).

P5. ***Epistemic agent*** (*in short: the* ***agent***) *is an object at the input or output of a system of communicating.*

P6. *The following* ***aspects of knowledge*** *are distinguished:*

Common-sense knowledge — applied knowledge and habitual knowledge (operator of assertiveness);
Emotive knowledge — knowledge related to feelings (operator of feeling);
Sensual knowledge — sensual knowledge (operator of perception);
Empirical knowledge — knowledge attained in the way of experiences (operator of experience);
Rational knowledge — knowledge attained through thinking and reasoning (operator of understanding).

Postulates of administering knowledge

P7. ***Administering knowledge*** *is processing knowledge within information channels, in which there occurs communicating. It follows from the definition of the information channel and determining the agent that the input and the output of the information channel is a certain agent. Information channels which compose administering the knowledge are* ***dispositions of knowledge****. The fact that* *the agent knows something, encodes, decodes and represents knowledge, acquires knowledge, announces knowledge, is convinced (believes in something), is interpreted as making use of suitable dispositions of knowledge by the agent: possessing knowledge, encoding, decoding, etc. We call the whole of administering the knowledge the* ***state of administering knowledge*** *(in short: the* ***state****).*

P8. *In order to administer knowledge, a group of agents who realize a certain type of communicating accept an appropriate* ***protocol of processing knowledge*** *that implements this type of communicating.*

The presented rational postulates for *DEL* allow establishing sets S of all states of administering knowledge within the selected real system of interaction. Let P be a set of atomic propositions expressing knowledge, and A be a set of agents. Relations of using — by agents — information channels, are then determined by the mapping $R_A : A \to \mathcal{P}(S \times S)$, and also the mapping $V^P : P \to \mathcal{P}(S)$ is known as it determines a set of states, in which for the given atomic proposition there occurs communicating that processes this atomic proposition. *A Kripke model for DEL is a structure* $M = \langle S, R_A, V^P \rangle$ (cf. [5]). Let us note that determining the real system of interaction is executed in a certain relational data basis. The above-mentioned postulates allow identifying attributes of this data basis and values of these attributes. This aspect of the research offers the possibility, in the case of vagueness in determining results of communicating, of applying the method of rough sets in Pawlak's sense [7] to describing this communicating. Administering sources of knowledge in social and economic systems of managing knowledge can be described in this sense as relational data bases, and then — by means of these bases — certain classes of Kripke models can be fixed for *DEL*. A result of such research can be fixing of this type of *DEL* for the given system of managing knowledge.

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Passive Structural Completeness

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Passive structural completeness ($\mathcal{PSC}$ for short), is satisfied for a logic if all rules with non-unifiable premises are derivable. Such rules were studied by V. Rybakov in [2] and this property was introduced (in an algebraic context) by A. Wroński under the name of non-overflow completeness, see e.g. his presentation *Overflow rules and a weakening of structural completeness* at the 51st Conference on the History of Logic, Kraków, 2005). This contribution is based on joint work with G. Metcalfe [1]. Unlike structural completeness $\mathcal{SC}$, $\mathcal{PSC}$ is a more 'stable' notion when one moves around the logical landscape; in particular, it is preserved in extensions and (special) fragments of a logic with $\mathcal{PSC}$. As $\mathcal{PSC}$ is clearly implied by $\mathcal{SC}$ (but not vice–versa as demonstrated by e.g. intuitionistic logic), disproving $\mathcal{PSC}$ provides a (usually) simple way of disproving $\mathcal{SC}$. In this talk we present basic definitions, separating examples, and properties of logics with $\mathcal{PSC}$. We also explore the relationship between $\mathcal{PSC}$ and a certain form of completeness theorem (w.r.t. the free algebra).

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Finitely generated quasivarieties and irreducibility of congruences

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§1. Triangular irreducibility

Let $\mathbf{Q}$ be a quasivariety, $\boldsymbol{A} \in \mathbf{Q}$ and $a_1, \ldots, a_m$ a finite sequence of elements of $\boldsymbol{A}$ (possibly with repetitions) of length $m \geq 3$. The following sequence of $\mathbf{Q}$-congruences on $\boldsymbol{A}$:

$$\langle \Theta^{\boldsymbol{A}}_{\mathbf{Q}}(a_i, a_j) : 1 \leq i < j \leq m \rangle$$

of length $m(m-1)/2$ is called the *triangular table* of relatively principal congruences corresponding to $a_1, \ldots, a_m$. The $\mathbf{Q}$-congruence $\bigcap_{1 \leq i < j \leq m} \Theta^{\boldsymbol{A}}_{\mathbf{Q}}(a_i, a_j)$ is called the *triangular intersection*.

Definition 1.1. Let $m \geq 3$ be a natural number. Suppose $\boldsymbol{A} \in \mathbf{Q}$ and $\Phi \in Con_{\mathbf{Q}}(\boldsymbol{A})$. Φ is said to be *m-triangularily irreducible* in the lattice $\boldsymbol{Con}_{\mathbf{Q}}(A)$ if for every sequence $a_1, \ldots, a_m$ of elements of $\boldsymbol{A}$ of length m, if $\bigcap_{1 \leq i < j \leq m} (\Theta^{\boldsymbol{A}}_{\mathbf{Q}}(a_i, a_j) +_{\mathbf{Q}} \Phi) = \Phi$ then $a_i \equiv a_j(\Phi)$ for some i and j, $1 \leq i < j \leq m$.

In particular, $\mathbf{0}_{\boldsymbol{A}}$ is m-triangularily irreducible in $\boldsymbol{Con}_{\mathbf{Q}}(\boldsymbol{A})$ iff for every sequence $a_1, \ldots, a_m$ of elements of $\boldsymbol{A}$ of length m, if $\bigcap_{1 \leq i < j \leq m} \Theta^{\boldsymbol{A}}_{\mathbf{Q}}(a_i, a_j) = \mathbf{0}_{\boldsymbol{A}}$, then $a_i = a_j$ for some i and j, $1 \leq i < j \leq m$.

$\mathbf{Q}_{m-TRI}$ is the class of all members $\boldsymbol{A}$ of $\mathbf{Q}$ for which the congruence $\mathbf{0}_{\boldsymbol{A}}$ is m-triangularily irreducible in $\boldsymbol{Con}_{\mathbf{Q}}(\boldsymbol{A})$. Trivially $\mathbf{Q}_{RFSI} \subseteq \mathbf{Q}_{m-TRI}$. It follows that every quasivariety $\mathbf{Q}$ has enough algebras $\boldsymbol{A}$ with m-triangularily irreducible zero congruences $\mathbf{0}_{\boldsymbol{A}}$ in the sense that *every* algebra of $\mathbf{Q}$ is isomorphic with a subdirect product of a family of algebras from the class $\mathbf{Q}_{m-TRI}$, for each $m \geq 3$.

Let $\mathbf{K}$ be a class of algebras. $\boldsymbol{P_S}(\mathbf{K})$ denotes the class of isomorphic copies of subdirect products of families of algebras from $\mathbf{K}$.

Corollary 1.2. *Let $\mathbf{Q}$ be a quasivariety. For every positive integer m, $m \geq 3$,*

$$\mathbf{Q} = \boldsymbol{SP}(\mathbf{Q}_{m-TRI}) = \boldsymbol{P_S}(\mathbf{Q}_{m-TRI}).$$

Theorem 1.3. *Let $\mathbf{Q}$ be an arbitrary quasivariety and $m \geq 3$ a positive integer. The following conditions are equivalent:*

(i) $\mathbf{Q}$ *is generated by a finite class of algebras each of which has at most $m-1$ elements.*

(ii) *For every algebra* $\boldsymbol{A} \in \mathbf{Q}$ *and for any sequence* $a_1, \dots, a_m$ *of elements of* $\boldsymbol{A}$ *of length* m *(possibly with repetitions) it is the case that* $\bigcap_{1 \le i < j \le m} \Theta^{\boldsymbol{A}}_{\mathbf{Q}}(a_i, a_j) = \mathbf{0}_{\boldsymbol{A}}$.

(iii) *For every algebra* $\boldsymbol{A} \in \mathbf{Q}$, *for any* $\Phi \in Con_{\mathbf{Q}}(\boldsymbol{A})$, *and any sequence* $a_1, \dots, a_m$ *of elements of* $\boldsymbol{A}$ *of length* m *it is the case that* $\bigcap_{1 \le i < j \le m}(\Phi +_{\mathbf{Q}} \Theta^{\boldsymbol{A}}_{\mathbf{Q}}(a_i, a_j)) = \Phi$.

