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(EN) means that the talk is presented in English, (PL)—in Polish.

Euclid Parallel Axiom and Infinities

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We present a model of a semi-Euclidean plane. It is a subspace of Cartesian geometry over the non-Archimedean field of hyperreal numbers, in which angles in a triangle sum up to π yet the parallel axiom fails. Contrary to the standard Cartesian plane $\mathbb{R} \times \mathbb{R}$, where the arithmetic structure does not include infinity, in our model, infinite numbers are parameters in equations for straight lines and enable a critical discussion of the condition "being produced to infinity" included in Euclid's definition of parallel lines.

There are two historically motivated concepts of infinity: infinite number = not-finite (Cantor's cardinal and ordinal numbers), infinite number = inverse of infinitesimal (Euler). The first is related to the concept of natural numbers, the latter – to the Archimedean axiom. We introduce a third interpretation showing that Euclid's straight-line is finite while modern – infinite. In that case, duality builds on the Pasch axiom.

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Kripke Semantics for Intuitionistic Logic with Identity

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The aim of the talk is to study intuitionistic version of basic non-Fregean logic, ISCI (*Intuitionistic Sentential Calculus with Identity*) from a semantic point of view. In the context of classical logic propositional identity can be thought of as expressing the notion of sameness of situations described by sentences, but it is no longer the case in intuitionistic setting, where identity expresses the notion of sameness of derivations.

Two approaches to Kripke semantics for ISCI will be presented. One approach was described in [1], the other one has not yet been published.

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Proof-Theoretical Analysis of Intuitionistic Non-Fregean Logic and Its Extensions

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The proposed lecture will focus on proof theory for Intuitionistic Non-Fregean Logic (ISCI), mainly natural deduction systems, and then on its possible extensions with its own natural deduction rules.

Non-Fregean logics (NFLs) came to be through Roman Suszko's willingness to formalize Wittgenstein's *Tractatus* [3, 2]. NFLs owe their name to the rejection of the so called Fregean Axiom which says that the identity of referents of two given sentences holds whenever they share the same logical value [1]. In NFL semantic correlates of sentences are no longer their logical values, but rather situations. Newly introduced binary identity connective will grant us identity of sentences whenever they describe the same situations. Identity is characterised by four axioms:

($\equiv_1$) $A \equiv A$

($\equiv_2$) $(A \equiv B) \to (\neg A \equiv \neg B)$

($\equiv_3$) $(A \equiv B) \to (A \leftrightarrow B)$

($\equiv_4$) $((A \equiv B) \wedge (C \equiv D)) \to ((A \otimes C) \equiv (B \otimes D))$

Originally NFLs were based on classical logic, but it doesn't have to be the case. I will try to show how we can obtain intuitionistic analogue to SCI, namely ISCI. Since the intuitionistic setting requires a constructive interpretation of identity, in this section I'll discuss the notion of identity of proofs. Then I will try to present natural deduction systems for ISCI.

There are three, most studied extensions of SCI: WB, WT and WH. But it would certainly be beneficial to consider extensions of ISCI, as well. Two usual ways of introducing extensions of NFL are through the addition of axioms extending the properties of identity connective or by the addition of inference rules.

However, extending ISCI will not be as straightforward as adding the axioms of classical SCI extensions to it. Since, they're all based on WB, which allows for the law of excluded middle to be derivable – making our logic no longer intuitionistic.

Thus another approach is needed, one that does not affect the constructive character of the logic. We will consider two extensions: one related to the notion of propositional isomorphism, the other introducing a special case of the law of excluded middle.

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Obligation Based on Preference

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In the lecture I will present Henryk Elzenberg's system of formal axiology including the notions of value, ought and obligation. For this purpose I will use the language of sentential modal logic. This system is burdened with several difficulties, which its author himself points out. Having presented these difficulties, I propose a certain modification in the foundations of this system. As I will show, in such a modified system it manages to meet the discussed difficulties and at the same time to preserve those formal properties of ought and obligation which Elzenberg postulated for these concepts. To show this, I will express in the proposed formal language a certain theory of the logic of preference, which I will describe from the syntactical as well as the semantical point of view.

