XXVI Conference

Applications of Logic in Philosophy and the Foundations of Mathematics

Szklarska Poręba

Poland

8–12 May 2023

XXVI Conference

Applications of Logic in Philosophy and the Foundations of Mathematics

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The Conference is organized by:

Department of Logic and Methodology of Sciences, University of Wrocław

Institute of Mathematics, University of Silesia

Department of Mathematics, University of Opole

Under the auspices of:

Polish Association for Logic and Philosophy of Science

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First published 2023

Publisher:
Department of Logic and Methodology of Sciences, University of Wrocław, Wrocław

ISBN 978-83-940690-7-0

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Editorial note

(EN) means that the talk is presented in English, (PL)—in Polish.

Infinity of Euclid's Straight Line and Circular Inversion

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1. Mathematics tamed the concept of infinity through numbers: Cantor's cardinal and ordinal numbers or inverses of infinitesimals developed by Euler. Sharing a similar understanding of finitude, be it a structure of natural numbers, or an Archimedean field, these two approaches diverge regarding infinity: Cantor ordinal arithmetic does not satisfy standard rules, e.g., commutativity, while inverses of infinitesimals comply with all the laws of an ordered field. The field of Conway numbers includes ordinal numbers and infinitesimals in one structure. Thus Euler's idea of infinity as an inverse of infinitesimals prevailed over Cantor's arithmetic of ordinal numbers [1], [2], [4]. We aim to implement the idea of infinity as the inverse of infinitesimals into Euclid's geometry.

2. The concept of infinity (apeiron) occurs in the definition of parallel lines and the Fifth Postulate, which evokes a line "being produced infinitely". Some view this proviso as potential infinity, meaning reiterated prolongation of a straight line [7]. We present a model of a semi-Euclidean plane to demonstrate that potential infinity does not guarantee straight lines satisfy the parallel axiom. It is a subspace of the Cartesian plane over the non-Archimedean field of hyperreal numbers in which angles in a triangle sum up to π, and the parallel axiom fails [3].

Standard models of non-Euclidean plane involve a non-Euclidean representation of straight lines (Poincare) or angles (Klein), in our model, both straight lines and angles are Euclidean. As all triangles in our model are also Euclidean, locally, it is the Euclidean plane, yet straight lines are "too short" to meet the parallel postulate. We propose a characteristic of infinite straight lines in terms of Euclid's geometry alone with no reference to the concept of "being produced infinitely" or a number.

3. Since a semi-Euclidean and Archimedean plane satisfies the parallel postulate [6], what makes straight lines "too short" are infinitesimal lines and angles. We introduce infinitesimals by negating Aristotle's axiom and thus do not refer to numbers [5]. Then we study inverses of infinitesimals.

The geometric counterpart of a multiplicative inverse operation in a field is the construction of circular inversion (Elements, III.37). We show that it guarantees straight lines meet the parallel postulate and present an equivalent version of that construction. It is that for any base and acute angle, there exists an isosceles triangle. It is an equivalent version of the Fifth Postulate showing inverses of infinitesimals exists in a plane.

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The Legacy of Euclid

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The talk aims to address the following topics (on demand by the public due to time constraints):

1. Brief history of the *Elements*. Theories making up the *Elements*.

2. Specific Euclid theorems still present in modern mathematics: the sum of angles in a triangle, Pythagoras theorem, the law of cosines, Thales theorem, prime factorization, there are infinitely many primes, Euclid's algorithm, the algorithm of successive subtraction (continued fractions, Baire space), and regular solids.

3. *Elements* in mathematics education. Christopher Clavius and the Jesuit system of schools. *Elements vs.* Aristotle's logic. *Elements* in contemporary school education.

4. Methodology of mathematics. *Elements vs. Grundlagen der Geometrie Hilbert*:

a) Models of Euclid's and Hilbert's systems of geometry. Cartesian planes over Euclidean and Pythagorean fields.
b) The role of linear order in both systems.
c) Finite *vs.* infinite straight line.
d) Two concepts of mathematical proof.

5. Mathematical techniques.
5.1 Geometry. Sum of angles in a triangle. Area of a triangle ($\frac{1}{2}ah$, $\frac{1}{2}ab\sin\alpha$). Squaring a figure. Area of similar triangles. Angles in a circle. Circular inversion.
5.2. Euclidean induction.
5.3 Similar figures

a) The theory of proportion as a basic technique of Greek mathematics (Archimedes, Heron, Ptolemy).
b) From the theory of proportions to an ordered field (Descartes' 1637 revolution).
c) Euclidean geometry in Newton's derivation of series for sine and arcsine (1700).
d) Trigonometry encodes the theory of similar figures. Euler's formula $e^{ix} = \cos x + i\sin x$. The rise of the modern trigonometry (1748).
e) *Elements* and 1872's constructions of real numbers.
f) *Elements* and Hilbert axioms of real numbers.
g) Equivalent versions of the parallels postulate: sum of angles in a triangle $= \pi$, Pythagorean theorem. The Euclidean metric and inner product.
h) Euclid's geometry and the calculus: $lim_{x=0}\frac{\sin x}{x} = 1$, definition of the number π. Tangent.

