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Abstracts

Editorial note

(EN) means that the talk is presented in English, (PL)—in Polish.

Representing Everyday Predictions in the Thin Red Line Semantics

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The Thin Red Line semantics is one of the two main approaches to solving the problem of future contingents in an indeterministic manner [2]. The theory claims that there is an actual future, but it is not determined. Despite the development of the TRL semantics in recent years, the theory seems to fail to represent any intuitive account of prediction-making. As the TRL function assumes a "God's eye" perspective and is not meant to serve any epistemic goals [1], it appears that there is no way to accommodate everyday reasonings into the TRL semantics.

In my paper, I argue that this does not have to be the case. I show that there exists a function $next$, such that branching-time (BT) structures $\mathbb{M} = \langle M, <, TRL \rangle$ and $\mathbb{N} = \langle M, <, next \rangle$ are definitionally equivalent [3]. My proof applies to discrete-time structures with usual BT axioms and TRL defined for every counterfactual moment [4, 5]. I axiomatize the function $next$ by a single postulate: $next^k(m_1) = m_2 \Rightarrow back^k(m_2) = m_1$, where $<$ turns out to be enough to define $back$. Finally, I argue that $next$ is closer to representing everyday expectations and predictions than TRL: when one flips a coin, she predicts that it will actually land either heads or tails ($next$ function), and does not predict every past and future state of affairs (TRL function).

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Conceptual Engineering and Semantic Holism

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Conceptual engineering is a recent project in analytic philosophy that contrasts with its traditional method of conceptual analysis. Instead of analyzing, conceptual engineers seek to re-shape, ameliorate or eliminate those concepts that, in one way or another, seem to function in a problematic way. That includes philosophical and scientific concepts as well as those used in a natural language. The project gained significant attention in roughly the last decade, resulting in many attempts to engineer concepts such as e.g. truth, responsibility, beauty, gender, race etc. (for an overview see: Cappelen 2018; Burges, Cappelen & Plunkett 2020). Besides dealing with the actual concepts engineering, there is a lot of attention paid to the theoretical foundations of the project. One of the most pertinent issues concerns the so-called metasemantic problem: conceptual engineering aims at a change of meaning, yet to understand how such change is even possible one needs to develop a full-blown conception of what makes our words (and other representational tools) have the meaning they have. In my talk I am going to argue that the appropriate theory in question should be seen as a holistic one: targeting language as a whole, as opposed to the semantic atomism often tacitly assumed by conceptual engineers. In particular, I will argue that among classical semantic theories, some versions of the directival theory of meaning developed by Kazimierz Ajdukiewicz (1934/1978) and inferentialism developed by Robert Brandom (1994) are best fitted as a metasemantic framework for conceptual engineering.

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Some Extensions of Intuitionistic Logic with Propositional Identity

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We introduce new Kripke semantics for intuitionistic variant of the weakest non-Fregean logic (ISCI system, studied in [3, 2]) and some of its extensions (non-Fregean logics based on classical logics has been introduced by Suszko, see [1]). We discuss constructive interpretation of propositional identity by providing a variant of BHK-interpretation. We show how this new semantics adequately reflects our understanding of identity in constructive framework. Finally we prove completeness and discuss disjuntion property.

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The Use of Strict, Binary Algebraic Formulas in Proving the Laws of Propositional Calculus

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The aim of the talk is to present strict binary algebraic formulas of the logical functors of negation, conjunction, disjunction, implication and equivalence used to perform strict proofs of propositional formulas consisting of any number of propositional variables.

The basic axiom is the assumption that every simple logical sentence, e.g. p, q, r, s, is in fact a bit, more precisely speaking, the appropriate coefficient located before the power of 2 in the power series expansion of any natural number. In turn, logical functors used in logic are the exact equivalent of bitwise operators used in electronics (logic gates) and in programming languages. This leads to the conclusion that all logical complex sentences consist of bit polynomials and are therefore subject to a new algebra — binary algebra, which describes this type of algebraic structures. By converting functor symbols into algebraic formulas inside logical sentence formulas, performing only basic arithmetic operations such as addition, subtraction, multiplication and exponentiation, you can easily, without any additional assumptions or evaluation of sentence variables, carry out strict proofs of any, even very complex, laws of calculation. A sufficient condition for the analyzed sentence formula to be a tautology or counter-tautology is the calculation result equal to 1 and 0, respectively. However, values different from 1 and 0 describe a different logical state of the formula, which, however, cannot be considered a random state, i.e. arbitrary, because it can be limited by for example, the number of simple sentences. It can also be proven that a given formula is a tautology if and only if the number of all partial sentences in the truth table is a power of 2 and the logical value of these logical sentences is equal to 1.

In order to be able to use the above formulas unambiguously and without any problems, you must first appropriately replace the sentence variables p, q, r, s appearing in the propositional law formulas with the corresponding bit variables a_0, a_1, a_2, a_3. A new, extended definition and in-depth interpretation of logical zero and logical one will also be presented, adapted to new calculations based on binary algebra. To sum up, the main advantage of this method is its universality, simple calculations and the ability to learn about the structure of formulas and logical structures from the IT side. Currently, however, the computational complexity of the new proof method is not known, and thus its performance cannot be compared with other proof methods and the scope of its applicability cannot be determined for very complex logical structures.

Keywords: classical logic, tautology, contradiction, truth table, bits, binary algebra

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Goldbach's Conjecture is True

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Goldbach's conjecture states that every even natural number greater than 2 is the sum of two prime numbers (repetitions are allowed).

Thus $4 = 2+2$, $6 = 3+3$, $8 = 3+5$, $10 = 3+7 = 5+5$, $12 = 5+7$, $14 = 7+7 = 3+11$ etc.

It is shown that Goldbach's conjecture is true. In fact, a stronger theorem is proved:

Every even number greater than 6 *is the sum of two odd different primes.*

Accordingly, $16 = 3+13 = 5+11$, $18 = 5+13 = 7+11$, $20 = 3+17 = 7+13$, $22 = 3+19 = 5+17$, $24 = 5+19 = 7+17 = 11+13$, $26 = 3+23 = 7+19$ etc.

The proof of this fact employs the methods based of Rasiowa-Sikorski Lemma and Boolean valuations of the language of Peano arithmetic PA.

The key role is played by the notion of a Rasiowa-Sikorski set, derived from this lemma. The central idea consists in constructing appropriate countable models of Peano arithmetic by means of Rasiowa-Sikorski sets. It then follows that Goldbach's Conjecture is true in the standard model $\boldsymbol{N}$ of arithmetic.