The Finite as the New Infinite

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The infinite has puzzled philosophers from the time of the Ancient Greece to our days. When it started mixing with mathematics, it brought huge controversies, mostly about the difference between the potential and the actual infinite. From its beginning with Cantor in the 19th century, set theory was associated with the revolutionary actual infinite, an infinite that can be named rather than viewed as a limit. This approach has had a lot of success in the sense of really understanding the large infinite sets, in set theory, in model theory and in many other areas where set theory has been applied. This includes topology, analysis and notably, philosophy through the work of Alain Badiou. However, something might have been lost in the process: the connection between the finite and the infinite. It seems that the combinatorial properties of the finite and of the infinite objects are so different, that there is no connection between them. After all, an infinite set can be bijective with a proper subset of itself, so how much worse can this get?

A recent trend in mathematics and in theoretical computer sciences is to bridge this gap by studying 'reasonable infinite objects'. This means the infinite objects which are built out of the finite ones in some precise way: as a Fraïssé limit, a result of some infinite automaton computation, a morass, an ultraproduct, a graphon... There have been several breakouts in making such connections, which we shall review. Then we shall talk about a possible connection with the most abstract of the infinite: abstract elementary classes.

Our thesis is that the study of the 'reasonable infinite' closes the controversy between the potential infinite built as a limiting structure of some finite processes and the actual infinite. It provides a third way.

Disjunction Property for Some Systems of Leśniewski's Ontology

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T. Kubiński in the paper "Vague Terms" (SL 1958) defines and studies theories of minimal *quasi*-ontologies. The author employs his constructions in semantics of vague terms. So-called limited disjunction property is analyzed. In the talk, we pose a problem of how to generalize this property to other systems.

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Ontological Versions of Temporal Logics in Wolniewicz Structures of Lattices (Including Topological Interpretation)

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In the 1980s, Bogusław Wolniewicz, a Polish philosopher and ontologist, gave his interpretation of the ontological theorems of Wittgenstein's *Tractatus* using lattices of elementary situations (cf. Wolniewicz [2], [3], [4]). I was able to generalize such lattices to lattices composed of topological spaces (cf. Kaczmarek [1]). This generalization allows us to develop Wolniewicz's ideas – and perhaps those of Wittgenstein and Russell (remember that before the publication of *Tractatus Logico-Philosophicus* Russell and Wittgenstein worked together on problems whose result we notice in Wittgenstein's *Tractatus*).

I will show that various temporal logics can be defined based on Wolniewicz's structures and generalized structures. In particular, I will give temporal logics satisfying the axioms Kt, CL (N. Cocchiarella logics), SL (D. Scott) and PL (A. Prior). These logics are correlated with ontological studies, hence I call them ontological versions of temporal logics.

I will also present such temporal logics in which the law: there always was α, there always is α and there always will be α (which is the view of the fact – sentence – necessary in Aristotle's view) will be important. The abstract is given in informal language, but the eventual presentation of the problems proposed here will be presented using the language of logic, algebraic terminology and concepts of general topology.

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On Translation from Intuitionism to Brouwer's Modal Logic

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We consider the Brouwer modal logic **KTB**, which is defined as normal extension of the minimal normal modal logic **K** as follows:

$$\mathbf{KTB} := \mathbf{K} \oplus T \oplus B$$

where the new axioms are the following: $T := \Box p \to p$ and $B := p \to \Box\Diamond p$. The set of rules consists of the *modus ponens*, the *rule of uniform substitution* and the *rule of necessitation*.

Axiom T is called the *axiom of necessity*, whereas axiom B is known as the *Brouwerian axiom*. As for Brouwerian axiom we paraphrase here the following justification of this name given by G.E. Hughes and M.J. Cresswell in [2], p. 57. As it is known, L. Brouwer is the founder of the intuitionist school of mathematics. The law of double negation does not hold in intuitionistic logic. Exactly it holds that (i) $\vdash_{INT} p \to \neg\neg p$ but (ii) $\nvdash_{INT} \neg\neg p \to p$. Suppose that negation has a stronger meaning – necessarily negative. Hence $\neg p$ may be translated as $\Box\neg p$. The corresponding modal formula to (i) is $p \to \Box\neg\Box\neg p$, which gives us $p \to \Box\Diamond p$ and obviously $\vdash_{KTB} p \to \Box\Diamond p$. If we translate (ii) in this way, we obtain: $\Box\Diamond p \to p$, which is not a thesis even of the system **S5** defined below. Hence $\nvdash_{KTB} \Box\Diamond p \to p$. Further, G.E. Hughes and M.J. Cresswell write: 'Thus although the connection with Brouwer is somewhat tenuous, historical usage has continued to associate his name with this formula.'