(iv) *For any sequence* $x_1, \dots, x_m$ *of* m *different free generators of the free algebra* $\boldsymbol{F} := \boldsymbol{F}_{\mathbf{Q}}(\omega)$ *and any* $\Phi \in Con_{\mathbf{Q}}(\boldsymbol{F})$, $\bigcap_{1 \le i < j \le m}(\Phi +_{\mathbf{Q}} \Theta^{\boldsymbol{F}}_{\mathbf{Q}}(x_i, x_j)) = \Phi$.

Note. 1. Implication (i) $\Rightarrow$ (ii) also holds for finitely generated *varieties*, i.e., for varieties $\mathbf{HSP}(\mathbf{K})$, where $\mathbf{K}$ is a finite set of finite algebras — see McKenzie [1987], Thm. 2.13.

Corollary 1.4. *Suppose that* $\mathbf{Q}$ *is a quasivariety generated by a finite class of algebras each of which has at most* $m-1$ *elements,* $m \ge 3$. *For any algebra* $\boldsymbol{A} \in \mathbf{Q}$, *the following conditions are equivalent:*

(1) $\boldsymbol{A} \in \mathbf{Q}_{m-TRI}$,

(2) $\boldsymbol{A}$ *has at most* $m-1$ *elements.*

Theorem 1.5. *Suppose that* $\mathbf{Q}$ *is a quasivariety generated by a finite class* $\mathbf{K}$ *of algebras each of which has at most* $m-1$ *elements,* $m \ge 3$. *Then* $\mathbf{Q}_{m-TRI}$ *is a finitely axiomatizable class.*

§2. Quasivarieties with equationally definable m–triangular meets of (relatively) principal congruences

Proposition 2.1. *Let* $m \ge 3$ *be a natural number and* $\Lambda = \Lambda(x_1, x_2, \dots, x_m, \underline{u})$ *a set of equations in variables* $x_1, x_2, \dots, x_m$ *(and possibly some parameters* $\underline{u}$*). For any quasivariety* $\mathbf{Q}$ *the following conditions are equivalent:*

(1) *For all* $\boldsymbol{A} \in \mathbf{Q}$ *and for any sequence* $a_1, \dots, a_m$ *of elements* $\boldsymbol{A}$ *of length* m,

$$\bigcap_{1 \le i < j \le m} \Theta^{\boldsymbol{A}}_{\mathbf{Q}}(a_i, a_j) = \Theta^{\boldsymbol{A}}_{\mathbf{Q}}((\forall \underline{e})\Lambda^{\boldsymbol{A}}(a_1, a_2, \dots, a_m, \underline{e}));$$

(2) *For all* $\boldsymbol{A} \in \mathbf{Q}$ *and for any sequence* $a_1, \dots, a_m$ *of elements* $\boldsymbol{A}$ *of length* m,

$$\bigcap_{1 \le i < j \le m} \Theta^{\boldsymbol{A}}_{\mathbf{Q}}(a_i, a_j) = \mathbf{0}_{\boldsymbol{A}} \ \Leftrightarrow \ \boldsymbol{A} \models (\forall \underline{u}) \bigwedge \Lambda^{\boldsymbol{A}}(x_1, x_2, \dots, x_m, \underline{u})[a_1, a_2, \dots, a_m];$$

(3) $\mathbf{Q}_{m-TRI}$ *validates the first–order sentence*

$$(\forall x_1)(\forall x_2) \dots (\forall x_m)((\forall \underline{u}) \bigwedge \Lambda(x_1, x_2, \dots, x_m, \underline{u}) \Leftrightarrow \bigvee \{x_i \approx x_j : 1 \le i < j \le m\})$$

Definition 2.2. Let $m \ge 3$ be a natural number. A quasivariety $\mathbf{Q}$ is said to have *equationally definable m-triangular meets* of (relatively) *principal congruences* (m–EDTPM, for short) if there is a set of equations $\Lambda(x_1, x_2, \dots, x_m, \underline{u})$ such that $\mathbf{Q}$

satisfies any of the equivalent conditions of the above proposition.

Theorem 2.3. *Let $m \geq 3$ be a natural number. For any quasivariety $\mathbf{Q}$ the following conditions are equivalent:*

(i) *$\mathbf{Q}$ has m–EDTPM.*

(ii) *For every algebra $\boldsymbol{A} \in \mathbf{Q}$, for any sequence $a_1, \ldots, a_m$ of elements of $\boldsymbol{A}$ of length m and any congruence $\Phi \in Con_{\mathbf{Q}}(\boldsymbol{A})$, $\Phi +_{\mathbf{Q}} \bigcap_{1 \leq i < j \leq m} \Theta^{\boldsymbol{A}}_{\mathbf{Q}}(a_i, a_j) = \bigcap_{1 \leq i < j \leq m} (\Phi +_{\mathbf{Q}} \Theta^{\boldsymbol{A}}_{\mathbf{Q}}(a_i, a_j))$.*

(iii) *For any sequence $x_1, \ldots, x_m$ of m different free generators of the free algebra $\boldsymbol{F} := \boldsymbol{F}_{\mathbf{Q}}(\omega)$ and any congruence $\Phi \in Con_{\mathbf{Q}}(\boldsymbol{F})$, $\Phi +_{\mathbf{Q}} \bigcap_{1 \leq i < j \leq m} \Theta^{\boldsymbol{F}}_{\mathbf{Q}}(x_i, x_j) = \bigcap_{1 \leq i < j \leq m} (\Phi +_{\mathbf{Q}} \Theta^{\boldsymbol{F}}_{\mathbf{Q}}(x_i, x_j))$.*

Corollary 2.4. *Let $m \geq 3$ be a natural number. Let $\mathbf{Q}$ be a quasivariety with m–EDTPM with respect to a set of equations $\Lambda = \Lambda(x_1, x_2, \ldots, x_m, \underline{u})$. Then for any algebra $\boldsymbol{A} \in \mathbf{Q}$, the following conditions are equivalent:*

(1) *$\boldsymbol{A} \in \mathbf{Q}_{m-TRI}$.*

(2) *For every sequence $a_1, \ldots, a_m$ of elements of $\boldsymbol{A}$, if $\Theta^{\boldsymbol{A}}_{\mathbf{Q}}(\Lambda(a_1, a_2, \ldots, a_m, \underline{e})) = \mathbf{0}_{\boldsymbol{A}}$, for all $\underline{e} \in \boldsymbol{A}_k$, then $a_i = a_j$ for some $1 \leq i < j \leq m$.*

It immediately follows from Theorems 1.3 and 2.3 that *every finitely generated quasivariety has m–EDTPM for some $m \geq 3$.*
The following observations supplement Theorem 1.3:

Theorem 2.5. *Let $\mathbf{Q}$ be an arbitrary quasivariety and let $m \geq 3$ be a fixed natural number. The following conditions are equivalent:*

(1) *$\mathbf{Q}$ is generated by a finite class of algebras each of which has at most $m - 1$ elements.*

(2) *$\mathbf{Q}$ has m–EDTPM and $\bigcap_{1 \leq i < j \leq m} \Theta^{\boldsymbol{F}}_{\mathbf{Q}}(x_i, x_j) = \mathbf{0}_{\boldsymbol{F}}$ for any sequence $x_1, \ldots, x_m$ of m different free generators of the free algebra $\boldsymbol{F} := \boldsymbol{F}_{\mathbf{Q}}(\omega)$.*

In view of the above theorems, the property of having m–EDTPM for some m is essentially weaker than the property of being a finitely generated quasivariety. It follows from Theorem 2.3.(ii) that every RCD quasivariety $\mathbf{Q}$ has m–EDTPM for all $m \geq 3$. But $\mathbf{Q}$ need not be finitely generated.