N denotes the set of natural numbers (including zero). N^+ denotes the set of positive natural numbers.

$L_0 = \{0, S, +, \cdot\}$ is the language of Peano arithmetic PA. The axiom system of PA is defined in the standard way.

Numerals are constant terms S^n0 of L_0, where n ranges over natural numbers. They are defined inductively as follows: S^00 is the symbol 0, $S^{n+1}0 := S(S^n0)$ for any n. Numerals are the names of the consecutive standard natural numbers in any model of PA.

L is the extension of the standard language L_0 of arithmetic by adding one unary predicate Pr. The formula $Pr(x)$ states that x is a prime number.

Goldbach's Conjecture in the stronger version is expressed in L as the first-order sentence

$$(\forall x)(S^30 < x \to (\exists x_1)(\exists x_2)(Pr(x_1) \wedge Pr(x_2) \wedge S^20 < x_1 \wedge x_1 < x_2 \wedge (2 \cdot x \approx x_1 + x_2))),$$

where the variables x, x_1, x_2 are different. ($S^n0 < x$ abbreviates the formula $(\exists u)(S^n0 + Su \approx x)$ and $x_1 < x_2$ is an abbreviation for $(\exists u)(x_1 + Su \approx x_2)$. (The formula $(\exists v)(y \approx x + Sv)$ defines the strict order $x < y$ in the models of PA. However, the binary predicate $<$ is not included into the vocabulary of L.)

The theory T is the extension of PA obtained by adding the following defining axiom that characterizes the predicate Pr:

$$(\forall x)(Pr(x) \leftrightarrow ((\exists u)(S^2 0 + Su \approx x) \wedge (\forall y)(\forall z)(x \approx y \cdot z \rightarrow x \approx y \vee x \approx z))).$$

(Pr is restricted throughout to *odd* primes.) The subformula $(\exists u)(S^2 0 + Su \approx x)$ states that x is strictly greater than 2, i.e, $S^2 0 < x$.

T is a conservative extension of PA in the language L.

Daniela Gromska's Influence on Polish Philosophy

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Certainly, one of the most notable representatives of the Lvov-Warsaw School is Daniela Gromska. Although she is primarily remembered as a translator, particularly of Aristotle's works, her translations remain relevant today. Her dedication and actions also helped to preserve the memory of the Lvov-Warsaw School and its founder, Kazimierz Twardowski. However, it is also worthwhile to remember Gromska's original philosophical writings, which included discussions on the theory of judgment. In my presentation, I aim to analyze Gromska's philosophical work and discuss her views on logic in general, placing them in the context of Polish philosophy. Gromska's classical education, as a classical philologist, played an important role in shaping her philosophical approach. Her teachers were Wartenberg, Łukasiewicz, and Witwicki, all of whom influenced her interests. Although her written work is scarce, she was undoubtedly a philosopher in her own right, and her few existing texts deserve to be revisited. Gromska's attention was often focused on the texts of the ancient classics, while working as Twardowski's assistant and editing the 'Ruch Filozoficzny'. Despite the limited sources available, I will attempt to reconstruct her philosophical path and encourage modern logicians to study Daniela Gromska.

When Epsilon Meets Lambda: Extended Leśniewski's Ontology

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Leśniewski's ontology LO is the expressive calculus of names. It provides a basis for mereology but allows also for direct formalisation of reasoning in natural languages. Recently its elementary part was characterised by means of the cut-free sequent calculus GO. In this talk we investigate its extended version ELO which introduces lambda terms to represent complex descriptive names. The hierarchy of three systems is formalised in terms of sequent calculi which satisfy cut elimination and the subformula property.

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Bisequent Calculi for Neutral Free Logic with Definite Descriptions

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Within the framework of neutral free logic, we provide a cut-free bisequent calculus for the minimum theory of definite descriptions. In this logic, formulas with non-denoting terms do not have a truth value, essentially making this logic three-valued. Definite descriptions are expressed using the ι-term-forming operator applied to a variable x and a formula A, leading to a term $\iota x A$, following a tradition that traces back to Russell, though we define them with the help of Lambert's axiom. Neutral free logics seem to be an especially useful framework for the consideration of improper definite descriptions. We combine two previous proof-theoretic results and enrich them by identity and definite descriptions: the paper [1] introduces cut-free bisequent calculi for various propositional three-valued logics and the paper [3] presents a calculus equivalent to a bisequent one for neutral free logics based on strong and weak Kleene's logics [2]. We formulate the bisequent rules for identity and definite descriptions, show their soundness and prove with their help Lambert's axiom which implies completeness. We present a constructive cut elimination proof.

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Blocking Fitch's Paradox: A Three-Valued Approach

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Fitch's Paradox is a sort of reasoning which, based on the elementary laws of classical, modal and epistemic logic, leads from the acceptance of the so-called Knowability Principle (KP), read informally that every truth is knowable, to the acceptance of the so-called Omniscience Principle (OP), read informally that every truth is known. The OP is a thesis definitely too strong, incompatible with our basic intuitions that there are true propositions that will never be known.

It seems, however, that drawing OP conclusion on the basis of two-valued logic, and certainly on the basis of classical logic, is from a semantic point of view unfair attitude towards anti-realism. Bivalent semantics assumes that each proposition has one of two logical values, regardless of the subject's knowledge. This is inconsistent with the philosophical doctrine of anti-realism, which associates the truth of a proposition with its warranted assertibility. For a more balanced approach, we therefore propose an analysis of Fitch's paradox on the basis of a logic in which propositions can take a third logical value. In this approach, propositions do not have to be true or false regardless of the subject's knowledge. This seems to be closer to the anti-realist position than bivalence, especially when the third logical value is properly interpreted.

The aim of our work is therefore to present an approach how to block paradoxical reasoning by changing the basic logic from classical to three-valued. By introducing a third logical value, we will show that, while it is possible to assume KP, it is not necessary to accept OP. In three-valued models, if logical connectives are properly interpreted, these principles are logically independent.

In the first part of the presentation, we discuss Fitch's paradox, pointing out its philosophical and logical background. In the second part, we propose a new approach based on Łukasiewicz's three valued logic. In the bivalent semantics, the truth value of propositions is determined independently of the knowledge of the subject. For the sake of counterbalance, we ground our considerations in a three-valued semantics, in which we interpret the third value as indeterminacy. Thanks to this shift, we are able to analyze the trivalent interpretation of modal connectives, preserving Łukasiewicz's interpretation for extensional connectives. The special role is played by Łukasiewicz's implication. We show that there are models in which the premises of Fitch's reasoning are true, but the counter-intuitive conclusion (OP) is not true. Consequently, we can claim that in a semantics more akin to anti-realist position, assuming Fitch's premises (and in particular KP) does not lead to a paradox. Finally, we describe an adequate tableau system with respect to the three-valued semantics used.