This combining Brouver's axiom with the intuitionistic logic will be a motivation for our research. Following Hughes and Cresswell we define some translation, which is completely different than the Gödel-McKinsey-Tarski one (see [1], [4]). We shall limit ourselves to a language with one propositional variable and shall consider the Rieger-Nishimura lattice. Then we shall translate this lattice. It will not be possible to interpret the whole lattice, however, we will be able to obtain an infinite upper sublattice. From this translation we obtain many theorems combining intuitionistic logic of one variable with the same fragment of the modal Brouwer logic.

Further, we shall find the connection between the height of the upper sublattice and the degree of branching the considered KTB-frames.

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Statements and Open Problems on Decidable Sets $\mathcal{X} \subseteq \mathbb{N}$

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We summarize the article available at `http://ssrn.com/abstract=3978669`. Let $f(1) = 2$, $f(2) = 4$, and let $f(n+1) = f(n)!$ for every integer $n \geqslant 2$. Edmund Landau's conjecture states that the set $\mathcal{P}_{n^2+1}$ of primes of the form n^2+1 is infinite. Landau's conjecture implies the following unproven statement Φ: $\mathrm{card}(\mathcal{P}_{n^2+1}) < \omega \Rightarrow \mathcal{P}_{n^2+1} \subseteq [2, (((24!)!)!)!]$. Let B denote the system of equations: $\{x_j! = x_k: \ j,k \in \{1,\dots,9\}\} \cup \{x_i \cdot x_j = x_k: \ i,j,k \in \{1,\dots,9\}\}$. We write some system $\mathcal{U} \subseteq B$ of 9 equations which has exactly two solutions in positive integers $x_1,\dots,x_9$, namely $(1,\dots,1)$ and $(f(1),\dots,f(9))$. No known system $\mathcal{S} \subseteq B$ with a finite number of solutions in positive integers $x_1,\dots,x_9$ has a solution $(x_1,\dots,x_9) \in (\mathbb{N} \setminus \{0\})^9$ satisfying $\max(x_1,\dots,x_9) > f(9)$. For every known system $\mathcal{S} \subseteq B$, if the finiteness/infiniteness of the set $\{(x_1,\dots,x_9) \in (\mathbb{N} \setminus \{0\})^9 : \ (x_1,\dots,x_9) \textit{ solves } \mathcal{S}\}$ is unknown, then the statement $\exists x_1,\dots,x_9 \in \mathbb{N} \setminus \{0\} \ ((x_1,\dots,x_9) \textit{ solves } \mathcal{S}) \wedge (\max(x_1,\dots,x_9) > f(9))$ remains unproven. We write some system $\mathcal{A} \subseteq B$ of 8 equations. Let Λ denote the statement: *if the system $\mathcal{A}$ has at most finitely many solutions in positive integers $x_1,\dots,x_9$, then each such solution $(x_1,\dots,x_9)$ satisfies $x_1,\dots,x_9 \leqslant f(9)$.* The statement Λ is equivalent to the statement Φ. It heuristically justifies the statement Φ. This justification does not yield the finiteness/infiniteness of $\mathcal{P}_{n^2+1}$. We present a new heuristic argument for the infiniteness of $\mathcal{P}_{n^2+1}$, which is not based on the statement Φ. Algorithms always terminate. The next statements and open problems justify the title of the linked article and involve epistemic and informal notions. We explain the distinction between existing algorithms (i.e. algorithms whose existence is provable in ZFC) and known algorithms (i.e. algorithms whose definition is constructive and currently known). For a set $\mathcal{X} \subseteq \mathbb{N}$ whose infiniteness is false or unproven, we say that a non-negative integer k is a known element of $\mathcal{X}$, if $k \in \mathcal{X}$ and we know an algebraic expression that defines k and consists of the following signs: 1 (one), + (addition), − (subtraction), · (multiplication), ^ (exponentiation with exponent in $\mathbb{N}$), ! (factorial of a non-negative integer), ((left parenthesis),) (right parenthesis). No known set $\mathcal{X} \subseteq \mathbb{N}$ satisfies Conditions `(1)`-`(4)` and is widely known in number theory or naturally defined, where this term has only informal meaning. `(1)` *A known algorithm with no input returns an integer n satisfying* $\mathrm{card}(\mathcal{X}) < \omega \Rightarrow \mathcal{X} \subseteq (-\infty, n]$. `(2)` *A known* *algorithm for every* $k \in \mathbb{N}$ *decides whether or not* $k \in \mathcal{X}$. (3) *No known algorithm with no input returns the logical value of the statement* $\mathrm{card}(\mathcal{X}) = \omega$. (4) *There are many elements of* $\mathcal{X}$ *and it is conjectured, though so far unproven, that* $\mathcal{X}$ *is infinite.* (5) $\mathcal{X}$ *is naturally defined. The infiniteness of* $\mathcal{X}$ *is false or unproven.* $\mathcal{X}$ *has the simplest definition among known sets* $\mathcal{Y} \subseteq \mathbb{N}$ *with the same set of known elements.* Conditions (2)-(5) hold for $\mathcal{X} = \mathcal{P}_{n^2+1}$. The statement Φ implies Condition (1) for $\mathcal{X} = \mathcal{P}_{n^2+1}$. We define a set $\mathcal{X} \subseteq \mathbb{N}$ which satisfies Conditions (1)-(5) except the requirement that $\mathcal{X}$ is naturally defined. We present a table that shows satisfiable conjunctions of the form $\#$(Condition 1)$\wedge$(Condition 2)$\wedge\#$(Condition 3)$\wedge$(Condition 4)$\wedge\#$(Condition 5), where $\#$ denotes the negation $\neg$ or the absence of any symbol. No set $\mathcal{X} \subseteq \mathbb{N}$ will satisfy Conditions (1)-(4) forever, if for every algorithm with no input, at some future day, a computer will be able to execute this algorithm in 1 second or less. The physical limits of computation disprove this assumption.