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Thought Experiments Are Not Deductive Arguments in Disguise: A Model of the Method of Cases in Analytic Philosophy

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Thought experiments are considered to be one of the essential tools in philosophers' argumentative repertoire. Moreover, they are exceptionally popular in contemporary analytic philosophy. Their prevalence stems from the fact that they proved to be an effective method for conceptual analysis. Once philosopher wants to provide a sounding account of a given concept p, the best way to control its applicability is to check it against some imaginary cases. Such cases often take a form of elaborative narratives, frequently involving far-fetched, outlandish or science-fiction scenarios. Although their importance in philosophy is generally acknowledged, their actual function is a subject to many metaphilosophical debates. According to one popular view, developed most notably by John D. Norton in his series of papers (1996, 2002, 2004), thought experiments are simply (deductive) arguments disguised by its narrative or pictorial form. Even though Norton's account is compelling and sheds an original light on our understanding of the function of thought experiments, in my talk I am going to argue that his account is wanting and does not illustrate in a full-scale the actual function of thought experiments. In order to show why his argumentative account does not hold, instead I am going to propose a model of thought experiments, showing their logical structure and making room for amendment of Norton's omissions.

On Prime Numbers

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The talk is concerned with the problem of building countable models for first-order languages from the perspective of the classic paper of Rasiowa and Sikorski [6]. The notion of a Rasiowa-Sikorski set of formulas of an arbitrary countable language L is introduced. Rasiowa-Sikorski sets form a dense subset of the family of Lindenbaum sets of a given theory equipped with the well-known compact Hausdorff topology. Each Rasiowa-Sikorski set defines a countable model for L. Conversely, *each* countable model for L is determined, up to isomorphism, by some Rasiowa-Sikorski set. Consequences of these facts are presented.

Rasiowa-Sikorski sets enable one to build substitutional semantics for first-order logic. This is due to the fact that the satisfaction relation in the model $\boldsymbol{A}_\Delta$ corresponding to a Rasiowa-Sikorski set Δ is expressed in a straightforward way in terms of "double" substitutions of variables in the formulas of L.

The problem of defining Rasiowa-Sikorski sets is crucial. Lindenbaum-Tarski algebras of elementary theory as well as Rasiowa-Sikorski Lemma are the key ingredients here. Constructions of Rasiowa-Sikorski sets are based on the complete Boolean algebras of regular open sets that arise from refined posets over Lindenbaum-Tarski algebras. The main principles of Boolean valuations of first-order languages in the algebras of regular open are presented. The above purely model-theoretic and algebraic constructions are applied to the elementary Peano arthmetic and number theory. Some consequences are shown.

This investigation makes use of the facts presented in [1]–[7].

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Few Words on Metric Ultraproduct

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joint work with **Krzysztof Majcher** and **Martin Ziegler**

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The ultraproduct construction is playing an important role in logic, model theory, group theory, and algebra. During my talk, I will explain a more general construction of the metric ultraproduct, especially the metric ultraproduct of groups equipped with an invariant metric. Its importance to group theory became apparent recently and the topic is intensively studied. I will concentrate, in this context, on some properties of groups (simplicity and amenability). I will explain, in elementary terms, the uniform metric simplicity of groups; including examples and potential applications.

Outstanding Female Philosophers from Lviv-Warsaw School and Their Accomplishments

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Research on the Lviv-Warsaw School has increased in recent years. More initiatives are being created to commemorate and bring to light the achievements of Polish philosophers, heirs to the ideas of Kazimierz Twardowski. However, a topic still overlooked is the scientific activity of women associated with the LWS. In order to fill this gap in the research on the history of Polish logic, I have devoted my doctoral dissertation to this idea. Its topic is as follows: "A host of women — outstanding female philosophers from the Lviv-Warsaw School" and is being written under the supervision of Professor Andrzej Indrzejczak.

Although the number of women associated with the LWS may seem small, by comparison we can point to the Vienna Circle where only two women are mentioned. Tracing the fate of women in the history of philosophy and higher education, the number of Twardowski's female disciples may be surprising. Taking scientific achievements as a criterion, we can consider Eugenia Blaustein, Izydora Dąmbska, Daniela Gromska, Maria Kokoszyńska-Lutmanowa, Seweryna Łuszczewska-Romahnowa, Helena Sloniewska, Janina Kotarbińska, Maria Ossowska, Janina Hossiason-Lindenbaum and Deborah Vogel as the most important representatives of the LWS. In my paper, I will introduce the profiles of Polish women philosophers who were disciples of Twardowski — an advocate of women's education. Therefore, I will not omit the topic of women's access to higher education in Poland, as I believe it is important to emphasize the difficulties faced by the women I mentioned above. I would like to devote a significant part of my paper to presenting their achievements in the field of logic. I will briefly recall the variety of topics covered by them. Some of their findings remain forgotten today, so my speech may prove to be an inspiration for further research. There is no doubt that the pre-war scientific activities of Janina Kotarbińska (Dina Sztejnbarg) or Janina Hossiason are examples of outstanding achievements in Polish logic. I will conclude by trying to answer the question: can modern logic learn something from the women pioneers of Polish philosophy?