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Generalization of Classical Syllogistic: Applications and Tableaux

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Classical syllogistic **CS** whose origin lies in the philosophy of Aristotle, is a logic of four kinds of sentences called *categorial sentences* (see [1], [5]):

UA: All T_1 are T_2 $\quad T_1\mathbf{a}T_2$

UN: No T_1 are T_2 $\quad T_1\mathbf{e}T_2$

PA: Some T_1 is/are T_2 $\quad T_1\mathbf{i}T_2$

PN: Some T_1 is/are not T_2 $\quad T_1\mathbf{o}T_2$.

A model for the language of **CS** is a structure $\mathfrak{M} = \langle W, \mathsf{d} \rangle$, where:

- W is a non-empty set,
- d is a function of denotation, i.e. $\mathsf{d}\colon \mathsf{TERM} \mapsto \mathcal{P}(W)$,

with the following truth-conditions for all $X, Y \in \mathsf{TERM}$:

(**a**) $\mathfrak{M} \models X\mathbf{a}Y$ iff $\forall_{x\in W}(x \in \mathsf{d}(X) \Rightarrow x \in \mathsf{d}(Y))$ (or just $\mathsf{d}(X) \subseteq \mathsf{d}(Y)$)

(**e**) $\mathfrak{M} \models X\mathbf{e}Y$ iff $\forall_{x\in W}(x \in \mathsf{d}(X) \Rightarrow x \notin \mathsf{d}(Y))$ (or just $\mathsf{d}(X) \cap \mathsf{d}(Y) = \emptyset$)

(**i**) $\mathfrak{M} \models X\mathbf{i}Y$ iff $\exists_{x\in W}(x \in \mathsf{d}(X) \,\&\, x \in \mathsf{d}(Y))$ (or just $\mathsf{d}(X) \cap \mathsf{d}(Y) \neq \emptyset$)

(**o**) $\mathfrak{M} \models X\mathbf{o}Y$ iff $\exists_{x\in W}(x \in \mathsf{d}(X) \,\&\, x \notin \mathsf{d}(Y))$ (or just $\mathsf{d}(X) \not\subseteq \mathsf{d}(Y)$).

In the paper we extensively examine generalized syllogistic (**GS**) that is a logic of four kinds of sentences called *generalized categorial sentences*:

UA: All T_1 are related to T_2 $\quad T_1\mathbf{a}T_2$

UN: No T_1 is related to T_2 $\quad T_1\mathbf{e}T_2$

PA: Some T_1 is/are related to T_2 $\quad T_1\mathbf{i}T_2$

PN: Some T_1 is/are not related to T_2 $\quad T_1\mathbf{o}T_2$.

A model for the language of **GS** is a structure $\mathfrak{M} = \langle W, \mathsf{d}, R \rangle$, where:

- W is a non-empty set,

- d is a function of denotation, i.e. $\mathsf{d}\colon \mathsf{TERM} \mapsto \mathcal{P}(W)$,
- $R \subseteq W^2$,

with the following truth-conditions for all $X, Y \in \mathsf{TERM}$:

- $\mathfrak{M} \models X\mathbf{a}^{\mathbf{g}}Y$ iff $\forall_{x\in\mathsf{d}(X)}\exists_{y\in\mathsf{d}(Y)}R(x,y)$
- $\mathfrak{M} \models X\mathbf{e}^{\mathbf{g}}Y$ iff $\forall_{x\in\mathsf{d}(X)}\forall_{y\in\mathsf{d}(Y)} \sim R(x,y)$
- $\mathfrak{M} \models X\mathbf{i}^{\mathbf{g}}Y$ iff $\exists_{x\in\mathsf{d}(X)}\exists_{y\in\mathsf{d}(Y)}R(x,y)$
- $\mathfrak{M} \models X\mathbf{o}^{\mathbf{g}}Y$ iff $\exists_{x\in\mathsf{d}(X)}\forall_{y\in\mathsf{d}(Y)} \sim R(x,y)$.

It is obvious that by imposing some constraints on the relation R we obtain different systems of syllogistic. For example, when we consider all models where R is an identity relation $=$, we obtain **CS**. However, other applications are possible by modelling properties of R. We can make use of sentences with causal, temporal, structural or other interpretations, like: *Each poison causes death*, *Some morning is after a day* or *No city lies by some river*. They also presume some properties of relation between subject and predicate.

In the paper we present selected applications of **GS** with certain relational constraints. We also provide appropriate tableau systems and investigate the problem of decidability of these systems.

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General Recursion Theorem with a Bunch of Quite Simple Examples

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In a standard course on the <u>Fundations of Mathematics</u>, we are taught some fragments of set theory, mostly declared as the one of Ernest Friedrich Ferdinand Zermelo (1871–1953) and Adolf Abraham Halevi Fraenkel (1891–1965). There, however, students are rarely told what indeed the natural numbers are. Instead of that, they are served some set of first order axioms, commonly called Peano's Axiomatics, where one of the postulates is the so-called <u>Induction Axiom</u>:

$$\Phi\left[{}^{x}/_{0}\right] \,\&\, \forall_x \left(\Phi \to \Phi\left[{}^{x}/_{Sx}\right]\right) \to \forall_x \Phi$$

Later on, some of us are fortunate enough to be taught set theory in a slightly broader scope, and then they learn that natural numbers are those sets that belong to every inductive class (that is classes having the empty set 0 as an element and closed under the successor operation: $x \mapsto S(x) \overset{\mathbf{df}}{=\!=} x \cup \{x\}$), and that the smallest of these classes ω is a set (the <u>Axiom of Infinity</u>) and for so defined natural numbers, the axiom of induction is derived in a trivial way. Further during this course, we are shown how to define the operations of addition, multiplication and exponentation of natural numbers, and subsequently, the following <u>Recursion Theorem</u> is being proven:

Theorem. *Let A and P be any sets, let $g\colon P \to A$ and let $h\colon P \times A \times \omega \to A$. Then there exists a unique function $f : P \times \omega \to A$ satisfying:*

$$f(p,n) = \begin{cases} g(p) & n = 0 \\ h(p, f(\widetilde{n}), n) & n = S(\widetilde{n}) \end{cases}$$

This theorem can be generalized in many ways, one of which involves expanding it beyond the realm of natural numbers. Actually to the class of all <u>Ordinal Numbers</u>, that is, the class **On** of all those sets α which are:

1. Transitive: $\forall_{x\in\alpha}(x \subseteq \alpha)$
2. Connected: $\forall_{x,y\in\alpha}(x = y \vee x \in y \vee y \in x)$
3. Well founded: $0 \neq A \subseteq \alpha \to \exists_{c\in A}(A \cap c = \emptyset)$

The last condition is often considered unnecessary due to the so-called <u>Well Founding Axiom</u>, which claims that all sets are well founded. It turns out however that one does not need this as a separate axiom for practising 'everyday' mathematics, which actually fits in the <u>von Neumann Universe</u>, where this principle is true inherently.