Existential Judgments

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The subject of this paper will be selected issues concerning the logical structure of existential judgments and the logical form of sentences used as linguistic equivalents of such judgments. The problem here is that we all sometimes hold beliefs that, for example, *electrons exist* (*or do not exist*), that *minds exist* (*or do not exist*), or that *Pegasus exists* (*or does not exist*). In such cases we may sometimes have some ambiguity about the logical form of judgments expressing such beliefs. Therefore, we should try to determine what logical structure a judgment stating that *a exists*, where *a* is a letter that represents an arbitrarily determined object, should take. We refer here to remarks made by Stanisław Leśniewski in his work entitled *Przyczynek do analizy zdań egzystencjalnych* (*A contribution to the analysis of existential propositions*).

The Connective "I. . . i. . . " in Polish

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Looking for the best natural language equivalents to various functors in logic I turned my attention to the Polish connective "I. . . i. . . ". The Polish connective "I. . . i. . . " may be a suitable equivalent to the functor of conjunction in logic.

Connexive Logics and Relating Semantics

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There is a common agreement that each connexive logic should satisfy the Aristotle's and Boethian Theses (AB). However, the sole AB theses don't guarantee any common content or other form of "connexions" as they are true in binary matrix $\{1, 0\}$ with distinguished value of 1, with classical material implication and negation defined as $\sim 1 = \sim 0 = 1$. Similarly, AB are true in a binary matrix with classical negation and implication defined as $x \Rightarrow y = 1$ iff $x = y$.

It show that the sole AB theses are very weak and should be strengthen in some way. We can eliminate first counterexample by assuming that negation behaves in a classical way. It brings us to the notion of Boolean connexive logic. By a *minimal Boolean connexive logic* we mean the least set of sentences containing all classical tautologies expressed by means of $\neg, \wedge, \vee$, (A1), (A2), (B1), (B2), $(A \to B) \supset (A \supset B)$ and closed under substitutions and modus ponens with respect to $\supset$. $\supset$ denote material implication.

In [1] we characterized Boolean connexive logics by means of relating semantics. Then Mateusz Klonowski proved that the class JT determines minimal Boolean connexive logics. The class $JT^{\neg}$ determines the least Boolean connexive logics satisfying the following two axioms: $(A \to B) \supset (\neg\neg A \to \neg\neg B)$, $(A \to B) \supset ((\neg A \to \neg B) \vee (\neg A \wedge B))$.

Malinowski and Arturo Nicolas Francisco in [2] analyzed a number of properties added to AB in terms of relating semantics for Boolean connexive logics. In particular we show that Minimal Boolean Connexive Logic (or alternatively the logic determined by JT) is Abelardian, strongly consistent, Kapsner strong and antiparadox. We also construct examples showing that it is not simplificative, neither conjunction-idempotent nor strongly inconsistent logics.