Towards a General Proof Theory of Term-Forming Operators

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Complex names are very important components of communication. Term-forming operators, like those forming descriptions or set abstracts, are often used as formal tools for building such terms in artificial languages. However, the role of complex terms is totally neglected in modern logic. So far, two different attempts to develop a general theory of such operators were provided. One is due to Scott, Corcoran, Hatcher and Da Costa, and the second was provided by Tennant. In the talk we will sketch a proof theoretic approach to such theories and its possible specification to some concrete cases.

Relating Logics: Theory and Applications

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When examining reasoning in logic, we usually consider affirming a logical relationship between the premises and the conclusion so that any situation which assigns a meaning of true to the premises, must assign a meaning of true to the conclusion. However, in many cases, there are non-logical relationships that can greatly contribute to the recognition of reasoning.

Such relationships can influence the logical value of a sentence (the meaning of a logical constant) but are different from that determined by only using the logical values of its components. Consider the following standard example:

$$(\diamondsuit) \qquad \frac{\text{If the thief tries to rob your house, you call the police} \qquad \text{If you call the police, the thief starts to run away}}{\text{If the thief tries to rob your house, the thief starts to run away}}$$

The inference $(\diamondsuit)$ can be seen as an instance of the transitivity of classical material implication if we read "if. . ., then. . ." as the material implication. Thus, in a classical setting, in which the truth values of the implications are determined only by the truth values of the subformuls, the inference $(\diamondsuit)$ is correct. But clearly, the conclusion of ($\diamondsuit$) is bizarre as there is no direct logical connection between the thief trying to rob your house and the thief's running away! Thus the classical material implication is unable to account for the causal relationship between the thief's actions and your actions which leads the thief to run away. Our challenge is to accommodate extra, non-logical, relationships such as "causation", to block ($\diamondsuit$). But "causation" is just one example of a non-logical relationship.

In many inferences, similar relationships of a non-logical nature also appear. These include not only causal relationship but also temporal, analytical, content-based, preferential, structural relationships etc. These are intensional relationships, because they are irreducible to the properties of their elements. To express additional intensional relationships, new connectives can be added to the language, which, besides finding dependencies between logical values, also allow for stating the existence of other non-logical relationships. Technically, in interpreting such connectives in the model, in addition to the logical value of individual sentences, we include the valuation of a pair of sentences. The connectives of this kind are called *relating connectives*. The basic idea behind relating connectives is that the logical value of a given complex proposition is dependent on two factors:

(i) the logical values of the main components of the compound proposition,

(ii) a valuation of the relation between these components.

The latter element is a formal representation of an underlying intensional relationship that exists between the main components of the proposition, as in (i), but which may not depend upon their logical values. Thus (i) and (ii) together give rise to a connective which is non-extensional. Including such non-extensional relating connectives in the language allows us to represent non-logical relationships, such as causation, in the syntax of our logic. We can then use the traditional connectives to form extensional combinations of these non-extensional relationships. That is, if we define a logic with relating connectives, we are in fact able to cover some relationships that are not extensional; however, the logic itself is extensional.

Relating Logic is a logic of relating connectives (just as Modal Logic is a logic of modal connectives). The basic approach to Relating Logic is two-valued and with one relation in a model to interpret relationships between sentences. However, more complicated implementations are also possible.

In the presentation, we would like to discuss the following problems related to relating logic (with some selected references):

1. motivations
2. the outline of history ([4], [10], [7], [5], [6], [30], [31], [36], [37], [14], [29])
3. the proper definition ([21])
4. possible semantic structures ([21], [15])
5. the fundamentals of proof-theory ([2], [19], [3], [34], [20], [33], [1], [9], [16], [35], [18], [28], [27], [26])

6. applications of relating logic ([32], [8], [23], [22], [13], [12], [24], [11], [17], [19])
7. the process of institutionalization ([25]).

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Tableau Metatheory for Propositional Logic

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In the presentation we explore the relationships between widely understood propositional logic and the tableau proofs. We set out a methodology for constructing adequate tableau systems for different sorts of propositional logic that can be determined with generalized relational semantics. The similar strategy, but relative to the context of syllogistic logic was implemented in the paper [8] where the tableau metatheory was developed for syllogistic languages.

The initial and basic idea that motivates our approach is the following observation: the syntax and semantics of a given logic always determine a minimal syntax and structure of a tableau system for the logic along with other properties. So, it seems reasonable to propose some metatheory for a class of logics that are very similar from a syntactical and semantic viewpoint. One can naturally ask: how can we benefit from such a theory? We show how the metatheory we propose makes the process of defining adequate tableau systems much easier. Since we define very general notions that cover all tableau issues concerning a very wide class of propositonal logics and demonstrate crucial relationships between them, we can apply them automatically in particular cases, concentrating only on remaining details, specific to the particular propositional logic under examination. The ideas behind the presented approach were outlined relative to certain contexts in [4], [5], [7], [6]. They have been developed and improved here to cope with propositional logic. Additional, and probably more important advantages of this metatheory that open future interesting research areas in tableau-proofs-approach are listed at the end.