The theorem in question is the following TRANSFINITE RECURSION PRINCIPLE:

Theorem 1 For any class function $G : \mathbf{Set} \to \mathbf{Set}$ there exists a unique class function $F : \mathbf{On} \to \mathbf{Set}$, for which

$$F(\alpha) = G(F|_{\alpha}) \ , \quad \alpha \in \mathbf{On}.$$

The standard examples of using this theorem include:

1. Definition of the von Neumann universe:

$$\mathfrak{V}(\alpha) = \begin{cases} \mathfrak{V}(\delta) \cup \wp(\mathfrak{V}(\delta)) \ , & \alpha = S(\delta) \\ \bigcup_{\delta<\alpha} \mathfrak{V}(\delta) \ , & \alpha = \bigcup \alpha \end{cases} , \quad \alpha \in \mathbf{On}$$

2. Operations on ordinal numbers:

$$\begin{cases} \alpha + \lambda = 0 \ , & \lambda = 0 \\ \alpha + \lambda = S(\alpha + \beta) \ , & \lambda = S\beta \\ \alpha + \lambda = \bigcup_{\gamma<\lambda} (\alpha + \gamma) \ , & \lambda = \bigcup \lambda \neq 0 \ , \end{cases} \qquad \begin{cases} \alpha \cdot \lambda = 0 \ , & \lambda = 0 \\ \alpha \cdot \lambda = \alpha \cdot \beta + \alpha \ , & \lambda = S\beta \\ \alpha \cdot \lambda = \bigcup_{\gamma<\lambda} (\alpha \cdot \gamma) \ , & \lambda = \bigcup \lambda \neq 0 \ , \end{cases}$$

$$\begin{cases} \alpha^{\lambda} = 1 \ , & \lambda = 0 \\ \alpha^{\lambda} = \alpha^{\beta} \cdot \alpha \ , & \lambda = S\beta \\ \alpha^{\lambda} = \bigcup_{\gamma<\lambda} (\alpha^{\gamma}) \ , & \lambda = \bigcup \lambda \neq 0 \ , \end{cases}$$

$$\alpha, \beta, \lambda \in \mathbf{On}$$

⋆⋆⋆

It turns out that some other interesting and well-known theorems can be quickly proven using that principle. These are:

- Hausdorff's Chain Extension Theorem
- Tuckey's Lemma
- Zorn's Lemma
- Ultrafilter Theorem
- Uniform Enumeration for sets
- Existence of bases in vector spaces
- ...
- Hahn-Banach Theorem

Let us show as an example the Zorn's Lemma. So let $A \neq \varnothing$ and $\bullet \notin A$ (for instance $\bullet \overset{\mathbf{df}}{=} \min \mathbf{On} \setminus A$) and let $\xi \colon \wp(A) \to A \cup \{\bullet\}$ be such that $\xi(X) \in X$, $X \in \wp(A) \setminus \{\varnothing\}$ (then $\xi(\varnothing) = \bullet$). Let now $G(h) \overset{\mathbf{df}}{=} \xi(A \setminus \mathbf{Rg}(h))$, for $h \in \mathbf{Set}$, and let $F \colon \mathbf{On} \to \mathbf{Set}$ be such that $F(\alpha) = G(F|_{\alpha})$, $\alpha \in \mathbf{On}$. Then, there exists the least ordinal $\alpha^{\star}$ with $F(\alpha^{\star}) \notin A$ (if such did not exist, the set A would be an one-to-one image of the proper class $\mathbf{On}$ which is impossible for sets, due to Replacement Axiom) and then the function $\eta = F|_{\alpha^{\star}}$ is a one-to-one mapping (so called ENUMERATION) from $\alpha^{\star}$ onto A which trivially defines a well ordering on the set A.

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Temporal Mereology without the Notion of Time

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Mereology is the study of parts and wholes. Polish logician Stanisław Leśniewski formulated what is now called classical mereology in 1927.

The talk will begin with a brief introduction of classical mereology. Then it will be explained why there is a need for mereology to be time-sensitive. In the main part of the talk, a proposal of axiomatization of such mereology will be presented.

There are two major approaches to mereology and time: perdurantism and endurantism. Perdurantism assumes the existence of temporal parts, e.g., Mona Lisa today and Mona Lisa at the same day a hundred years ago are two seperate objects in the perdurantist view, and both are temporal parts of Mona Lisa as a whole. Endurantism treats the parthood relation as a relation-in-time. The presented system is perdurantist or, in other words, four-dimensional. Unlike most contemporary systems, it does not impose any structure on time — atomicity and atomlessness (gunkness) of temporal parts are independent from the presented axioms, just like atomicity and gunkness of "ordinary" parts are independent from the axioms in classical mereology.

The presented system also differs from contemporary systems in that it does not use the notion of time, which, in the speaker's opinion, does not comply to Leśniewski's nominalistic views. Instead of times, we use relation between the objects, taking inspiration from Tadeusz Batóg's works on axiomatization of phonology.

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Hypersequents and the Method of Socratic Proofs for Propositional Linear Temporal Logic

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Propositional Linear-Time Temporal Logic (PLTL) has been considered in a number of proof systems, ranging from tableaux methods [12, 5], natural deduction [3, 4] to sequent calculi [1, 6]. The sequent calculi and tableaux methods approaches are based on the similar reasoning: usually involving state-prestate rules and facing similar problems around loop-generating formulas. However, the labelled natural deduction approach employs a different reasoning that involves relational judgements, and thus, faces different obstacles, revolving around (the principle of) induction.