Theorem 2.6. *Let $m \geq 3$ be a natural number. Let $\mathbf{Q}$ be a quasivariety with m–EDTPM with respect to a set of equations $\Lambda = \Lambda(x_1, x_2, \ldots, x_m, \underline{u})$. The following conditions are equivalent:*

(1) *$\mathbf{Q}$ is generated by a finite class of algebras each of which has at most $m - 1$ elements.*

(2) *$\mathbf{Q}$ validates the equations $\Lambda(x_1, x_2, \ldots, x_m, \underline{u})$.*

We know that every finitely generated quasivariety $\mathbf{Q}$ has the m–EDTPM property for sufficiently large m. But, more interestingly, $\mathbf{Q}$ has m–EDTPM with respect to a *trivial* set of finite equations:

Theorem 2.7. *Let $m \geq 3$ be a natural number. Suppose that $\mathbf{Q}$ is a quasivariety generated by a finite class of algebras each of which has at most $m-1$ elements. Then $\mathbf{Q}$ has m–EDTPM with respect to the following finite set of equations $\Lambda(x_1, x_2, \ldots, x_m) := \{x_1 \approx x_1, x_2 \approx x_2, \ldots, x_m \approx x_m\}$ (no parameters).*

The above theorem shows that one cannot expect much from the m–EDTPM in general while studying specific properties of *finitely generated* quasivarieties $\mathbf{Q}$ - the equations of $\Lambda(x_1, x_2, \ldots, x_m)$ from Theorem 2.7 are not conjoined with the intrinsic structure of the algebras of $\mathbf{Q}$. Consequently, the m–EDTPM property trivializes for $\mathbf{Q}$. But if one imposes further constraints on m–EDTPM e.g. by requiring that $\mathbf{Q}$ has m–EDTPM *with respect to a certain specific set of equations* $\Lambda(x_1, x_2, \ldots, x_m)$, the problem becomes less trivial. E.g. in the case of quasivarieties with the additive equational commutator the situation differs — there are sets of equations determining m–EDTPM whose properties are strictly linked with the commutator. This issue will not be discussed here. On the other hand, the fact that a quasivariety $\mathbf{Q}$ has m–EDTPM with respect to a set of equations $\Lambda(x_1, x_2, \ldots, x_m, \underline{u})$ need not imply that $\mathbf{Q}$ has m–EDTPM with respect to $\{x_1 \approx x_1, x_2 \approx x_2, \ldots, x_m \approx x_m\}$. For instance, as $\mathbf{Q}$ one may take any non-finitely generated RCD quasivariety.

§3. Congruence–modularity and triangular irreducibility

Given a quasivariety $\mathbf{Q}$ of type τ, we let $\mathbf{Q}^{eq\models}$ denote the *consequence operation* on the set $Eq(\tau)$ of τ-equations determined by $\mathbf{Q}$. Thus, for $\{\alpha_i \approx \beta_i : i \in I\} \cup \{\alpha \approx \beta\} \subseteq Eq(\tau)$,

$$\alpha \approx \beta \in \mathbf{Q}^{eq\models}(\{\alpha_i \approx \beta_i : i \in I\} \text{ iff, for every } \boldsymbol{A} \in \mathbf{Q} \text{ and every } h \in Hom(\boldsymbol{Te}_\tau, \boldsymbol{A}),$$
$$h(\alpha) = h(\beta) \text{ whenever } h(\alpha_i) = h(\beta_i) \text{ for all } i \in I.$$

$\alpha \approx \beta \in \mathbf{Q}^{eq\models}(\{\alpha_i \approx \beta_i : i \in I\})$ is read: $\alpha \approx \beta$ *follows from* $\{\alpha_i \approx \beta_i : i \in I\}$ *relative to* $\mathbf{Q}$.

$\alpha \approx \beta \in \mathbf{Q}^{eq\models}(\emptyset)$ means that the equation $\alpha \approx \beta$ is valid in $\mathbf{Q}$. There is an obvious translation of $\mathbf{Q}^{eq\models}$ into the language of quasi-identities over $\boldsymbol{Te}_\tau$: $\alpha \approx \beta \in \mathbf{Q}^{eq\models}(\{\alpha_i \approx \beta_i : 1 \leq i \leq n\})$ if and only if the implication $\alpha_1 \approx \beta_1 \wedge \ldots \ldots \wedge \alpha_n \approx \beta_n \to \alpha \approx \beta$ is valid in $\mathbf{Q}$.

m–EDTPM can be neatly expressed (in an equivalent way) as a property of $\mathbf{Q}^{eq\models}$, viz.

$$(*) \qquad \mathbf{Q}^{eq\models}(X \cup \bigcap_{1 \leq i < j \leq m} \mathbf{Q}^{eq\models}(x_i \approx x_j)) = \bigcap_{1 \leq i < j \leq m} \mathbf{Q}^{eq\models}(X \cup \{x_i \approx x_j\}),$$

for any set (*equivalently – for any finite set*) *of equations* X.

(Here $x_1, x_2, \ldots, x_m$ is an arbitrary but fixed sequence of m different individual variables.)

If $\mathbf{Q}$ is RCM, the equational consequence $\mathbf{Q}^{eq\models}$ satisfies a weaker form of m–EDTPM, for all $m \geq 3$:

Theorem 3.1. *Let $\mathbf{Q}$ be an arbitrary RCM quasivariety. Then*

$$\mathbf{Q}^{eq\models}(X) \cup \bigcap_{1\leq i<j\leq m} \mathbf{Q}^{eq\models}(x_i \approx x_j) = \bigcap_{1\leq i<j\leq m} \mathbf{Q}^{eq\models}(X \cup \{x_i \approx x_j\}),$$

for any finite set of equations X and any sequence $x_1, x_2, \ldots, x_m$ of m different individual variables not occurring in the equations of X.

(The difference between m–EDTPM and the thesis of the above theorem consists in the additional assumption of separation of the variables $x_1, x_2, \ldots, x_m$ from the equations of X.) The problem whether any RCM $\mathbf{Q}$ satisfies m–EDTPM for all $m \geq 3$ (thus without the above separation requirement) is open.

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Problem dylematów moralnych

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Celem referatu jest prezentacja oraz pewne przeformułowanie tzw. argumentu „z logiki deontycznej" w kwestii sporu o rzeczywiste istnienie dylematów moralnych.

W pierwszym kroku krótko omawiam samo zagadnienie dylematu moralnego — definicję, istotę problemu istnienia dylematów oraz główne argumenty obu stron sporu. Następnie ściśle prezentuję standardową definicję dylematu moralnego, by dalej poddać ją formalizacji w języku multimodalnej logiki deontyczno–aletycznej — tak jak ma to miejsce w literaturze przedmiotu. Pokazuję również dwa przykłady funkcjonujących w literaturze rozumowań prowadzonych w tym języku, które pozwalają wykazać, że już w samej naturze dylematu moralnego tkwi sprzeczność. Szczególnie eksponuję zasady do których trzeba się w tych rozumowaniach odwołać, w szczególności omawiam ich problematyczność (bardzo mocne założenia dla systemu w którym miałby się znaleźć, filozoficzne kontrowersje).

Dalej przechodzę do omówienia własnej propozycji — wpierw nieznacznej zmiany formalizacji definicji dylematu, będąc zupełnie w zgodzie z pewnymi racjami filozoficznymi. Następnie pokazuję, że przy takiej reinterpretacji można wykazać sprzeczność w samej istocie dylematu moralnego na gruncie pewnej logiki deontycznej (prezentuję ją zarówno od strony syntaktycznej jaki i semantycznej), słabszej od standardowej logiki deontycznej, a przez to „odpornej" na znane paradoksy logiki deontycznej typu paradoks dobrego samarytanina czy Alfa Rossa. Pokazuję, że aby udowodnić sprzeczność wystarczy odwołać się do dwóch „prostych" zasad: formuły D oraz pewnej wersji zasady Elzenberga.

Unification and Structural Completeness in extensions of S4 modal logic

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Key words: unification, projective unification, modal logics $S4$, $S4.3$.

Given a formula α, a *unifier for* α in a logic L is a substitution σ such that $\vdash_L \sigma(\alpha)$. A *formula* α *is unifiable* in L, if such σ exists. If τ, σ are substitutions, then $\tau \preceq \sigma$, if there is a substitution θ such that $\vdash_L \theta(\sigma(x)) \leftrightarrow \tau(x)$. Classical propositional logic CL has unitary unification. It means that every formula α, unifiable (= consistent) in CL, has a mgu, i.e. a substitution σ such that $\vdash_{CL} \sigma(\alpha)$ and that every unifier τ for α is a special case of σ, i.e. $\vdash_{CL} \theta(\sigma(x)) \leftrightarrow \tau(x)$, for some θ. Unification type of a logic can be unitary ("best"), finitary, infinitary or nullary depending on the number of maximal unifiers, see e.g. [1], [2], [3].

A particular splitting of the lattice of extensions of $S4$ modal logic, $ExtS4$, given by the splitting pair $(L(f_2), S4.2)$, where f_2 denotes the "fork", determines unification types for logics in $ExtS4$, see [4].