At the beginning let us notice that in the theory of tableaux, we can distinguish three kinds of approaches to their construction.

First, tableau proofs are either with a signed or unsigned language. *Signed* means that in a language of tableau proofs, additional symbols to denote logical values are used, while *unsigned* means that in the tableau proofs we do not use any such symbols for logical values. This is a traditional division. It is worth mentioning that the first appearance of the tableau method in Beth [1] used signed tableaux (see also [11]).

Second, tableaux either can be built with languages that contain labels that denote possible worlds (points of relativization), or can be built with languages without labels (see [2], [3]).

Third, a way of construction of tableaux (and branches) can be divided into *nodes based on formulae* or *nodes based on sets of formulae*. While the former seems to be usually of didactic form (this approach is for example extensively outlined in [13]), the latter is more paradigmatic (see for example [2] and [3]) and has a strong connection to sequent calculi.

In our paper we set out a generalization of all these aspects of tableau theory for propositional logic that can be tweaked to any of the more specific forms mentioned above. We obtain the required generality by using *generalized labels* (but we call them *labels*). The generalized labels can obviously code points of relativization, but also other important semantic (and not only semantic) aspects, such as logical values, an object-property of belonging/non-belonging to a denotation of a given term or possibly other things. Other possible uses of generalized labels can be: (a) tracking the origin of decomposed formulae (see for example case for relevance tableaux [12] or paraconsistent tableaux [9], [10]), (b) quasi-negation (-)/quasi-assertion (+) in many valued tableaux or FDE tableaux, including Routley Star (see for example [13]). Moreover, since we are developing a metatheory, we should be open to new roles for labels, which may only emerge when we use the metatheory for particular cases of new propositional logics.

Last but not least, our approach is of the nodes-based-on-sets-of-formulae kind but different to others of this kind in at least two ways. First, decomposed expressions are collected rather than deleted, so at any stage of a tableau branch, the full information on the proof is still available. So additional constraints on tableau rules can be imposed directly on the inputs of rules (since in our approach tableau rules are sets of n-tuples of sets such that the input is always a proper subset of the output). Second, in fact we do not use direct tree structures with nodes, since branches are strictly monotonic sequences of input/output sets, while tableaux are sets of such branches (selected with some additional conditions). However, the remaining approaches (with nodes based on formulae as well as sets of formulae) can be defined using our apparatus, since our approach is more abstract.

In the presentation we determine a language that includes what is common to any propositional logic, whilst not excluding richer grammatical constructions, since we want our theory to cover all possible propositional languages.

Then we introduce general semantic structures for propositional logics, as well as notions of satisfiability which together form models. It turns out that models semantically determine particular propositional logics, which is illustrated by examples. In keeping with the spirit of generality, we propose a syntax for our tableau language to describe properties of general semantic structures in tableau proofs. Finally, we introduce a notion of a set of satisfied expressions.

The next part is completely devoted to problems of what is generally a tableau rule, a branch, a tableau etc. A multistage, set-theoretical construction of these notions is proposed. As a novelty in the field of propositional logic a branch consequence relation as the tableau counterpart of a semantic consequence relation for a given logic is also introduced.

All important relationships between these notions are proved to present general connections between the tableau notions and general semantics. This establishes sufficient conditions for a complete and sound tableau system that are in this section formulated.

In the last part some perspectives for the further development of the tableau metatheory are presented.

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Topological Interpretation of Wittgenstein's Logical Atomism and Perzanowski's Theory of Analysis and Synthesis

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In this paper I consider some of Wittgenstein's theses contained in his *Tractatus Logico-Philosophicus* concerning the ontology presented there. The work on logical atomism is a joint work of Russell, Wittgenstein and to some extent Whitehead. The present paper is a continuation of the research contained in (Kaczmarek 2019a and 2019b). In the second half of the 20^{th} century, some formal interpretations of Wittgenstein's concept emerged. The proposal of Wolniewicz, through which the basic theses of the *Tractatus* are interpreted by means of the lattice of elementary situations as well as the proposal of Perzanowski, who proposed his combinational ontology are examples of this. Both theories were inspired by the ontology of Leibniz and Wittgenstein. Perzanowski, within the framework of his theory, defined the basic concepts of Leibniz's and Wittgenstein's ontology, including: situation (state of affairs), possible world, co-possibility, God, monad and others. In the paper we show that Wittgenstein's approach can be put in the language of lattice theory, but also in the language of general topology. The topological approach allows us to consider an atomistic ontology and one in which we have no atoms. I call lattices constructed from topological spaces in which atoms can be indicated and spaces in which atoms are not present hybrid lattices. In the final section, I will give a fragment of the topological account of combinational ontology and some theorems of topological ontology in the hope that they shed some light on how to understand and interpret the theorems of Wittgenstein's ontology.