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Subnormal Modal Logics

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Systems of modal logic which are most often studied by logicians are extensions of the system K, determined by the axiom $\Box(\alpha \to \beta) \to (\Box\alpha \to \Box\beta)$ and the necessitation rule $\alpha/\Box\alpha$. There are many ways of weakening K, and one of the least common ways is rejecting the above axiom while holding to the necessitation rule. Such a system of modal logic, here called 'CM', was first studied by Fitting, Marek and Truszczyński in [1]. The purpose of this talk is to present semantic methods of studying CM and its extensions different from those presented in [1], and to demonstrate how these methods can be used to describe a large class of systems intermediate between CM and K by providing suitable completeness theorems.

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Model Theory, a Survey with Particular Emphasis on Topological Methods

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Model theory was founded by Abraham Robinson and Alfred Tarski in mid-twentieth century. Over the years it grew in volume and depth. Now it is an established part of mathematics and mathematical logic. In the talk I will survey the development of model theory and will present some recent ideas in it, related to topological dynamics.

In the development of model theory there are some major stages. The turning point was the Morley categoricity theorem (1964), answering the conjecture of Jerzy Łoś. Gradually model theory incorporated more and more methods from various areas of mathematics and integrated with them. Also the nature of model theory was changing. So Keisler and Chang around 1970 gave a succint definition of model theory in the form of equation:

Model Theory = Universal Algebra + Logic

25 years later Wilfrid Hodges changed it into:

Model Theory = Algebraic Geometry – Fields

The change of definition reflects the change in model theory over those years, due to the contributions of several leading researchers, most notably Saharon Shelah, Boris Zilber, Anand Pillay, Ehud Hrushovski and many others. In the talk I will explain these definitions in greater detail.

Topological methods were present in model theory already in its early stages. In fact, they played a prominent role in the proof of the Morley categoricity theorem. The major idea there was to measure definable sets in models by means of Morley rank that is a variant of Cantor-Bendixson rank in the space of types. Later Shelah invetigated variants of Morley rank by combinatorial methods that led to his discovery of forking and development of geometric model theory.

In the years 2000 I suggested applying in model theory some stronger and more precise topological tools, coming from topological dynamics. These tools turned out to be useful to investigate deep nature of theories, like strong types, Galois groups and Borel complexity of various model-theoretic equivalence relations. I will survey this development.

Algebraic Completeness of Bi-intuitionistic Multilattice Logic

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In the paper [1], Kamide, Shramko, and Wansing introduced a logic $\mathbf{BML}_n$ which is a bi-intuitionistic version of Shramko's [2] multilattice logic $\mathbf{ML}_n$ (a logic of multilattices, lattices with n orders). Additionally, they studied a connexive variant of $\mathbf{BML}_n$ called $\mathbf{CML}_n$. Both logics were formulated in the form of sequent calculi and Kripke semantics. However, the very notion of bi-intuitionistic and connexive multilattices has not been presented and the algebraic completeness theorem for $\mathbf{BML}_n$ and $\mathbf{CML}_n$ has not been proven in [1]. In this talk, we would like to formulate the notions of bi-intuitionistic and connexive multilattices, to establish that $\mathbf{BML}_n$ and $\mathbf{CML}_n$ are complete with respect to these structures.

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One Hundred Years of Logic for Constructive Mathematics

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We clarify what is constructive mathematics without emotional coloring. There is no need to 'be' a constructivist. Well-known expounders of constructive mathematics include Brouwer, Markov, and Bishop. Classical mathematics has a formal logic associated with it, classical logic. Shortly before 1930 Heyting developed a logic for constructive mathematics. Almost from the beginning critics wondered whether this intuitionistic logic could be justified as the logic of constructive mathematics. Some, including Gödel, were not convinced that it was, or at least that it lacked a proper justification. We confirm that intuitionistic logic is not the logic of constructive mathematics. We present a new correct version of constructive logic.