A projective unifier for a unifiable formula α in a logic L is a unifier σ for α such that $\alpha \vdash_L \sigma(x) \leftrightarrow x$, see [6], [2]; every projective unifier is an mgu. L has *projective unification* if every unifiable formula has a projective unifier.

A logic L is *structurally complete* if:
($\star$) every (structural) admissible rule in L is derivable in L. In terms of unifiers: a rule A/B is admissible in L iff every unifier for A is a unifier for B, $\vdash_L \sigma(A)$ implies $\vdash_L \sigma(B)$.

We show that for a logic L in $ExtS4$, structural completeness of L and the fact that L has a particular unification type are independent of each other. On the other hand, logics having projective unification are structurally complete or almost structurally complete (i.e. ($\star$) holds for rules with unifiable premises).

In a joint paper [5] we prove that a modal logic (containing $S4$) has projective unification iff it contains modal logic $S4.3$ (an analogous theorem for intermediate logics was proved by A. Wroński, see [7], [8]). A simple description of structurally complete logics over $S4.3$ and an effective procedure for structural completeness follow.

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Dyskretny urok rachunku relacji

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Mój referat jest referatem historycznym. Analizuję pracę A. Tarskiego „On the calculus of relations", JSL z 1941 roku. Przedstawiono tutaj dwie różne metody w opisie rachunku relacji. W pierwszej metodzie wykorzystano język pierwszego rzędu z dwoma rodzajami zmiennych. Jest to metoda naturalna, ale mało elegancka. W drugiej metodzie, prostszej, wykorzystano tylko jeden rodzaj zmiennych (zmienne relacyjne) i to w języku bez kwantyfikatorów. Podana jest lista aksjomatów i reguł systemu CR. Podano detaliczne dowody kilkudziesięciu twierdzeń w nawiązaniu do monografii E. Schrödera „Algebra und Logik der Relative". Opisano model geometryczny dla systemu CR. System ten od strony algebraicznej jest rozmaitością. Autor rozważa pewne ważne metalogiczne własności tego systemu i na nich chciałbym się skoncentrować.

Advertising Generalized Satisfiability

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In 1991 R.H.Cowen introduced in [1] the notion of satisfiability with respect to a given hypergraph ($\mathcal{H}$–satisfiability). Alas in his definition he required that the satisfying set should be a *coloring* of the considered hypergraph. Then he recalls the notion of $\mathcal{H}$*–resolution* from [10] of Linial and Tarsi and proves the following soundness theorem:

Theorem.
Let $[\mathcal{A}]_{\mathcal{H}}$ be a resolution closure of $\mathcal{A}$ with respect to $\mathcal{H}$. Then if $\mathcal{A}$ is satisfiable with respect to $\mathcal{H}$, then $\emptyset \notin [\mathcal{A}]_{\mathcal{H}}$.

It turned out however that the reverse is not valid which means the lack of completeness of $\mathcal{H}$–resolution for checking $\mathcal{H}$–satisfiability and it was a disadvantage.

In the paper [7], A. Kolany shows, that if one skips the requirement of being a coloring in the definition of satisfiability, the theorem of Cowen can be reversed. From this time, speaking about hypergraph satisfiability we mean its weaker version proposed by Kolany.

One of the most surprising and important properties of $\mathcal{H}$–satisfiability is the following *duality* property:

$$\mathcal{A} \text{ is satisfiable with respect to } \langle\mathcal{V},\mathcal{E}\rangle \quad\Longleftrightarrow\quad \mathcal{E} \text{ is satisfiable wrt. } \langle\mathcal{V},\mathcal{A}\rangle,$$

which possibly gives new ways of checking the usual satisfiability of clauses (**SAT**), since the latter is a special case of $\mathcal{H}$–satisfiability.

The duality recalled above allows to notice (Cowen, [2]) a very straightforward relation between $\mathcal{H}$–satisfiability and existence of so called *transversals* (Schrijver, [11]). Moreover instead of speaking about satisfiability of one family with respect to another the duality suggests to consider satisfiability of a couple of families of sets $\{\mathcal{A},\mathcal{E}\}$, which requires only a small step to obtain the notion of *property* **S** introduced by Cowen in [2] as a generalization of Felix Bernstein property **B** (comp. [12,13]).

In [3] Cowen and Kolany present a set of rules which preserve property **S** (which we like also to call *generalized satisfiability*) of a couple of families $\{\mathcal{A},\mathcal{E}\}$ and which are a result of analysis of the three steps of the **DPLL** algorithm proposed in [6] by Davis and Putnam, and then enhanced in [5] by Davis, Logemann and Loveland.

In 1993 Cowen and Wayatt present in [4] a BREAKUP procedure which processes a given set of clauses $\mathcal{A}$ by separating them into so called then into *connected components* (with disjoin sets of variables) and whose application in many cases speeds up deciding satisfiability. Complexity of this procedure is $\mathcal{O}(n \times k)$, where n is the number of variables appearing in $\mathcal{A}$ and k the number of clauses in

$\mathcal{A}$. In [8] we present an algorithm of a similar application and the complexity of $\mathcal{O}(\#\mathcal{F} \times (\#\mathcal{C} + \mathbf{wd}(\mathcal{F}))) + \mathcal{O}(\#\mathcal{V} \times \#\mathcal{C} \times \mathbf{rk}(\mathcal{F}))$, where $\#\mathcal{A}$ is a number of elements of a family $\mathcal{A}$ (the cardinality of $\mathcal{A}$), $\mathbf{wd}(\mathcal{A})$ – is a cardinality of its biggest element, and $\mathbf{rk}(\mathcal{A})$ – the cardinality of the biggest subfamily of $\mathcal{A}$ with nonempty meet, which is based on the *Deep First Search* algoritm. In the case of **SAT**, where $\#\mathcal{F} = \#\mathcal{V}$, $\mathbf{rk}(\mathcal{F}) = 1$ and $\mathbf{wd}(\mathcal{F}) = 2$, this gives the complexity $\mathcal{O}(\#\mathcal{V} \times \#\mathcal{C})$, which is exactly the same as in the case of Cowen and Wyatt. Moreover, in [9] we show that using *Reverse Resolution Rule* we can reduce step by step the satisfiability of a couple of families to satisfiability of a couple $\{\mathcal{E}, \mathcal{F}\}$, where $\mathcal{E}$ contains at most 3–element sets and $\mathcal{F}$ consists of pairs, which seems being a very beautiful transfer of the classical result of reducing **SAT** to 3-**SAT** to the case of generalized satisfiability.

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On interpolation in $NEXT(\mathbf{KTB})$

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We examine the Brouwer logic **KTB**, and its extensions known as n–transitive logics. They are denoted as $\mathbf{T_n}$ and were defined by I. Thomas in 1964 (see [5]). The family $\{\mathbf{T_n},\, n \geqslant 1\}$ forms the following chain, ordered by inclusion:

$$\mathbf{KTB} \subset \ldots \subset \mathbf{T_{n+1}} \subset \mathbf{T_n} \subset \ldots \subset \mathbf{T_2} \subset \mathbf{T_1} = \mathbf{S5}.$$

In contrast to the logics laying in the interval **S4**–**S5**, which are very well characterized, the logics between **KTB** and **S5** are, in some way, neglected.

Our aim is to improve this situation. In this talk we pay attention to the Craig interpolation property (CIP) and Halldén completeness of extensions of **KTB**. Regarding (CIP), one may apply a very general method of construction of inseparable tableaux (see i.e. [1]) and get:

Theorem 1. *The logics* **KTB** *and* $\mathbf{T_n}$, $n > 1$ *have* (*CIP*).

The connection between the Craig interpolation property and Halldén completeness of modal logics is described in the following lemma due to G. F. Schumm [4]:

Lemma 1. *If L has only one Post-complete extension and is Halldén–incomplete, then interpolation fails in L.*

Although it is known that **KTB** is Halldén complete (see [3]), there are no results concerning this property in the case of logics $\mathbf{T_n}$, $n > 1$. From Theorem 2 and Lemma 1 we immediately obtain:

Corollary 1. *The logics* $\mathbf{T_n}$, $n \geq 1$ *are Halldén–complete.*

The next step in our investigation is to answer the question: 'how many logics in $NEXT(\mathbf{T_2})$ are Halldén–incomplete?'

In this case we take advantage of two constructions from [2]: infinite sequence of nonequivalent formulas in $\mathbf{T_2}$ and continuum of normal extensions distinguishable by these formulas. Then we will prove:

Theorem 2. *There are uncountably many extensions of* $\mathbf{T_2}$ *which are Halldén incomplete and hence — without* (*CIP*).