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Edge Colourings and Representations of Chromatic Algebras

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In an edge n-colouring of a complete graph, each triangle of edges consists of either one colour, two colours or three colours: monochromatic, dichromatic or trichromatic. We explore edge-colourings determined by disallowed triangle colour combinations, but also requiring others. Thus, disallowing monochromatic triangles restricts to edge-coloured complete graphs within the Ramsey bound $R(3,3,\ldots,3)$. But what if in addition to disallowing monochromatic triangles, we also impose the dual constraint that all remaining colour combinations (trichromatic and dichromatic) are present: is it possible to find such a network? These are natural combinatorial considerations in their own right, but there is an additional motivation by way of the algebraic foundations of qualitative reasoning, which finds wide application in AI settings around scheduling [1], navigation [6, 9] and geospatial positioning amongst others [10]. The constraint language underlying typical qualitative reasoning systems determines a kind of non-associative relation algebra, in the sense of Maddux [8], thus making the algebraic approach to constraint satisfaction available. This approach to qualitative reasoning is attracting considerable attention from a theoretical computer science perspective; see [2, 3, 5] for example. The inverse problem of deciding if a suitably defined non-associative algebra arises from a concrete constraint network is shown to be `NP`-complete in [3], whereas the same problem for associative relation algebras was shown to be undecidable in [4].

The present work focusses on a natural family of combinatorially intriguing cases, whose algebraic rendering we dub *chromatic algebras*. Representability of chromatic algebras boils down to existence of colourings satisfying various disallowed/required triangle constraints, that we find have nontrivial solutions, and provide some novel extensions of classically understood connections between associative relation algebras and combinatorial geometries, such as in Lyndon [7].

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Logic of Actions and Norms

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In the two part lecture I will present the fundamentals of formal action theory and elements of deontic logic that can be built as extensions of that theory. I will give an account of the main modeling principles formulating the folklore of the discipline and recall some well known results. I will also point out some works I was personally involved in over the last 15 years.

Part I *Deontic logic of single-step actions*

I will start this part with some philosophical remarks on actions, including various attempts to define what an action is and principles that underlie formalization of actions. Actions themselves, as well as their descriptions, consist of some simple (atomic) elements. I will present how complex actions can be described using Boolean algebra. Then I will introduce deontic operators of prohibition, permission and obligation. I will show how the deontic characteristics of complex actions defined in terms of deontic operators rely on the deontic characteristics of elements of those complex actions. The study of relations between the deontic operators will follow. Finally, I will show some examples of applications of logics built in that way.

Part II *Sequentially composed actions and norms defined on them*

Actions usually form sequences. Formalization of sequentially composed actions enriches deontic action logic with new insights and problems. I will present an algebraic account of sequentially composed actions taking into account successful and unsuccessful realizations of actions. Then, I will show how the deontic properties of single steps may be extended to the properties of sequences of actions. Sometimes, however, the deontic properties of sequences reflect the whole sequence and its final result rather than their single step elements. As an example of a formal account of such phenomena I will introduce a goal oriented notion of obligation.

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Some Systems for Formalising Sentences Containing Definite Descriptions by a Binary Quantifier

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A definite description is an expression of the form 'the so-and-so'. On the face of it, definite descriptions are singular terms: they purport to refer to the sole so-and-so. Since Peano it is customary to formalise them by a term forming operator: ι takes an open formula F and forms the singular term ιxF, binding the variable x. This accords with the grammar ordinary English, where the definite article 'the' forms a a complex expression that can be the subject or object of a sentence, e.g. 'the present King of France', from a noun phrase, in this case 'present King of France'. It is probably the most common way of treating definite descriptions in formal logic.

There is, however, an alternative method that builds on a distinctly Russellian point. According to Russell, a definite description has no meaning in itself, but only in the context of a complete sentence. This is because upon Russell's celebrated analysis of sentences of the form 'The F is G' the definite description 'the F' disappears: 'The F is G' means 'There is exactly on F and it is G'. Two things are worth noting. (1) Russell, too, utilises the term forming operator, but its use is defined in the context of formulas only. $[\iota xF]\ G(\iota xF)$ is contextually defined as $\exists x(\forall y(Fy \leftrightarrow x = y) \wedge Gx)$. (2) the need for scope distinctions in the contextual definition.

The alternative method consists instead of formalising complete sentences containing definite descriptions with a binary quantifier I: I takes two open formulas and forms a formula out of them, binding a variable. $Ix[F, G]$ means: The F is G.

I will present a number of options for rules of inference governing I in natural deduction and sequent calculus, for intuitionist and classical, positive and negative free logic. Two systems follow an established account by Lambert, adjusted to natural deduction by Tennant, of the formalisation of definite descriptions with a term forming operator quite closely, with one crucial difference: the binary quantifier permits the marking of scope distinction, while the established account avoids these. The systems are therefore not directly comparable, but there is considerable overlap. The rules for I are suitable for classical and intuitionistic negative free logic. In the former case, I use natural deduction, in the latter sequent calculus. They are not, however, in the spirit of positive free logic. Thus I will also present a set of rather complicated rules suitable for classical and intuitionist positive free logic, once more in sequent calculus and natural deduction. The resulting theory is new, and thus of some interest, but it must be admitted that a much simpler theory is possible. It results by ignoring the existence assumptions in the rules for negative free logic. I will present this theory at the end of the talk and briefly consider the effect of adding the rules to modal logic.

