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(EN) means that the talk is presented in English, (PL)—in Polish.

Open Problems in a Logic of Gossips

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Gossip protocols aim at arriving, by means of point-to-point or group communications, at a situation in which all the agents know each other secrets. In [4] a dynamic epistemic logic was introduced in which distributed epistemic gossip protocols could be expressed as formulas.

In [1] a simpler modal logic was proposed that is sufficient for reasoning about correctness of such protocols. This logic was subsequently studied in a number of papers. In particular, in [3] decidability of its semantics and truth for a limited fragment was established and in [2] its extension with the common knowledge operator was considered, for which the analogous decidability results were established.

However, several, often deceptively simple, questions about this logic remain open. The purpose of this talk is to present and elucidate these questions and provide for them an appropriate background information in the form of partial or related results.

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Where the Mathematical Proof Comes From

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1. Modern logic emerged from the reform of Aristotle's syllogisms through algebra and other techniques of symbolic manipulation. It was also shaped by the 19$^{\text{th}}$ century developments of mathematics, specifically the foundations of geometry, the search for rigor in the calculus, and the axiomatization of set theory. By mathematical logic (ML) we mean the branch of logic that has arisen from foundational studies in mathematics.

ML developed two models of mathematical proof (MP): (1) sequence of formulae $F_1, \ldots, F_n$, where F_k is either an axiom or is obtained from the previous formulae F_i, F_j by the *modus ponens* rule, see (Hilbert, 1922), (2) twofold composition that includes, on the one hand, a sequence of formulae, and on the other, a sequence of signs explaining the status of each formula in the first sequence in terms of axioms, definitions, and references to other theorems or formulae, see (Peano, 1956), (Russell & Whitehead, 1910–1913). While the first model is rather speculative, it gave rise to a branch of ML called proof theory; the second model seems to emulate mathematical practice.

We focus on the historical roots of MP and show their Euclid origins. More precisely, we show how editions and commentaries on *The Elements*, starting with the Late Renaissance and Early Modern ones, via the Peano *Formulario Mathematico* program, have paved the way to the second model of MP.

2. There are two components of Euclid's proposition: the text and the lettered diagram. The Greek text is linearly ordered as sentence follows sentence, from left to right, and from top to bottom. Diagrams consist of line segments and circles. Capital letters on the diagram are located next to points; they name ends of line segments, intersections of lines, or random points.

Text of a proposition is a schematic composition consisting of six parts: *protasis* (stating the relations among geometrical objects by means of abstract and technical terms), *ekthesis* (identifying objects of protasis with lettered objects), *diorisomos* (reformulating protasis in terms of lettered objects), *kataskeue* (a construction part which introduces auxiliary lines exploited in the proof that follows), *apodeixis* (proof), *sumperasma* (reiterating diorisomos). References to axioms, definitions, and previous propositions are made via *prostasis* technical terms and phrases.

In modern developments of Euclid geometry enunciations of Euclid's propositions are redeveloped in *protasis* or *diorisomos* form, while *ekthesis* and *sumperasma* are omitted, therefore construction parts refer to axioms rather than straight-edge and compass *Postulates*.

3. Latin tradition, beside extensive commentaries, has introduced a third part into Euclid's proposition, namely marginalia, containing references to definitions, axioms and other propositions. Starting with the 17^{th} century editions, these references have been included in the linear structure of the text in square brackets. In modern translations, references are treated on par with any other interpolations, and they are included in the linear structure of the text, also in square brackets. Next to marginalia, the tradition of commentaries introduced yet another part into Euclid's proposition: symbols representing some notions and relations; these symbols were included in the linear structure of the text simply in the place of words.

Peano has introduced a technique of purely symbolic representation of Euclid's propositions from Books V, and VII to IX. He has managed to formulate Euclid's propositions (*protasis* or *diorismos* parts) without a single word of natural language. Peano has followed the same technique of symbolic representation in the foundations of calculus and geometry. Occasionally, he has applied this technique to proofs of propositions. Still, his symbolic propositions, next to a sequence of formulae, have included a system of references. Peano's technique of symbolic representation of mathematical sentences has been adopted in (Russell & Whitehead, 1910–1913), and due to the great influence of *Prinicipa Mathematica* on logic, it has become a standard model of MP.

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Cut-Elimination in Constructive SCI

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In [1] two sequent calculi for Suszko's *Sentential Calculus with Identity* ([2]) were introduced along with a cut-elimination theorem for one of them. Those sequent systems were obtained from an axiomatic description of SCI by means of a certain strategy described by Negri and von Plato [3]. In our talk we will describe two versions of SCI based on minimal and intuitionistic logic along with corresponding sequent systems for them obtained as a result of employing the aforementioned strategy. We will discuss the problem of admissibility of structural rules, *cut-rule* in particular.

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Action Systems and Agency

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The talk is concerned with the issue on how one can meaningfully and consistently speak of deontology of actions performed by agents. In other words, the focus is on the meanings attached to statements of the form "a definite agent is permitted (is obliged) to perform an action A in a given situation s". These statements are paraphrased in an equivalent form as "An action A is permitted (is obligatory) in a situation s for a definite agent". The talk offers a solution of this problem from the viewpoint of situational action systems in the sense of [1]. Agents of actions are treated as specialized constituents of situational envelopes of elementary action systems. More specifically, the talk presents a bunch of remarks on the relationship between actions and their agents from the perspective of context-free grammars in Greibach normal form (GNF) (see [2]). The situational interpretation of context-free grammars offers a coherent picture of the deontology of concerted actions performed by a collection of agents.