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Logiczne metody uzasadniania wiedzy w buddyzmie

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Logika w Indiach, ze względu na przyznawaną jej doniosłą funkcję poznawczą, była zaliczana do epistemologii. Rozumowanie (*anumāna*) zazwyczaj uważano, za jedno ze źródełwiedzy (*pramāṇa*). Ponadto, podobnie jak w przypadku logiki zachodniej, jej rozwojowi towarzyszyła refleksja nad regułami sprawnej argumentacji, przydającej się w debatach doktrynalnych.

„Okres buddyjski" w dziejach logiki indyjskiej przypada na czas, w którym koncepcje wysuwane i bronione przez przedstawicieli tej szkoły były dominujące i wywierały spory wpływ na poglądy myślicieli wywodzących się z innych tradycji. Za datę rozpoczynającą ów okres przyjmuje się przełom V i VI w. n.e. odpowiadający działalności wielkiego mędrca i świętego buddyjskiego — Dignagi (*Dignāga*). Natomiast moment największej świetności, gdy logika buddyjska osiągnęła najbardziej znaczącą i dopracowaną formę, wiąże się z postacią mnicha Dharmakirtiego (*Dharmakirti*) żyjącego w VII w. n.e., który znacząco rozwinąłi dopracowałpomysły swojego wielkiego poprzednika.

W buddyzmie, zgodnie z ogólnymi tendencjami panującymi w filozofii indyjskiej, logika również była ściśle podporządkowana swojej funkcji poznawczej, a rozumowanie (*anumāna*) było uważane z pośrednie źródło poznania (*prāmaṇa*). Jego istota polegała na dostarczaniu wiedzy o stanach rzeczy niemogących stanowić obiektu doświadczenia bezpośredniego, tj. percepcji (*pratyakṣa*), na podstawie obiektów bezpośrednio doświadczanych — znaków (*liṅga*) oraz na skutecznym przekazywaniu tej wiedzy. Wynikiem tak przeprowadzonego podziału funkcji rozumowania było oddzielenie od siebie jego dwóch poziomów: pierwszy, to rozumowanie przeprowadzane dla siebie (*svārthānumāna*) na poziomie mentalnym, czyli źródło pośredniej wiedzy o świecie; drugi — to poziom rozumowania dla innych (*prārthānumāna*), które charakteryzuje się usystematyzowaną strukturą omawianą w traktatach logicznych.

Celem *prārthānumāna* jest przekonanie osoby trzeciej, że poznanie uzyskane drogą *svārthānumana* jest prawdziwe. Buddyści sformułowali również metalogiczne reguły (*trairūpya hetu*), które muszą być spełnione przez rację (*hetu*), aby rozumowanie na niej oparte można uznać za poprawne.

W odczycie opieramy się na dwóch ważnych buddyjskich dziełach logiczno–epistemologicznych — *Nyāya-bindu* Dharmakirtiego oraz komentarzu *Nyāya-bindu-ṭīkā* Dharmottary (*Dharmottara*). Do naszych analiz stosujemy *teorię lokacji własności*, tj. metodę interpretacji logiki indyjskiej, w której stosuje się kategorie ontologiczne i syntaktyczne zaczerpnięte bezpośrednio z filozoficznej tradycji indyjskiej. Są to, m.in.: *własności* (*dharma*) oraz *lokacje* (*pakṣa*, *dharmin*), w których własności są *ulokowane*. Naszym celem jest pokazanie, iż wypracowane przez buddystów reguły, znajdujące swój wyraz w strukturze wzorcowych schematów poprawnie przeprowadzonych rozumowań, mają za zadanie pokazać osobom trzecim tożsamość (*sarūpya*) konceptualnego modelu rzeczywistości, z modelem empirycznym, jakiemu odpowiada określony stan rzeczy zachodzący poza podmiotem poznającym. Podejście takie wynika z warunków nakładanych na wiedzę w buddyzmie:

> Wiedza właściwa, to wiedza nie zaprzeczona w doświadczeniu. (Dharmottara, *Nyāya-bindu-ṭīkā* I.3.5.)

Wynika z tego, iż reguły, o których mowa, a za nimi schematy rozumowania, nie posiadają charakteru dedukcyjnego każącego konsekwentnie uznawać za prawdziwe pewne zdania, jeśli tylko wcześniej zaakceptowało się jakieś inne. Logika buddyjska pozostaje na poziomie epistemologii i swój cel stara się osiągnąć nie przez eksploatację niezawodnych schematów wnioskowań opartych na strukturze prawd logicznych, lecz przez ustalenie zasad prawidłowego odwoływania się do doświadczenia bezpośredniego (percepcji) oraz pewnych powszechnie uznawanych, niebudzących wątpliwości prawd ogólnych.

Characteriziation of Medvedev's Logic by Means of Kubiński's Frames

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Medvedev's logic of finite problems ML is the only known structurally complete intermediate logic with the disjunction property. ML in its semantical version can be given as follows:

$$ML = \bigcap\{\boldsymbol{L}(\sigma_n) : n \geq 1\},$$

where σ_n stands for n-atomic Boolean cube without unit.

Tadeusz Kubiński, investigating semantics of fuzzy names, has defined the lattices $(R_n, \leq)$ as follows: for each natural n, let $E_n = \{1, 2, \ldots, n\}$ and $P_n = \mathcal{P}(E_n) \times \mathcal{P}(E_n)$. Putting:

$$R_n = \{(A, B) \in P_n : A \cap B = \emptyset\},$$

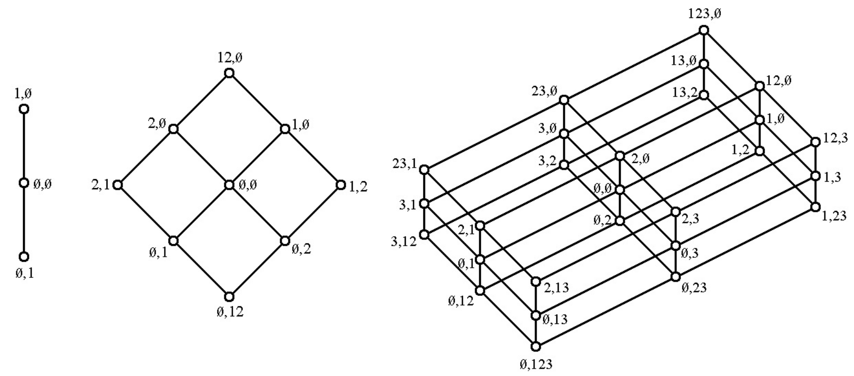
and

$$(A, B) \leq (C, D) \;\Leftrightarrow\; A \subseteq C \;\&\; D \subseteq B,$$

we get the lattice with operations supremum and infimum:

$$(A, B) \vee (C, D) = (A \cup C, B \cap D), \qquad (A, B) \wedge (C, D) = (A \cap C, B \cup D),$$

zero — $(\emptyset, E_n)$, and unit — $(E_n, \emptyset)$. (The lattice R_n can be regarded as sum of Boolean cubes B_n, and skeleton B_n, too.)



Picture 1: Kubiński's lattices R_1, R_2 and R_3 (e.g. $12, \emptyset$ denote $(\{1, 2\}, \emptyset)$).

Since posets $K_n = R_n \setminus \{(E_n, \emptyset)\}$ (with $\leq$ restricted) are Kripke frames, deterime intermediate logics:

$$\boldsymbol{L}(K_1) \supseteq \boldsymbol{L}(K_2) \supseteq \boldsymbol{L}(K_3) \supseteq \ldots$$

and intersection:

$$KL = \bigcap \{\boldsymbol{L}(K_n) : n \geq 1\}.$$

Then we have:

Theorem 1 $KL = ML$.

The inclusion $KL \subseteq ML$ is clear, since $\boldsymbol{L}(K_n) \subseteq \boldsymbol{L}(\sigma_n)$ (restricted model). The converse, follows from $\boldsymbol{L}(\sigma_{2n}) \subseteq \boldsymbol{L}(K_n)$. We prove it, constructing special p-morphism from σ_{2n} on K_n. Finally, we ask, does exists some way to generalize this technique.