On Formalisation in Logic and Philosophy. Notes on the Margins of the Work of Catarina Dutilh Novaes

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The presentation will address issues related to the philosophical foundations of formalisation. Problems of understanding the notions of formality, formalisation, symbolisation will be presented. Two research fields will be analysed: the foundations of formal logic and the formalisation of philosophical arguments. The starting point of the presentation is the analysis contained in Catarina Dutilh's book Novaes *Formalizing Medieval Logical Theories*, Springer 2007. On this basis, a discussion of some of these problems will be presented, especially in the light of achievements of other, in particular Polish, authors.

Investigations into Boolean Non-Fregean Logic WB

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Logic WB is a Boolean non-Fregean logic introduced by Roman Suszko [2]. It is an extension of his *Sentential Calculus with Identity*, SCI. The latter is very restrictive as far as identity is concerned: hardly anything can be stated about the identity of situations in SCI, since all valid equations are of the form '$\alpha \equiv \alpha$'.

WB is a logic strengthening SCI by allowing $\equiv$ to have some Boolean properties; for example, '$(\alpha \wedge \beta) \equiv (\beta \wedge \alpha)$' is a validity in WB. Still, $\equiv$ in WB is not truth-functional equivalence.

Little has been established in the field of non-axiomatic proof theory of this logic. In the talk I present a sequent system for WB (based on an idea by Agata Tomczyk) together with a proof procedure by means of which positive decidability of WB is shown. I also introduce a new semantics of truth valuations for WB.

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On Elements $\to$-Irreducible in Finite Heyting Lattices

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In the talk I will summarize my previous results regarding $\to$-irreducible elements in finite Heyting lattices, sketch further questions and formulate open problems.

Logical Analysis of Truth and Some Concepts Related to It

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The lecture deals with several problems related to the concepts of truth, assertion and denial, as used in relation to statements, in connection with the concepts of state of affairs and propositional content of statements. It gives an axiomatic characterization of these concepts within the framework of multimodal propositional logic, and then, presents a semantic analysis of these concepts. This semantics is a slight modification of the standard relational semantics for normal modal propositional logic. It then discusses the classical formulas characterizing the concept of truth from Plato and Aristotle, as well as the formulations of some 20^{th}-century philosophers.

Tautologies

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In the paper, I attempt to find a common ground set by the notions of tautology.

Subnormal Modal Logics — Continued

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In the talk some systems of modal logic weaker than **K**, which yet hold to necessitation rule but do not hold to the monotonicity rule, are presented together with the appropriate semantic theory. Suitable completeness theorems are provided and some further model-theoretical results are demonstrated, like (modified) Łoś's theorem or Halldén-completeness.

Normalisation for Some FDE*-Style Logics*

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Belnap and Dunn's four-valued logic **FDE** can be viewed as an implication-free fragment of Nelson's logic **N4**. The possibility of a normalisation proof for **N4** (and as a result for **FDE**) is mentioned by Prawitz [5]. The proper proof one may find in [3]. In this talk, we plan to present a generalisation of this result: first we prove normalisation for a negation fragment of **FDE** as well as its any extension by n-ary four-valued tabular operators; then we show that normalisation can be proved for the logics based on fifteen other four-valued negations which together with the negation of **FDE** are treated by Omori and Wansing [4] as the only ones which in the four-valued case keep the condition '$\neg A$ is true iff A is false', but violate from the understanding of falsity as untruth.

However, we need a more general approach. So as a starting point we take Kooi and Tamminga's [1] sequent calculi for the negation fragment of **FDE** extended by n-ary four-valued tabular operators and transform them into natural deduction systems such that all their rules are either generalised introduction or generalised elimination ones, in the terminology of von Plato [6]. Then we modify the rules from [4] for the rest fifteen negations into generalised introduction and elimination ones. After that we show that the rules for n-ry connectives (they depend on negation) are still sound and complete if we change the negation of **FDE** to any of these 15 negations.

Finally, using the developed by Kürbis [2] technique we prove normalisation theorem for all the logics in question. Additionally, we discuss the possibility of the extension of this approach to some non-tabular operators: e.g. **S5**-style modalities. The possibility of the extension of the calculi in question by quantifiers and term operators, such as definite descriptions, is also considered.

Acknowledgements. This research is funded by the European Union (ERC, ExtenDD, project number: 101054714). Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the European Research Council. Neither the European Union nor the granting authority can be held responsible for them.