We are going to present two sequent-based proof systems for PLTL that follow that relational reasoning. The first one is a hypersequent calculus [7, 2] and the second uses finite sequences of sequents (called seqsequents [11], to distinguish these structures from hypersequents, as seqsequents work conjunctively). Seqsequents are used in proof systems known under the label *the method of Socratic proofs* [9, 8]. Both of the discussed proof systems use right-sided sequents and labels. We are going to address the problem of induction and looping formulas as dealt with within the two calculi. Another goal of this work in progress is to provide a comparison between the two systems, and possibly a translation between them.

Last but not least, the fact of using erotetic calculi (the method of Socratic proofs) allows for an analysis of the mentioned systems in the context of Inferential Erotetic Logic [10], hence opening the perspective for the modelling of reasoning concerning time and involving questions.

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Existence, Reality and Fiction

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As Willard Van Orman Quine wrote, *a curious thing about the ontological problem is its simplicity. It can be put in three Anglo-Saxon monosyllables: 'What is there?' It can be answered, moreover, in a word—'Everything'—and everyone will accept this answer as true. However, this is merely to say that there is what there is. There remains room for disagreement over cases; and so the issue has stayed alive down the centuries.* (*On What There Is, 1948*) In fact, the answer to this question is not so simple at all. For when asking what is, or in other words, what exists, one also asks what does not exist. As the Stranger of Plato's *Sophist* asserted, what exists is that which possesses any sort of power to affect another or to be affected by another, if only for a single moment, however trifling the cause and however slight the effect. (247 D, E) To exist, then, is to be capable of action or to be able to be subject to action. If, therefore, we merely present an object to ourselves, we do not grasp it immediately as something that exists. Existence requires something more, that something happen to this object, that it be entangled in some action, that it be the subject of something that happens. Therefore, the claim that a certain object exists is based on the conviction that this object is the subject of a certain state of affairs that obtains. So, one could say that object **a** exists, if and only if a certain state of affairs whose subject is object **a**, obtains. A slightly different question is the question of what is real. Real objects are contrasted here with fictional objects, virtual objects, *etc.* Of course, not every real object exists, while the question of whether every existing object is real is much more complicated. The concept of a real object can be defined using the concept of a state of affairs, which is necessary for a certain fixed object. One could say that object **a** is a real object if and only if every state of affairs which is necessary for object **a** obtains.

The lecture deals with several problems concerning notion of existence, reality and fiction as well as related ontological notions of possibility and necessity, existential dependence, *etc.* It provides an axiomatic characterization of these concepts within the framework of a multi-modal propositional logic and then, presents a semantic analysis of these concepts. The semantics is a slight modification to the standard relational semantics for normal modal propositional logic.

A Few Comments About the Connective "Aczkolwiek" in Polish

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We can connect two sentences with the Polish word "aczkolwiek" (equivalent to "although" in English). The question therefore arises: can "aczkolwiek" be the linguistic equivalent of one of the logical functors?

The paper is a search for the answer to this question.

History of Studia Logica

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We have been working on piecing together the history of Studia Logica for a while now. Our aim is to re-create the journal's history from its very beginning in 1934, to the first volume that initiated the series in 1953, up to the present day, chronicling its complicated — yet successful — journey to becoming a truly international periodical. We wish to share our progress along that path and present an up to date picture of our efforts.

Trends in Logic Conferences — A History

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This presentation tells the story of the conference series called Trends in Logic. The initial conferences held under that name were related to the fiftieth anniversary of Studia Logica. This (2024) year marks the twenty-fourth conference in that series. The history of Trends in Logic is intertwined with the history of Studia Logica, which we try to piece together for a while now.

Modal Structures over Lattices

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In this talk, we will present a modal framework that integrates modal worlds governed by different logical systems, each defined by sublattices within a shared lattice structure.

Dealing with worlds operating in different logic systems poses a challenge for the standard principles in normal modal logic. When evaluating a modal sentence $\Box A$ in such setting, it is not sufficient to inspect the truth of A in accessed worlds (possibly in different logics). Rather, ways of transferring more subtle semantic information between logical systems must be established.

We will introduce modal structures that accommodate communication between logic systems by fixing a common lattice L that contains as sublattices the semantics of each world. Our approach redefines necessity and possibility, based on a comparative analysis of assignment values across accessible worlds and the base lattice. The value of a formula $\Box A$ in a world with lattice L' will be defined in terms of the values of A in accessible worlds relativized to L' using the common order of L. We will explore simple instances where a formula can be said to be necessary/possible even though all the accessible worlds falsify it.

We will also examine frames that characterize dynamic relations between logic systems: classically increasing, classically decreasing, and dialectic frames. To exemplify the kind of issue one should face in this framework, we formalize the semantics of considering worlds operating in classical logic or logic of paradox.

Subnormal Modal Logics and Hyperintensionality

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Hyperintensionality as a property which might be attributed to some modal operators manifests itself withing a logical system as its not being closed under the extensionality rule, and in consequence as its not being closed under the necessitation rule or unprovability of Kripke's axiom in the system. Within the context of hyperintensionality we will place the questions regarding subnormal systems of modal logic characterized by the necessitation rule, and in particular the question of intermediate systems between the system **N** and the system **K**.

Acceptance and the Weak Assertions

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In my talk, I will discuss selected topics around the so-called assertion norms. The dominant position in this discussion is the epistemic approach, recognizing that knowledge is the norm of assertion (there are also many variations of this position, e.g. taking into account the perlocutions of the act of assertion or its social aspects). At the same time, in the theory of speech acts (but also more broadly in linguistic pragmatics), it is pointed out that the person making the assertion suggests in some way (implicate) that he believes, which may constitute evidence for the doxastic norm of assertion (or some iterated version of it).

My position is that the primary normative role for assertion is played by acceptance and not by knowledge or belief, which play such roles only secondarily in some contexts (in which we impose an epistemic or doxastic norm on acceptance). This approach does not undermine competing positions but assimilates them to some extent while avoiding some of their problems.

I will briefly analyze the concept of acceptance and contrast it with the concept of belief (I am based here mainly on Jonathan Cohen's approach from the turn of the 1980s and 1990s). Then, I will outline selected positive arguments for acceptance as the norm of assertion: from the principle *Ought Implies Can*, from the mental equivalent of the assertion and from the orientation of the assertion towards the recipient.

The main goal of the talk, however, is to point out the advantages of the acceptance-based approach when considering different types of 'weak assertions' (in a broad sense). The following types of such assertions I will discuss: *selfless assertion* (Lackey), *proxy assertion* (Ludwig), *group assertion* (Lackey, Tollefsen), *weak assertion* (in a narrow sense; Mandelkern & Dorst), *hedged assertion* (Benton & Van Elswyk), *bullshit assertion* (Kotzee).