Ontologika modalna

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Rozważania dotyczące związków logicznych pomiędzy tym, co konieczne, możliwe i niemożliwe znajdujemy już w *Hermeneutyce* oraz w *Analitykach Pierwszych*

Arystotelesa. Stagiryta utrzymywał jednak, że każdy pojedynczy sąd ma strukturę podmiotowo–orzecznikową. Zatem twierdzenie, że *jest konieczne* (*lub możliwe*), *że każde* α *jest* β powiada, że β *musi* (*lub może*) *być orzekane o każdym przedmiocie, o którym* α *jest orzekane* lub że β *musi* (*lub może*) *przysługiwać każdemu* α . Związki koniecznościowe zachodzą więc w samych rzeczach, mogą być natomiast ujmowane poznawczo i wyrażane w zdaniach. Dlatego, jak pisze Krąpiec, [*k*]*onieczność w systemie Arystotelesa można bardziej ściśle określić jako: „to, czego negacja jest negacją bytu”.* Pojęcia modalne od samego początku były więc ściśle związane z pojęciami ontologicznymi. W referacie przedstawimy propozycję logicznej analizy pewnych pojęć ontologicznych, przyjmując jednak za punkt wyjścia współczesny sposób analizy pojęć konieczności i możliwości. W tym celu wykorzystamy dwie multimodalne logiki zdaniowe wzbogacone o ontologiczne pojęcia *istnienia*, *możliwości*, *lokalności*, *ufundowania*, i *fuzji*.

Spójniki zewnętrzne a spójniki wewnętrzne

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Referat będzie pewnym spojrzeniem logiczno–językoznawczym na struktury, w których występują tzw. spójniki wewnętrzne jak i na struktury, w których występują tzw. spójniki zewnętrzne oraz na pewne zależności między tymi strukturami.

Structural Completeness and Admissible Rules in (Fragments of) Intuitionistic Logic

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Following Lorenzen, a rule is said to be admissible for a logic (a finitary structural consequence relation) if it can be added to a proof system for the logic without producing new theorems. While the admissible rules of classical propositional logic CPC are also derivable — that is, CPC is structurally complete — this is not the case for intuitionistic propositional logic IPC. The goal of this talk is a description of the landscape of admissible rules and structural completeness for fragments of IPC and their axiomatic extensions.

Structural completeness for the implicational fragment of IPC (and its axiomatic extensions) was established by Prucnal, and a similar proof method extends also to the implication-conjunction and implication–conjunction–negation fragments. Full IPC is not structurally complete but the set of admissible rules is decidable (proved by Rybakov) and, moreover, as shown independently by Iemhoff and Roziere, admits an elegant infinite basis (axiomatization by admissible rules). Curiously, as observed by Wroński, the implication–negation fragment is also not structurally complete. Again, however, an elegant infinite basis can be found for the admissible rules of this fragment and indeed of any of its axiomatic extensions.

Giles's Game and the Proof Theory of Łukasiewicz Logic

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In the 1970's Robin Giles introduced a two–player dialogue game based on betting on physical experiments for atomic formulas, and Lorenzen–style dialogue rules for compound formulas. Remarkably, the existence of winning strategies for this game corresponds directly to the validity of formulas in the infinite–valued logic introduced by Jan Łukasiewicz in the 1920s.

The aim of this talk is to describe the relationship between Giles's game and the proof theory of Łukasiewicz logic. In particular, a correspondence will be described between proofs in a hypersequent calculus for Łukasiewicz logic and "disjunctive strategies" for Giles's game. Variants of the game and extensions to the first–order level will also be considered.

An Axiomatization of the Propositional Version of Kovac's Logic KC

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In [1], S. Kovač defined semantically a logic—called **KC**—referring to Kant's formulation of the principle of contradiction. **KC** was formulated in the first order predicate language. Among others, Kovač indicated an adequate tableau system for **KC**.

In the present paper we will consider only propositional version of the logic **KC**, retaining the symbol '**KC**' for the sake of simplicity.

While defining **KC** we will use a mapping from the set of all formulas For in the propositional language with the logical connectives $\{\sim, \wedge, \vee, \rightarrow, \leftrightarrow\}$ into the modal formulas $\mathrm{For_m}$ built up from the functors $\{\neg, \wedge, \vee, \rightarrow, \leftrightarrow, \Diamond, \Box\}$, which is a simplified version of Kovač's original transformation.

Definition 1. *Let $\mathcal{KC}$ be a function from* For *into* $\mathrm{For_m}$ *that for any* $A, B \in$ For *fulfils the following conditions:*

$$\begin{array}{lcl}
\mathcal{KC}(a) & = & a, \textit{ for any propositional variable } a, \\
\mathcal{KC}(\sim A) & = & \neg\mathcal{KC}(A), \\
\mathcal{KC}(A \wedge B) & = & \Diamond\mathcal{KC}(A) \wedge \Diamond\mathcal{KC}(B), \\
\mathcal{KC}(A \vee B) & = & \Box\mathcal{KC}(A) \vee \Box\mathcal{KC}(B), \\
\mathcal{KC}(A \rightarrow B) & = & \Diamond\mathcal{KC}(A) \rightarrow \Box\mathcal{KC}(B), \\
\mathcal{KC}(A \leftrightarrow B) & = & \Box(\Diamond\mathcal{KC}(A) \wedge \Diamond\mathcal{KC}(B)) \vee \Box(\Diamond\neg\mathcal{KC}(A) \wedge \Diamond\neg\mathcal{KC}(B)).
\end{array}$$

The propositional version of logic's Kovač has got the form:

Definition 2. $\mathbf{KC} = \{A : A \in \mathrm{For},\ \mathcal{KC}(A) \in \mathbf{S5}\}$.

In the presentation we give a Hilbert style axiomatization of the above defined logic.

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Finite axiomatization and term–equivalence

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Two algebras are *term–equivalent* if the operations of each of them can be expressed by terms in the language of the other. Obviously, if two algebras are term–equivalent then the existence of a finite basis for identities (or quasi–identities) of one of them implies the same for the other. It is not known if a similar implication holds for finite bases (or finite axiomatizations) of logical matrices with non–*protoalgebraic* consequence operations. The problem was proposed by W. Rautenberg in the 90'ties. It was shown in [1] that for 2–valued consequence operations the finite basis and finite axiomatization properties are preserved under term–equivalence; all of them are finitely based (independently of their classification by term–equivalence — the Post classification) and therefore also finitely axiomatizable. The question of Professor Rautenberg is open even for the case of three–element matrices. However, for some particularly simple three–element matrices, some of which are not finitely axiomatizable, it has been checked that the finite axiomatization and finite basis properties are preserved under term–equivalence. An example is known that adding a constant (not definable in terms of other operations) to the signature of a finitely axiomatizable matrix may spoil this property.

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A general concept being a part of a whole. Mereology and intransitive parthood

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The transitivity of the notion *of a part of a whole* is often questioned. But it is among the most basic principles of mereology. In this paper we present a general solution of the problem of transitivity of parthood which may be satisfactory for both its advocates and its opponents. We formulate also a general approach to the concept *being a part of a whole.*

Structural Completeness: A Look Back

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Completeness is one of the fundamental concepts in the theory of formal systems. In the most general setting, we can say that a system is complete if it captures all correct schemes of argumentation. However, it turns out that there are many possible variants of the property of being complete.

Historically, one of the most important ways of approaching completeness is due to E. Post. According to his definition, a logical system is complete if it cannot be extended to another consistent system by adding new formulas as its axioms. However, Post-completeness turned out to be a rather uncommon property among logical systems and one could hope for a better suited notion. Forty years ago a very important refinement of Post-completeness was discovered. It was the notion of structural completeness introduced by W.A. Pogorzelski in 1971 in his article *Structural Completeness of the Propositional Calculus.*

In this talk we present this notion, explain motivations behind it, and give some natural examples of structurally complete and incomplete systems. Then we discuss the most important results on structural completeness of the first years of investigations.

Admissibility and structural completeness in algebraic logic

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This talk will deal with admissible rules and related completeness notions from the point of view of abstract algebraic logic, while stressing concrete examples and applications. Treatments of structural completeness are often framed in the context of *algebraizable* logics, but many natural examples of admissibility involve non–algebraizable systems. For example, the necessitation rule is admissible in quasi-normal modal logics, and cut rules in suitable cut–free sequent calculi, but unless we add these rules as postulates, the systems just mentioned are not algebraizable. The 'sub–algebraizable' levels of the Leibniz hierarchy (protoalgebraic, equivalential, etc.) will therefore be included in our remit. Thus, we can indicate which tools of abstract algebraic logic are really needed at various stages of the extant theory of admissibility, while also supplying some new results, of both the abstract and the concrete kind.