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Justifying the Explosion Principle

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In this talk, we consider the ex falso quodlibet rule (EFQ; also known as the explosion principle, the law of Pseudo-Scotus, or simply the falsity or absurdity rule)

$$\frac{\bot}{A}\ \text{EFQ}$$

as a derived rule and propose a new justification for it based on a rule we call the collapse rule (we assume $\vee$ is commutative)

$$\frac{A \vee \bot}{A}\ \text{collapse}$$

The collapse rule is a mix between EFQ and disjunctive syllogism (DS) and, informally, it says that a choice between a proposition A and $\bot$, which is understood as nullary disjunction, is no choice at all and it defaults to A ("the implosion principle"). Furthermore, we show that the collapse rule can be also used to justify DS and that all these three rules have the same deductive strength: they are all interderivable. Thus, the discussions about the acceptability of EFQ or DS can be reduced to a discussion about the acceptability of the collapse rule. Finally, we consider the computational meaning of the collapse rule with the help of the Curry-Howard correspondence ([1],[2]).

More specifically, using the collapse rule and the standard disjunction introduction rule, we can derive EFQ as follows

$$\cfrac{\cfrac{\bot}{A \vee \bot}\ \vee\mathrm{I}_r}{A}\ \text{collapse}$$

And to derive the corresponding ex falso formula $\bot \to A$, all we need to do is apply the implication introduction rule to the last step of the above derivation.

The DS can be derived as follows (in addition to the collapse rule, we also need negation elimination, and disjunction introduction and elimination)

$$\cfrac{A \vee B \qquad [A]^1 \qquad \cfrac{\cfrac{\cfrac{\neg B \qquad [B]^2}{\bot}\ \neg\mathrm{E}}{A \vee \bot}\ \vee\mathrm{I}_r}{A}\ \text{collapse}}{A}\ \vee\mathrm{E}_{1,2}$$

And to capture the computational meaning of the collapse rule, we introduce a new noncanonical eliminatory operator $\mathsf{collapse}$ that behaves similarly to EFQ's abort

$$\frac{c : A \vee \bot}{\mathsf{collapse}(c) : A}$$

but while the abort function has no instructions for computation (since $\bot$ can never be true), our function $\mathsf{collapse}$ has instructions (since $A \vee \bot$ can be true).

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On a Certain Form of Ad Hominem Argumentation

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The paper concerns this kind of ad hominem argumentation, in which one accuses someone of incompatibility between proclaimed views and behaviour. In addition to presenting the ways in which the premises of such an argumentation can be formulated, our aim is to analyse various, possible conclusions, namely those expressing moral judgements. As a basis for consideration, we take Kant's categorical imperative, assuming the legislative character of free will postulated by it. In the course of the consideration, we distinguish three types of attack on the rhetorical ethos of an opponent: (i) on their logical-cognitive abilities, (ii) their communicative intentions and competencies, and (iii) their volitional-moral characteristics.

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An Attempt at Formal Approach to the Philosophy of Boredom. Notes in the Margin of the Works of Evagrius Ponticus

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The feeling of boredom may arise in cases of:

a) a boring message,
b) a lack or an overabundance of messages,
c) activities in which we find no joy, especially repetitive tasks.

The value of a message may be:

1) utilitarian (expressed by e.g.: 'this may be useful to me'),
2) cognitive ('this is interesting', 'now I understand it')
3) self-affirmative ('I think so as well', 'the others agree with me')
4) aesthetic ('this is beautiful')

Let us assume that the system of knowledge in the mind of a person X is a relational structure.
Let M — a set of information in the mind of the X;
R — a set of various relations defined in M;
W — the system of knowledge of the X.
Then $W = (A, R)$ where $A \subset \mathcal{P}(M)$ (A is contained in the set of subsets of M).
Let p — a message conveying a piece of information 'm';
X considers p boring iff A.

That means X is unable to include the new piece of information into the system of knowledge they possess. To X such message has no (cognitive) value, therefore X considers it boring. Which leads to the conclusion that the smaller the knowledge pool of the person X, the more messages they classify as boring.

Evagrius Ponticus (4^{th} Century) as one of the Desert Fathers, battled boredom and its consequences on daily basis. Application of basic means of formal logic to his thorough analysis allows us to understand better the content of this term.

A Tour around the Applications of Paraconsistent Logic in Machine Learning and Artificial Intelligence

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The recent years were fruitful for the researchers in the field of paraconsistent logic and their discoveries yielded a bountiful crop not only in the area of logics and philosophy, but also in other, seemingly less relevant domains. Among the latter is the domain of artificial intelligence and machine learning, as more and more scientists become aware of usability of paraconsistency while working with uncertain data. During this talk we will acquaint the Auditors with the recent advances in applying paraconsistent logics in the field of machine learning and explainable AI, as well as presenting some prospects of future research in this topic.

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On the Logic of Good

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Here, the subject of analysis is the notion of *good* and the notion of *evil* linked with it. The analysis is inspired by Władysław Tatarkiewicz's work *On the Absoluteness of the Good*. The elementary expressions "*x is good a of kind b for y*" and "*x is evil a of kind b for y*" are adopted here as primary. They are characterised axiomatically. The whole analysis is founded on elementary ontology. The functors of good and evil in abstract sense are adopted by definition.