Discussion of the Notion of Co-topos

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Category theory and topos theory in particular have interesting and profound connections with logic and philosophy, more generally. Moreover, they are also considered in the context of the foundations of mathematics. It is well known that toposes are very closely connected with higher order intuitionistic logic. However, in recent years there appeared some proposals, *inter alia* Mortensen *Inconsistent Mathematics* (1995), and Estrada-González *Complement-Topoi and Dual Intuitionistic Logic* (2010), that suggest certain dualization of the logic of a topos, changing it from intuitionistic into some kind of a paraconsistent logic. The categories that emerge in this dualization process were labeled by these authors as complemented-toposes (or co-toposes in short). If this process turned out to be valid it would be highly fruitful as the connections of toposes with intuitionistic and intermediate logics are so manyfold. However, this proposal also raises some concerns. I want to discuss the validity of the notion of co-topos and examine briefly their dualization process which supposedly transforms certain Heyting algebras of the topos into co-Heyting ones, which would make it

possible to relate these algebraic structures to those paraconsistent logics which are sometimes called dual to intuitionistic logics. In my talk, I shall investigate the question of possible interpretations of generic subobject, i.e. distinguished by Ω-axiom arrow $1 \to \Omega$. Is its meaning as truth imposed by the very structure of the topos or is it open to different interpretations, especially as falsity, as proposed by the authors of papers on co-toposes? If it can be interpreted as falsity, what would be the consequences of such an interpretation? I shall try to face these questions and will offer some examples. My results are partial and show some possibility of the proposed dualization especially for propositional logics, but without prejudging the correctness of this approach, especially due to possible problems for higher-order logics.

Sequent Calculus for a Boolean Extension of Non-Fregean Logic SCI

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The aim of the talk is to present $\mathsf{G3}_{\mathsf{WB}}$: a sequent calculus for a Boolean extension of the weakest non-Fregean logic proposed by Roman Suszko, SCI (*Sentential Calculus with Identity*). In non-Fregean logics we reject the so called *Fregean Axiom*—an assumption that the sameness of logical values of two given sentences constitutes identity of their semantic correlates. Suszko disagreed with this idea and proposed a number of non-Fregean systems by means of an addition of an identity connective, which is stronger than material equivalence and which expresses the sameness of the situations denoted by two sentences. [2]

WB is obtained through an addition of six axioms to set comprised of Classical Propositional Calculus axioms and axioms characterizing identity in SCI. This way we extend the set of tautological identities—in SCI it consisted of a singular scheme $\phi \equiv \phi$, whereas in WB we will consider tautological identities $\phi \equiv \chi$ such that $\phi \leftrightarrow \chi$ has been obtained from set of Truth-Functional Tautologies. In $\mathsf{G3}_{\mathsf{WB}}$ we formalize this notion and extend the calculus $\ell\mathbf{G3}_{\mathsf{SCI}}$ introduced in [1] by means of one right-sided identity rule. Additionally, in order to control and restrict the application of identity rules, we add markers labelling whole sequents. However, through these particular modifications, even though cut elimination was proven for $\ell\mathbf{G3}_{\mathsf{SCI}}$, we are unable to prove it for $\mathsf{G3}_{\mathsf{WB}}$. We will identify issues regarding cut elimination procedure and distinguish a class of formulas requiring cut application.

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Chwistek's "On Axioms" as a Foundational Text in the History of Polish Logic

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We wish to discuss the contents of Leon Chwistek's PhD thesis entitled "On Axioms" (1906). This little known work is the starting point of Chwistek's logical career and has not been the subject of any in-depth study so far. The analysis of the text provides a fascinating insight into the origins of Chwistek's interest in themes and thinkers, some of which lasted his lifetime. Leon Chwistek is a problematic figure in the history of Polish logic and analytic philosophy. While not considered a member of the Lvov-Warsaw school of logic, he was nevertheless interacting with the members of the school and was very much interested in some of the themes that were important to e.g. Łukasiewicz, Leśniewski or Tarski. Chwistek is perhaps best known among English speaking historians of the history of logic for the mention of his paper "The Theory of Constructive Types" in the introduction to the 1925 second edition of *Principia Mathematica* and, notoriously, for his competition with Alfred Tarski over an appointment as professor at the Polish university in Lwów. Chwistek's success was in part due to a brief letter of reference from Russell that is sometimes seen as a scandalous error of judgement on Russell's part. "On Axioms" contains seeds of Chwistek interest in and indeed a critique of Russell's ideas. After sketching the history and philosophy surrounding the study of axioms, Chwistek moves on to focus on two issues. The first is the rebuttal of the synthetic a priori sentences and the second is the issue of the axiomatic foundations of geometry. Both relate to Russell's early work, namely to his "An Essay on the Foundations of Geometry" from 1897. In fact, this book is the only text by Russell that Chwistek refers to in his thesis. What is also notable, is Chwistek's reliance on the principle of contradiction as the most important law of logic. This conviction was no doubt shaken, if not shattered by the 1910 publication of Łukasiewicz's *On the Principle of Contradiction in Aristotle*, which upended the perception of axiomatic foundations of logic. One sees the evolution of Chwistek's ideas under the influence of Łukasiewicz in "The Law of Contradiction in the Light of Recent Investigations of Bertrand Russell", which Chwistek published in 1912. In there, he mounts a defence of the principle of contradiction also – as in "On Axioms" – making use of Russell's ideas. But this time, his focus shifts from "An Essay on the Foundations of Geometry" to Russell's more logically oriented work, most notably *Principia Mathematica*. "On Axioms" is an important historical document allowing one to trace the origins of many ideas and intellectual fascinations present in Chwistek's more mature publications.