From the semantic point of view, a rule r is admissible in an arbitrary deductive system S iff every matrix model $\mathcal{A}$ of S is a homomorphic image of one that validates r. Under the surjective homomorphism, the property of being designated is preserved, but not necessarily reflected, and no assumptions about finitarity of S or finiteness of r are made. When $\mathcal{A}$ is a *reduced* matrix model of S, there is no guarantee that its r–validating pre–image can be chosen reduced as well, but that can be achieved when S is *protoalgebraic*. In the context of substructural logics, this fact and some constructions of pre–images for free reduced matrix models are a source of interesting admissible rules where, in some cases, syntactic proofs of admissibility are not easy.

A protoalgebraic finitary system will be hereditarily structurally complete if all of its relatively subdirectly irreducible reduced matrix models are weakly projective. (A partial converse, essentially due to Gorbunov, holds for *equivalential* systems.) The result can be used to account for hereditary structural completeness in the Gödel logic **G** (Dzik & Wroński, 1973), and in the positive fragment of $\mathbf{RM^t}$ (Olson & Raftery, 2007); the latter case is not susceptible to 'Prucnal's trick'. Wroński's recent characterization of *overflow completeness* (2005) can be carried out for *finitely equivalential* systems; it rules out (even a weak form of) structural completeness for a large class of fuzzy and substructural logics, not all of which are algebraizable. The non–algebraizable fragments of relevance logic are an interesting case study, as they are *order–algebraizable*. In particular, the open problem of structural completeness for the implication fragment of **R** will be analysed.

Idempotent residuated structures and finitness conditions

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A class K of similar algebras is said to have the *finite embeddability property* (briefly, the *FEP*) if every finite subset of an algebra in K can be extended to a finite algebra in K, with preservation of all partial operations. If a finitely axiomatized variety or quasivariety of finite type has the FEP, then its universal first order theory is decidable, hence its equational and quasi–equational theories are decidable as well. Where the algebras are residuated ordered groupoids, these theories are often interchangeable with logical systems of independent interest. Partly for this reason, there has been much recent investigation of finiteness properties such as the FEP in varieties of residuated structures.

A residuated partially ordered monoid is said to be *idempotent* if its monoid operation is idempotent. In this case, the partial order is equationally definable, so the structures can be treated as pure algebras. Such an algebra is said to be *conic* if each of its elements lies above or below the monoid identity; it is *semiconic* if it is a subdirect product of conic algebras. We prove that the class SCIP of all semiconic idempotent commutative residuated po-monoids is locally finite, i.e., every finitely generated member of this class is a finite algebra. It turns out that SCIP is a quasivariety; it is not a variety.

The lattice–ordered members of SCIP form a variety SCIL, provided that we add the lattice operations $\wedge, \vee$ to the similarity type. This variety is not locally finite, but the local finiteness of SCIP facilitates a proof that SCIL has the FEP. In fact, we show that for every relative subvariety K of SCIP, the lattice–ordered members of K form a variety with the FEP. It turns out that SCIL has a continuum of semisimple subvarieties.

The variety SCIL contains all Brouwerian lattices, i.e., the algebraic models of positive intuitionistic logic. SCIL also includes all positive Sugihara monoids; these algebras model the positive fragment of **R**–mingle. The results here give a unified explanation of the strong finite model property for many extensions of these and other systems. They generalize Diego's Theorem, as well as the fact that the variety generated by all idempotent commutative residuated *chains* is locally finite (Raftery, 2007).

We do not know whether the quasivariety SCIP is finitely axiomatized. Motivated by this question, we consider the larger quasivariety IP consisting of all idempotent commutative residuated po–monoids, and we show that a relative subvariety of IP consists of semiconic algebras if and only if it satisfies a certain equation that is not itself an identity of SCIP. Thus, SCIP is not a relative subvariety of IP.

Uwagi na temat oceny argumentów

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Referat składa się z dwóch części. W pierwszej rozważamy strukturalne własności argumentów. Wprowadzamy w tym celu bardzo szeroką definicję, wedle której argument to skończony zbiór sekwentów, tj. uporządkowanych par o postaci: ⟨ skończony zbiór zdań, zdanie ⟩. Pokażemy, że przy tym określeniu można ugruntować wszystkie pojęcia, za pomocą których opisuje się budowę argumentów (w szczególności da się wyrazić tradycyjnie przyjmowany podział na argumentacje szeregowe i równoległe). Niektóre z tych pojęć mają jednak nieco odmienny, niż to się zwykle zakłada, charakter. Na przykład uważa się zazwyczaj, że argument powinien mieć dokładnie jedną konkluzję główną, natomiast proponowana definicja pozwala rozważać struktury, posiadające wiele konkluzji głównych lub nieposiadające ich wcale. Dlatego też definiujemy pewne ogólne, strukturalne własności argumentów, mianowicie: *zbieżność* i *spójność*, a także *cyrkularność*, wygodne przy wyróżnianiu i opisie struktur nietypowych lub wadliwych. Staramy się też wyjaśnić częste nieporozumienie, wedle którego argumenty to grafy (uważane zazwyczaj za drzewa), których wierzchołkami są przesłanki i konkluzje.

W części drugiej przedstawimy propozycję formalnej metody waluacji argumentów — pokazuje ona, jak obliczyć wiarygodność konkluzji, znając wiarygodność przesłanek pierwszych oraz siły inferencji przypisane poszczególnym, składającym się na dany argument, sekwentom. Przy opisie przedstawimy krytycznie istotne intuicje, które doprowadziły nas do sformułowania następujących zasad oceny argumentów:

Wiarygodność zdań określamy za pomocą liczb wymiernych z domkniętego przedziału $\langle 0, 1\rangle$. Liczba 1 oznacza całkowitą akceptację danego zdania, liczba 0 — całkowite odrzucenie, zaś liczba $\frac{1}{2}$ odpowiada postawie neutralnej. Siły inferencji przypisane sekwentom wyrażamy w tej samej skali, przy czym przypisana sekwentowi wartość oznacza maksymalny stopień wiarygodności, z jakim można by przyjąć jego konkluzję w sytuacji, gdyby wszystkie przesłanki były w pełni wiarygodne. Aby wyznaczyć wiarygodność konkluzji w świetle przesłanek pojedynczego sekwentu (tj. przy połączeniu szeregowym), obliczamy najpierw iloczyn (arytmetyczny) stopni akceptacji jego przesłanek. Jeżeli liczba ta nie jest większa niż $\frac{1}{2}$, to mamy do czynienia z błędem zwanym *petitio principii*, co uniemożliwia określenie wiarygodności konkluzji tego sekwentu i dyskwalifikuje tę część argumentacji lub nawet cały argument. W przeciwnym wypadku otrzymaną liczbę mnożymy przez wielkość wyrażającą siłę inferencji przypisaną sekwentowi, a otrzymana wartość (jeśli tylko jest większa niż $\frac{1}{2}$) oznacza stopień wiarygodności konkluzji pojedynczego sekwentu. W sytuacji, gdy dwa różne sekwenty mają tę samą konkluzję (tj. przy połączeniu równoległym), obliczamy najpierw osobno stopnie wiarygodności konkluzji w świetle przesłanek każdego sekwentu. Jeżeli w obu przypadkach konkluzja jest wystarczająco uzasadniona, to otrzymujemy dwie liczby większe niż $\frac{1}{2}$, powiedzmy a i b. Chcąc obliczyć wiarygodność konkluzji w świetle przesłanek z obu sekwentów (oznaczmy tę wartość: jako $a \oplus b$), korzystamy ze wzoru:

$$a \oplus b = 2(a + b - ab) - 1.$$

Działanie $\oplus$ jest przemienne i łączne, a więc można je stosować także do obliczania wiarygodności konkluzji wspieranej przez więcej niż dwie grupy przesłanek (tj. identycznej w trzech, czterech i większej liczbie różnych sekwentów), przy czym kolejność wykonywanych działań i kolejność branych pod uwagę wartości składowych nie jest istotna.