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Conditionals and Modus Ponens — Linguistic Intuitions vs. the Formal Model

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We analyze some interesting arguments from the literature, where ascribing probability 1 to a certain right-nested conditional $A \to (B \to C)$ leads to strong theses concerning conditionals: they serve as counterexamples to Modus Ponens. A classical example comes from the work of (McGee (1985)):

> *If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.*

According to McGee, this sentence is always true, which means that it has probability 1. Moreover, it is reasonable (taking into account the opinion polls and the general knowledge concerning the political situation) to accept the sentence:

A Republican wins the election.

However, it does not seem reasonable to accept the claim:

If it's not Reagan who wins it will be Anderson.

But this means that in spite of accepting two sentences of the form $\alpha \to \beta$ and α, we are not ready to accept β.

Contemporary, similar reasoning can be found in the papers of Santorio (2021) and Cantwell (2022). These examples are originally presented and analyzed without defining a probability space in which they are assigned probability in a standard mathematical sense. We can say that they are presented within an intuitive framework.

In the presentation we propose a rigorous probabilistic model in which the sentences in question have interpretations as events in a standard probability space (Wójtowicz K., Wójtowicz A. (2022). This allows to show that the assumption that their probability is 1 faces serious difficulties. As a result, the arguments put forward in the three cited works prove unconvincing.

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A Minimal Probability Space for Conditionals

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One of central problems in the theory of conditionals is the construction of a probability space, where conditionals can be interpreted as events and assigned probabilities. Van Fraassen (1976) discussed in great detail the solution in the form of Stalnaker Bernoulli spaces. These spaces are very complex — they have the cardinality of the continuum, even if the language is finite. A natural question is, therefore, whether a technically simpler (in particular finite) partial construction can be given. Obviously, we demand that the structure satisfy certain natural assumptions concerning the logic and semantics of conditionals. Here we take the fairly standard assumptions which have been discussed (for instance) in van Fraassen (1976):

(I)	$((A \to C) \wedge (A \to B)) \Leftrightarrow (A \to (C \wedge B))$;
(II)	$((A \to C) \vee (A \to B)) \Leftrightarrow (A \to (C \vee B))$;
(III)	$(A \wedge (A \to B)) \Leftrightarrow (A \wedge B)$;
(IV)	$(A \to A) = K$ (the set of all possible worlds);
(PCCP)	$P(A \to B) = P(B\|A)$;

PCCP is the acronym for "probability of conditionals is conditional probability" — i.e. that the probability of $A \to B$ is equal to the conditional probability $P(B|A)$ in the sample space.

A partial solution has been given in (Węgrecki, Wroński 2022). In the talk a new solution to the problem is provided. We show how — starting with a Boolean language L and a finite probability space $S = (\Omega, \Sigma, P)$ — to construct a finite probability space $S^{\#} = (\Omega^{\#}, \Sigma^{\#}, P^{\#})$ in which simple conditionals and their Boolean combinations (forming the conditional language L($\to$)) can be interpreted (Ω might be thought of as the set of possible worlds). The set of elementary events $\Omega^{\#}$ in the new space consists of all permutations of Ω; the probability measure $P^{\#}$ on $\Omega^{\#}$ is assigned in such a way that van Fraassen's conditions hold. The cardinality of $\Omega^{\#}$ is $n!$ and there are good arguments which show that this is the minimal possible size.

We can repeat the construction starting with the probability space $S^{\#}$. We obtain a new probability space $(S^{\#})^{\#}$ (and the expanded language containing nested conditionals). $S^{\#}$ is naturally imbedded into $(S^{\#})^{\#}$, all probabilities from $S^{\#}$ are preserved — and so on. The construction can be iterated, leading to a sequence of probability spaces. Every conditional (regardless of how nested and complicated it is) will sooner or later be interpretable in one of the spaces. This means that the construction has a universal character.

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Problem of Compatibility of the First-Order Quantifiers

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One well known problem regarding quantifiers, in particular the first-order quantifiers, is connected with their syntactic categories and denotations. The unsatisfactory efforts to establish the syntactic and ontological categories of quantifiers in formalized first-order languages can be solved by means of the so called principle of categorial compatibility formulated by Roman Suszko, referring to some innovative ideas of Gottlob Frege and visible in syntactic and semantic compatibility of language expressions. In the paper the principle is introduced for categorial languages generated by the Ajdukiewicz's classical categorial grammar. The first-order quantifiers are typically ambiguous. Every first-order quantifier of the type $k > 0$ is treated as a two-argument functor-function defined on the variable standing at this quantifier and its scope (the sentential function with exactly k free variables, including the variable bound by this quantifier); a binary function defined on denotations of its two arguments is its denotation. Denotations of sentential functions, and hence also quantifiers, are defined separately in Fregean and in situational semantics. They belong to the ontological categories that correspond to the syntactic categories of these sentential functions and the considered quantifiers. The main result of the paper is a solution of the problem of categories of the first-order quantifiers based on the principle of categorial compatibility.

ISBN 978-83-940690-7-0