The Calculus of Names — The Legacy of Jan Łukasiewicz

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With his research on Aristotelian syllogistic, Jan Łukasiewicz (1929; 1934; 1939; 1951/57) initiated a branch of logic called the calculus of names. This section deals with axiomatic systems that analyze various fragments of the logic of names, i.e., one that study various forms of names and functors acting on them, as well as logical relationships between sentences in which these names and functors occur. In this work, we want not only to present the genesis of the calculus of names and its first system created by Łukasiewicz. We also want to deliver systems that extend the first. In this work, we will also show that, from the point of view of modern logic, Łukasiewicz's approach to syllogistic is not the only possible one. However, this does not diminish Łukasiewicz's role in the study of syllogism. We believe that the calculus of names is indisputably Łukasiewicz's legacy.

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A Logic of Judgmental Existence and Its Relation to Proof Irrelevance

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In the constructive and intuitionist tradition, we can identify two closely related but distinct notions of a proposition φ being true, let us call them $true_1$ and $true_2$, informally:

(1) φ is $true_1$ $\Leftrightarrow$ we have a concrete proof a of φ

(2) φ is $true_2$ $\Leftrightarrow$ there exists a proof of φ

At first glance, these notions might seem interchangeable but there is a subtle and important difference between them. To call something true according to (1) we need to have the actual proof a, i.e., to know its construction. However, to call something true according to (2), we just need to know that there exists some proof but what exactly its construction looks like is not important.

The notion of being true in the sense of (1) is tightly associated with the idea of proof-relevant approaches to logic and mathematics (= proofs are treated as proper mathematical objects: it is not enough to know that a proposition is true, we also need to have its proof, i.e., to know its structure), while the notion of being true in the sense of (2) is connected with proof-irrelevant approaches (= proofs are not treated as proper mathematical objects: it is sufficient to know that a proposition is true, the proofs themselves do not matter beyond the fact that they exist, i.e., their structure is unimportant). From this perspective, we can view $true_1$ and $true_2$ as capturing proof-relevant and proof-irrelevant notions of truth, respectively: $true_1$ cares about the structure of proofs, $true_2$ does not. So, let us call a proposition φ that is $true_1$ as *proof-relevantly true* or simply *true* and a proposition φ that is $true_2$ as *proof-irrelevantly true* or simply *just true*.

In this talk, we introduce a simple natural deduction system for reasoning with judgments of the form φ *just true* to explore the notion of judgmental existence in the sense of (2), following Martin-Löf's methodology of distinguishing between judgments and propositions. In this system, the existential judgment φ *just true* (or with explicit proof expressions as $e \therefore \varphi$) can be internalized into a modal notion of propositional existence denoted as $\triangle\varphi$. This modality is closely related to truncation modality, a key tool for obtaining proof irrelevance. We provide a constructive meaning explanation for existential judgments and computational interpretation in the style of the Curry-Howard isomorphism for the corresponding existence modality.

The investigation of judgmental existence is directly motivated by [2] who informally considers a new judgment of the form φ *exists* as expressing the notion of "bare existence". The logic of judgmental existence itself is inspired by [1] and their judgmental reconstruction of modal logic. Formally, our system shares the most resemblance with their possibility logic and lax logic, however, we also allow existence premises of the form φ *just true* for elimination rules, not only true premises of the form φ *true*. In its present form, our system deals only with a fragment of propositional logic containing the existence modality $\triangle$ and implication $\to$.

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The Principle of Proportionality in Research on Argument Strength

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The aim of the paper is to explicate and discuss a principle (postulate) that underlies the previously developed model for evaluating arguments (2014) as well as counter-arguments (2019). It may be called *the principle of proportionality*, for it states that *the strength of entire argument should vary proportionally to the variations of the values assigned to its components.*

The *values* assigned to sentences of a given language are understood as their *acceptability* (*credibility*), which — in order to be able to talk about proportions — should be expressed within some numerical scale. For this purpose, the scale of (rational) numbers from the closed interval $[0, 1]$ is employed. Formally, the computation involves a certain partial function, i.e., *function of evaluation*, which assigns the values to some sentences, including the first premises of an argument under consideration. Such a function is being extended, step by step, to the set containing the remaining sentences, which form the argument, i.e., to its *intermediate conclusions* and eventually up to the *main conclusion*. The *strength* of the entire argument is defined as the value of the main conclusion in this new, extended function of evaluation. The process is carried out according to the structure of an argument in question, while respecting the principle of proportionality. Algorithms used during computation can be illustrated by means of diagrams, where the values correspond to the lengths of intervals, and the suitable proportions can be read by using Thales' theorem. Thus, in addition to their explanatory property, such illustrations can be regarded as providing an autonomous *diagrammatic* (geometric) method for argument evaluation. The above model is in line with the requirements specified by Walton and Gordon (2015, p. 509) for a satisfactory formalization of informal logic, but still needs to be developed to fully meet all of them.

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Gödel's 'Ontologisher Beweis' Enriched. Positiveness and Quantity of Reality

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We start with the famous sketch "Ontologisher Beweis" from 1970 [3] and present another version of Gödel's ontological argument in which we introduce the relational concept of *being more perfect than* taken from Leibniz. The reference to Leibnizian metaphysics is justified by original Gödel's onto-theological considerations the main structure of the derivations carried out in [4]. According to the main idea of ontological argument formulated by Leibniz in 1676, the so-called *perfections* attributed to the Absolute are *positive*. Gödel takes this path: he uses the primitive predicate "is positive" predicated of properties and proves the necessary existence of the subject of all positive properties. Usually, it is supposed that $S5$ modalities are used. Leibniz, however, also gives in [4] the interpretation of perfection similar to Anselm's idea in *Proslogion*. As he writes, every perfection is "degree or [that is] the quantity of reality or essence, as intensity is degree of quality, and force is a degree of action" [4]; due to the "quantity of reality" conveyed by the properties, they can be compared as more or less perfect relative to each other. In our proposal, we follow this intuition and we link the concept of positiveness with the intensionally understood relationship of being more perfect than. We define the Absolute as the subject of all and only those properties that are more perfect than all those properties that are not positive (i.e. negative or indifferent). In the frame of the proposed modal two-sorted theory, we prove e.g. that all and only properties that are attributed to the Absolute are positive and that although each property necessarily implies identity property, identity is not more perfect than any positive property. We show a model for the resulting theory and compare it with the known versions of Gödel's argument by Scott [1] and Anderson [2].