Aby teraz ocenić wiarygodność konkluzji głównej w świetle wszystkich przesłanek danego argumentu, sprawdzamy przede wszystkim, czy jest on niecyrkularny (aspekt formalny) i czy jego przesłanki pierwsze są wystarczająco wiarygodne (aspekt merytoryczny). Jeśli któryś z tych warunków nie jest spełniony, uznajemy argument za niepoprawny, co uniemożliwia ocenę konkluzji, o ile nie da się usunąć wadliwego fragmentu argumentacji. Jeśli da się to uczynić lub oba warunki są spełnione, to (po ewentualnej eliminacji usterek) możemy przystąpić do obliczeń. Polegają one na tym, że wedle wzorów podanych powyżej zliczamy, sekwent po sekwencie, stopnie wiarygodności konkluzji pośrednich, aż do konkluzji głównej. Gdyby w międzyczasie któraś z konkluzji pośrednich okazała się niedostatecznie uwiarygodniona (tj. w stopniu nie większym niż $\frac{1}{2}$), musimy ją pominąć wraz ze wszystkimi innymi konkluzjami, do uwiarygodnienia, których jest ona niezbędna. Jeśli wśród nich znajdzie się również konkluzja główna, wówczas konkluzję tę musimy uznać za niedostatecznie uwiarygodnioną przez przesłanki badanego argumentu. W przeciwnym wypadku otrzymujemy poszukiwaną wartość — zawsze w skończonej liczbie kroków.

Husserlowska teoria całości i części

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Czy zabarwienie kartki jest częścią kartki? Czy jasność tego zabarwienia jest częścią zabarwienia? Czym różni się *bycie częścią* w przypadku zbioru i jego elementu od *bycia częścią* w takim sensie, w jakim noga stołu jest częścią stołu? Tego typu rozważania doprowadziły wielu filozofów (np. Kazimierza Twardowskiego, Romana Ingardena) do rozważenia ogólnej teorii całości i części. W referacie omówimy teoretycznie najlepiej opracowaną teorię, tzn. teorię Husserla wyłożoną w *Badaniach Logicznych*. Wyróżnimy, za Husserlem, części samodzielne (*kawałki*) oraz części niesamodzielne (*momenty*), abstrakty i konkrety, przedmioty proste i złożone, zdefiniujemy część i całość, poddamy analizie pojęcia części bliższych i dalszych, wskażemy na możliwe formy stosunków części w całości (leżenie obok siebie, przenikanie się) oraz na rodzaje całości (całości ekstensywne, całości łańcuchowe). Przytoczymy twierdzenia Husserla o fenomenach samodzielnośći/niesamodzielności. Omówimy także — szczególny dla niektórych całości — moment jedności.

Omawiając teorię Husserla wskażemy na próby jej formalizacji zawarte m.in. w [1], [2], [4], [5]. Będziemy jednak akcentować — nie zawsze podnoszone — związki rozważań o całościach i częściach z pojęciami domknięcia, spójności czy brzegu w topologii.

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Fragments of Intuitionistic Propositional Logic

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Notions from computational complexity can be used to explicate that (the decision problem of) intuitionistic propositional logic IPL is more complicated than that of classical propositional logic CPL. Important subclasses of *PSPACE*, the problems that are decidable in polynomial space, are *NP* and *coNP*, the problems decidable in non-deterministic polynomial time, and the complements of such problems respectively. While CPL is a problem in *coNP*, IPL is *PSPACE*-complete, which is generally accepted as an evidence that it is more complicated than any problem in $NP \cup coNP$, including CPL.

We will discuss some ideas in a *PSPACE*-completeness proof of IPL. Then we will consider fragments of IPL, resulting either by restricting the set of possible logical connectives, or by restricting the number of propositional atoms. It is known that IPL remains *PSPACE*-complete if implication is the only connective allowed in propositional formulas. It is also known from [2] that IPL remains *PSPACE*-complete even if the number of propositional atoms is restricted to two. However, one cannot restrict both: A. Urquhart [5] showed that, for each n, if the number of atoms is restricted to n and implication is the only connective then the number of non-equivalent formulas is finite. It can be easily deducted from Urquhart's result that the corresponding decision problem is then *not* *PSPACE*-complete. We will show an easy alternative proof of Urquhart's result using Kripke models only. We will make this result more detailed in cases that the number of atoms is 2 (with or without the symbol $\perp$ for contradiction) or 3: in these cases one can identify the universal models. With a little help from a computer, namely by using sql scripts, one can verify the models and generate a list of all possible formulas.

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Zarys logiki przypadku

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Od czasów starożytnych znane były dwa sposoby pojmowania przypadku. Pierwszy wywodził się od Demokryta i jego historii o żółwiu i traktował przypadek jako lokalną niewiedzę. Przypadek to fakt, którego źródeł nie znamy, choć gdybyśmy mieli większą wiedzę niczym by nas nie zaskoczył. Drugi sposób pochodzi od Epikura, który wprowadził dla swych atomów własność parenklizy. Istnieją zdarzenia, których przewidzieć nie potrafimy, nawet posiadając pełną wiedzę o wszystkich przyczynach. Właśnie ten drugi sposób pojmowania przypadku kierował Łukasiewiczem wprowadzającym logikę trójwartościową. W XX wieku ustaliło się jeszcze inne spojrzenie na przypadek. To fakt występujący w strukturach tak złożonych i bogatych, że nie wystarcza nawet teoretycznej mocy obliczeniowej, aby go przewidzieć. Dla wszystkich tych sposobów istnieje aparatura logiczna pozwalająca je opisać. Treść referatu stanowi omówienie tej aparatury.

Funktory asercji i koniunkcji sekwencyjnej

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Proponowana jest tu aksjomatyczna charakterystyka funktora *asercji sekwencyjnej*. Przy jego pomocy jest definiowany funktor *koniunkcji sekwencyjnej*. W interpretacji temporalnej funktory te są czytane odpowiednio: *następnie/potem* oraz *i-następnie/ i-potem*.

Dowodzi się, że w proponowanym systemie (**SAS**) i jego wzmocnieniu (**SAS***) zawierają się odpowiednio systemy von Wrighta **And Next** oraz **And Then**. Niesprzeczność i niezależność aksjomatów bogatszej z proponowanych konstrukcji (**SAS***) jest ustalana przez interpretację w czterowartościowym rachunku zdań.

Structural completeness in extensions of S4.3

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The lecture is based on a paper by Wojciech Dzik and Piotr Wojtylak, *Projective Unification in Modal Logic*, submitted to the Logic Journal of the IGPL. The main result of the paper is the following theorem

THEOREM *A modal logic L containing $S4$ has projective unification if and only if $S4.3 \subseteq L$.*

A substitution ε is said to be a *projective unifier* (the notion due to Silvio Ghilardi) of a formula A on the ground of a modal logic L if

(i) $\vdash_L A[\varepsilon]$;

(ii) $A \vdash_L x[\varepsilon] \leftrightarrow x$, for each variable x.

The logic L has *projective unification* if each unifiable formulas has a projective unifier. As an immediate corollary of the above theorem, we obtain,

COROLLARY *Every logic extending $S4.3$ is almost structurally complete.*

A rule A/B is called *passive*, if A is not unifiable in L. The logic $S5$ is not structurally complete, since the following rule is admissible but not derivable

$$P_2 : \frac{\Diamond A \wedge \Diamond \sim A}{B}$$

We say that a logic L is *almost structurally complete* if every admissible rule in L, which is not passive, is derivable in L.

COROLLARY *Let L be an extension of $S4.3$. Then the following conditions are equivalent:*
(i) L is structurally complete
(ii) $\vdash_L (\Box\Diamond x \to \Diamond\Box x)$, i.e. $S4.3M \subseteq L$.

Structural completeness in extensions of S4.3

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Propositional language of *graded modal logic* $\mathcal{GM}$ is defined in a following manner:

1. $\Pi \subseteq \mathcal{GM}$ where Π – set of propositional letters;
2. if $\phi, \psi \in \mathcal{GM}$, then $\neg\phi, \phi \vee \psi, \phi \wedge \psi, \phi \to \psi, \phi \leftrightarrow \psi \in \mathcal{GM}$;
3. if $\phi \in \mathcal{GM}$, then $\Diamond_{\geq C}\phi, \Diamond_{\leq C}\phi \in \mathcal{GM}$ for any non–negative integer C.

By means of Standard Translation from modal language in first–order language we can easily find a first–order counterpart of graded modal logic, namely a fragment of FOL with *counting quantifiers*. When we consider satisfiability problem within graded modal logic, it turns out that in comparison to standard modal logic its computational complexity raises in the case of the particular types of frames and remains unchanged in the case of others.
In my talk I would like to present main theorems on computational complexity of the Sat–$\mathcal{ML}_C$ problem for modal logics with counting and investigate the way in which adding counting operators to hybrid logic influences the Sat–$\mathcal{H}$ problem.

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