Some Operations on Flou Sets: A General Framework

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We define two new operations on so-called *flou* (that is, *nested* or *double*) sets. These sets have been introduced by Gentilhomme in [2]. In general, the idea is that a flou set on a non-empty universe X is just an ordered pair $A = [A^1, A^2]$ of subsets of X, such that $A^1 \subseteq A^2$. One can define binary union and intersection in the following way:

$A \cap B = [A^1 \cap B^1, A^2 \cap B^2]$,

$A \cup B = [A^1 \cup B^1, A^2 \cup B^2]$.

In this case one can obtain $1-1$ correspondence between flou sets and so-called intuitionistic sets which were introduced by Çoker in [1] as a crisp version of intuitionistic fuzzy sets invented earlier by Atanassov.

In our paper we define two other operations:

$A \odot B = [A^1 \cap B^1, A^2 \cup B^2]$,

$A \oplus B = [(A^1 \cup B^1) \cap (A^2 \cap B^2), A^2 \cap B^2]$.

We show certain advantages and limitations of this viewpoint. Moreover, we suggest an interpretation of our operations in terms of negotiations and decision making. As a result, we obtain a structure of discussion between several participants who propose their "necessary" and "possible" requirements or propositions. This framework can be fuzzified. An interesting observation is that these new operations form bisemilattice with only one law of absorption. Bisemilattices have been studied by some Polish (and not only Polish) authors in 80s and 90s.

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The Definite Negation and Intuitionistic Logic

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The philosophy of logic knows the distinction between the *ontological research attitude* and *epistemic research attitude*. On the other hand, there is the distinction between two types of negation: the *classical / external / indefinite* ($-$) and *non-classical / internal / definite* ($\neg$) one. The paper presents a propositional calculus with two types of negation ($-, \neg$), which includes both the classical and intuitionistic propositional calculus. We associate classical negation ($-$) with the ontological research attitude, whereas the definite negation ($\neg$) with the epistemic one. The last and the richest construction is accompanied by the *ontological-epistemic* research attitude.

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Operatory Counterparts of Reasoning

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Philosophical literature provides different classifications of reasoning. In the Polish literature on the subject, for instance, there are three popular ones accepted by representatives of the Lvov-Warsaw School: Jan Łukasiewicz, Tadeusz

Czeżowski and Kazimierz Ajdukiewicz (1974). The author of this paper, having modified them, distinguished the following reasonings: (1) deductive and (2) non-deductive, and additionally two types of them in each of the two, depending on the manner of combining their premises with the conclusion through the relation of logical entailment. Consequently, the four types of reasoning:

1.1. unilateral deductive (incl. its sub-types: deductive inference and proof),

1.2. bilateral deductive (incl. complete induction), and

2.1. reductive (incl. the sub-types: explanation and verification),

2.2. logically nonvaluable (incl. inference by analogy, statistic inference),

correspond to four operators of derivability. They are defined formally on the ground of Tarski's axiomatic theory of deductive systems, by means of the consequence operation Cn (Tarski 1930). Also, certain metalogical properties of these operators are given, as well as their relations with Tarski's consequence operations Cn^{+} ($\mathrm{Cn}^{+} = \mathrm{Cn}$) and dual consequences Cn^{-1} (Słupecki, Bryll, Wybraniec-Skardowska 1971) and Cn^{-} (Wójcicki 1973).

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