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Some Formal Aspects of the Concept of Stupidity

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The structure of intelligence, thus the structure of stupidity as well, is based on language. Therefore, this paper is focused on the analysis of a sentence, in particular questions, answers and decision chains treated as a sentence sequence. The intention is to examine whether it is possible to distinguish the sentences unquestionably stupid in the set of incorrect sentences by the means of simple formal tools (examples given come from school textbooks). Judging questions, as well as answers, as "stupid", depends on the informational content of a sentence, as well as the cost of acquiring such information. The reasons for making stupid decisions, though, are analogous to the reasons for considering certain events as random or chaotic.

Two Formal Interpretations of Edward Zalta's Abstract Object Theory. Comparison of Semantics by Scott and by Aczel

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Edward Zalta's Abstract Object Theory (AOT) is a formal system inspired by Meinong's ontology and applied to fragments of different known Platonic ontologies (e.g. Leibniz, Frege, Bolzano, ...). Zalta distinguishes between ordinary and abstract objects, introducing two kinds of instantiation of properties: exemplification and encoding. Zalta expresses the latter distinction in a modified language of second-order logic using two types of atomic formulas: Fx and xF respectively. Based on this, he constructs a deductive theory that assumes a comprehension schemata for both types of attribution. Comprehension schema for encoding states that for every property, there is an abstract object that encodes it. AOT has two different interpretations. The first semantics, invented by Scott, was designed for the monadic fragment of AOT. We will show its philosophically unsatisfactory properties and explain why they are so (e.g. it is extensional and enforces the truth of the sentence that for any property, either every abstract object exemplifies it, or none). Semantics inspired by Aczel's remarks is intensional and avoids some problems of the first semantics. However, to prevent violation of

the axiom of foundation, it requires to assume a set of urelements, which serve as 'proxies' of abstract objects. In my talk, I present the axiomatics of non-modal AOT and discuss the relations between both semantics. In particular, I explore the conditions under which Scott's and Aczel's semantics are equivalent. I also provide appropriate proofs for some semantic observations that are not present in Zalta's lecture of AOT.

Referring to Modal Worlds via Definite Descriptions

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We will present our recent results on modal logics with operators expressing definite descriptions. In particular we will introduce an extension of (propositional) modal logic with operator $@_\varphi$ indexed with an arbitrary formula φ, whose intended reading is "in the modal world in which φ holds". For example $@_p q$ states that q holds in the world in which p holds. Note that with such operators we can also express much more complex properties, for instance, $@_{(\neg\Diamond\top)}\top$ states that there exists exactly one modal world which has no outgoing accessibility relation.

This mechanisms of reference resembles approaches known from hybrid modal logics and modal logics with counting operators. As we show, these logics share some metaproperties, but there are also significant differences between them. We show that checking satisfiability of formulae in our logic has the same computational complexity as in the case of formulae with counting operators (with numbers encoded in binary).

Theorem 1. *Checking satisfiability of formulae in modal logic with definite descriptions is ExpTime-complete.*

On the other hand, if the "definite descriptions", that is, formulae in the subscript of our @-operators do not mention modal operators ($\Diamond$ and $\Box$), then the complexity drops to PSpace. Since satisfiability checking in basic modal logic is already PSpace-complete, it means that adding this kind of definite descriptions has no negative impact on complexity.

Theorem 2. *Checking satisfiability of formulae in modal logic, with definite descriptions not mentioning $\Diamond$ and $\Box$ in the subscripts of @-operators, is PSpace-complete.*

To analyse expressive power of definite descriptions, we introduce a tailored bisimulation notions. As we show, it satisfies the bisimulation invariance property (bisimilar worlds satisfy the same formulae) as well as the Hennessy-Milner property (in finitely branching models, worlds which satisfy the same formulae are bisimilar). This allows us to show, among others, the following result.

Theorem 3. *Modal logic with definite descriptions does not allow us to define the following operators: the "everywhere" (universal) operator, the difference operator, the "somewhere" operator, and the counting operators $\exists_{\leq n}$, for each $n \geq 2$.*

Knowledge in View of Multimodal Epistemic Logics

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Logics of knowledge and belief, often called multimodal epistemic logics, aim to characterize knowledge, belief, and their mutual dependencies. Usually, yet not always, they consider knowledge as a species of belief. Generally speaking, sometimes knowledge is defined in terms of belief (e.g. as true conviction or true justified belief), while in other cases some 'bridge' postulates/axioms connecting knowledge and belief are stipulated. In my talk I am going to use tools and results of multimodal epistemic logics in the analysis of the old philosophical idea, according to which knowledge is true belief 'plus something else'. What this 'something else' component is, remains controversial. After the famous Gettier's paper, philosophers are aware that it is not just mere justification (to be more precise, being justified in believing). However, it is not my aim to resolve this philosophical problem. What I am going to argue for is that regardless of how the problem is solved, we end with a doxastic concept of knowledge which absorbs the truth requirement. Of course, this conclusion is not absolutely binding, but it remains in place as long as the relevant claims of multimodal epistemic logics are taken into account.

Paradoxes of Multimodal Epistemic Logic: Doxastic Infallibility, Doxastic Omnipotence, and More

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No paradigmatic, commonly accepted logic of knowledge and belief has been elaborated so far. The existing proposals suffer from yelding paradoxical statements. These usually are not contradictions by themselves. To speak generally, their paradoxicality lies in ascribing to beliefs properties one usually would not be willing to ascribe, in particular features supposed to be exhibited only by knowledge. When a logic of knowledge and belief is interpreted as speaking about propositional attitudes of cognitive agents, the paradoxical statements ascribe to such agents powers that no human cognitive agent possesses. In my talk I will concentrate upon two paradoxical statements of this kind. The *paradox of infallibility* is the claim that beliefs yield the truth of what is believed. The *paradox of doxastic omnipotence* is symmetric to it: the truth of a proposition yields that the proposition is believed. Both paradoxes are provable in some multimodal epistemic logics. In my talk I am going to identify sources of these paradoxes and point out some of their equally paradoxical consequences.

On Disposition Terms

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Rudolf Carnap was the first who noticed the difficulties connected with defining disposition terms. An attempt at defining them in the classical way leads to paradoxes.

The present paper proposes a way out of this theoretical deadlock. The starting system is elementary ontology enriched with specific axioms (A1–A4) which characterise the phrase *x is subordinated-to y*. The language of this system is subsequently extended to include nominal variables (a, b, c) referring to objects which change in time. Successive axioms (B1–B4) determine the phrase *x is a subreference of a*. What is understood by the subreference of the name a is an object to which the name refers and which is only its temporary carrier. Thanks to a distinction between an object and its subreferences we shall avoid a theoretically problematic quantification with a temporary variable. The functors of disposition (D) and conditional disposition (DW) are introduced by definition. They allow to define disposition terms in the classical way free from the above paradoxes.

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Credence of Conditionals — A Network Approach

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In the talk we propose a network approach to the problem of computing probabilities of conditionals. We consider a system of agents, each of which has some beliefs concerning unconditional facts (their systems of beliefs differ), who want to form beliefs about conditionals. The agents take each other's opinions into account; i.e., they mutually influence one another. Their epistemic attitude towards conditionals is expressed as subjective probability (credence); i.e., the agents assign a real number from the interval $[0, 1]$ to each conditional in question.

We present a formal account describing this situation and construct inductively a family of random variables $\xi_\alpha : \boldsymbol{W} \to [0, 1]$ which represent the agents' individual attitudes towards the conditional α. This allows one to define the notion of the collective opinion of the system of agents about α. The model might be seen as a natural extension of the framework for modal logics: the influence matrix might be viewed as a generalization of the accessibility relation R in Kripke models, consisting in assigning (probabilistic) weights to the edges of the graph — and there is also a probability distribution $\mathbf{P}$ on the set of worlds $\boldsymbol{W}$.

The model is flexible enough to incorporate additional assumptions concerning the formation of beliefs, the mutual influences, the class of conditionals that is evaluated, etc. It can be used to describe how beliefs within the network of agents propagate, how mutual influences result in forming a state of opinion equilibrium — and, for instance, which of the agents exert the strongest overall influence on the network (i.e., are potential successful lobbyists). The model allows to analyze both the agents' individual beliefs and — as a result — also the "collective network beliefs". So the evaluation of the probability of conditionals is relative both to the individual agents' factual beliefs and to the states of beliefs of other agents (including their beliefs concerning conditionals). We also show that the network model might be viewed as a generalization of some known models as, in a special case, results like PCCP or the formula for conjoined conditionals can be proven.

The model does not prejudge what the truthmakers are for particular propositions (in particular — for conditionals) nor what exact laws are satisfied (in particular, whether PCCP holds). We might view it as being more general than some well-known models; indeed, if we make some additional (simplifying) assumptions, then the conclusions obtained with it will be equivalent to those of the Stalnaker Bernoulli model.

The Conditional Connective from the Point of View of Inferentialism

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Inferentialism is a position according to which the meanings of expressions of a language L are constituted by correct inferences in which these expressions occur. By inference, we understand the relation that occurs between a set of premises X (expressed by sentences of L) and a conclusion, which is a certain sentence A of L. In this convention, inferentialism implies that the meaning of an expression t can be established by identifying all pairs $\langle \mathrm{X}, \mathrm{A} \rangle$ such that $\mathrm{X} \vdash \mathrm{A}$ is a correct inference and t occurs in some sentence from the set X or in A.

Let's assume that we have thus characterized a certain language L in which the vocabulary consists exclusively of classical logical constants. Then we add a non-classical connective $\rightarrow$ to L, which we would like to interpret as a conditional connective, obtaining a broader language $\mathrm{L}(\rightarrow)$ in this way. The natural problem arises is which new inferences within $\mathrm{L}(\rightarrow)$ should be recognized as correct in order for the connective $\rightarrow$:

1. to have the intended interpretation as the conditional connective;
2. to be recognized as a logical constant.

In the presentation, we will attempt to provide at least a partial answer to this question. In particular, we analyze the relationships between inference rules in the extended language $\mathrm{L}(\rightarrow)$, which seem very natural, however, are not compatible.

A Solution to the Problem of the Meaning of 'Meaning'

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The word 'meaning', itself lacking precision as regards its meaning, requires formal-logical explication. Searching for its precision has been and still is the goal of numerous attempts undertaken in the literature pertaining to philosophy and logic of language. There exist different philosophical conceptions concerning the nature of meaning and various theories of meaning, but none of them is a general theory of meaning as a semantic-pragmatic theory.

The present work embarks on providing an answer to the question: What is the meaning of 'meaning'? The aim of the paper is to outline the foundations of a certain general, formal-logical theory of meaning and denotation which explicates these crucial notions of current general semantics and pragmatics. In the theory, according to the *token-type* distinction of Peirce, language is formalized as a creation of double ontological nature: first, at the *token*-level, as a language of *tokens* (understood as material, empirical objects, placed in time and space) and then, at the *type*-level, as a language of *types* (understood as abstract objects, as classes of *tokens*). The basic concepts of the theory, i.e. the notions *meaning* and *denotation* of well-formed expressions (*wfes*) of the language are defined at the *type*-level, however, by means of some primitive notions introduced on the *token*-level. The definition of the notion of meaning makes reference to the ideas of L. Wittgenstein and K. Ajdukiewicz, of treating the notion as a creation determined through the way of using expression-*type*. The meaning of a *wfe* is defined as an equivalent class of the *relation possessing the same manner of using wfe-types*. In accordance with the well-known differentiations *Sinn-Bedeutung* of G. Frege and *intension-extension* of R. Carnap, the notion of denotation differs from that of meaning and, in the paper, is defined by means of the *relation referring of wfe-types* to objects of reality described by the given language.

Complexity-Optimal Decision Procedure for Modal Logic with Identity

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Modal Sentential Calculus with Identity, MSCI in short, is the basic propositional modal logic K augmented with the identity connective $\equiv$, or, to put it differently, it is the well-known logic SCI proposed by Suszko and Bloom [1, 2] whose language is enriched with the necessity operator $\Box$. The semantics of MSCI reconciles both the Kripke-style semantics of K and the algebraic semantics of SCI.

In my talk I will present a tableau-based decision procedure for the satisfiability problem of MSCI, $\mathsf{TP}_{\mathsf{MSCI}}$. I will argue that its execution only requires polynomial space in the size of the input formula. Since MSCI inherits the PSpace lower bound for the satisfiability problem after K, the memory consumption of $\mathsf{TP}_{\mathsf{MSCI}}$ turns out to be optimal. Thus, as a byproduct of $\mathsf{TP}_{\mathsf{MSCI}}$ we obtain tight complexity bounds for MSCI-SAT.

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