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Consequence Operators for Logic Programs and the Classical Consequence

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The goal of our talk is to show the connection between the notion of the classical consequence and three consequence operators for logic programs: immediate consequence operator $T_{\mathcal{P}}$ proposed by van Emden and Kowalski [4], operator $\mathcal{T}_{\mathcal{P}}$ based on the Kleene's strong three-valued logic that was proposed by Stenning and van Lambalgen [3], and operator $\Phi_{\mathcal{P}}$ based on the Łukasiewicz three-valued logic that was proposed by Hölldobler [2]. Our motivation is the following: we are interested in modelling abductive reasoning in neural-symbolic systems that combine logic programs and artificial neural networks (e.g. [1]) and such systems in turn model the way consequence operators work for logic programs. The definition of one of the main concepts, i.e. the abductive goal, is usually based on the definition of classical consequence. However, using the above mentioned operators as the semantics for logic programs leads to different definitions of consequence of a logic program, and therefore to different definitions of abductive goals. In our talk we want to describe how those differences affect the definition of abductive goal and what are pros and cons of each solution.

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Elzenberg: Value and Ought

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In my paper I will present a relative preference semantics for multimodal logic of good and ought. The philosophical inspiration for this semantics comes from the axiological writings of Henryk Elzenberg. The central concept is an act of preference between alternative possibilities by metaempirical will, i.e. one which is guided only by pure reasons. In semantics, the act of this will is matched by an ordering relation between the possible worlds.

Elzenberg presents some proposals for the definition of formal relationships of good and ought. They will be formalised in this logic.

Decompositions of Modal Operators and Continuity in Zero

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We study the join-semilattice $\mathcal{M}(B) = \langle M(B), \vee, f^0, f^1 \rangle$ of (unary) modal operators on a nontrivial Boolean algebra $\langle B, +, \cdot, -, 0, 1 \rangle$, with the smallest element $f^{\mathbf{0}}$, which is the constant mapping $f \equiv 0$, and the largest element $f^{\mathbf{1}}$ which is the unary discriminator. Following Jonsson and Tarski [2], a map

$f : B \to B$ is a modal operator if f is additive, $f(a+b) = f(a) + f(b)$, and normal, $f(0) = 0$. Recall that in an upwardly bounded semilattice $\langle S, \vee, 1\rangle$, the *dual pseudocomplement* of x is the smallest y such that $x \vee y = 1$. We show that $\langle M(B), \vee, f^0, f^1\rangle$ is dually pseudocomplemented if and only if B is complete. A map $f^{\mathbf{1}} : B \to B$ is a *unary discriminator function*, if

$$f^{\mathbf{1}}(a) = \begin{cases} 0, & \text{if } a = 0, \\ 1, & \text{otherwise.} \end{cases} \tag{1}$$

We investigate, among others, the problem of decomposing the discriminator in the bounded semilattice $\langle M(B), \vee, f^0, f^1\rangle$. A pair (f, g) of modal operators on $\langle B, +, \cdot, -, 0, 1\rangle$ such that $f(a) + g(a) = f^{\mathbf{1}}(a)$ for all $a \in B$, is a *decomposing pair*. We study the proper decompositions of $f^{\mathbf{1}}$ i.e. decomposing pairs (f, g) such that $f \neq f^{\mathbf{1}} \neq g$. We give a condition saying that a modal operator $f : B \to B$ is *continuous in* 0 and we show, in particular, how this condition is related to the proper decompositions of $f^{\mathbf{1}}$. Several examples of join-semilattices $\mathcal{M}(B)$, in particular for some weakly transitive modal logics, are provided.

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A Logical-Conceptual Mode for Analyzing Propositions about God

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While to worship God and also to defend their beliefs, most theists need a conception of God, theist with philosophical inclinations need to find a conception proportionate to philosophical and logical presumptions. One of these presumptions, which is a result of the principle of non-contradiction, states that conceptions of God should be neither internally inconsistent nor inconsistent with other theistic assumptions.

In theology, normally God is assumed to be identifying with absolutely perfect being. While this conception of God can be considered as the classical one in western theism, different paradoxes surrounding the concept of absolute perfection usually taken as a sign of inconsistency in this conception.

In this article I try to answer the question that whether it is possible to find a framework for analyzing propositions containing divine attributes, in which paradoxes resolved. In order to answer this question I will suggest a logical-conceptual model with two main parts. First, applying the concept of "eternal perspective" implying God's being wholly other than creatures, and second, appealing to a new account of "absoluteness" as "metaphysically possible from eternal perspective".

The goal is to resolve paradoxes through using the suggested framework while keeping the classical concept of God as absolutely perfect being.

Keywords: Conception of God, Absolute Perfection, Omnipotence, Omniscience, Perfect Goodness, The Problem of Evil, The Paradox of the Stone.

Automated Proof Search for Modal Logic K *in Labelled Sequent Calculus*

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Traditional proof theory distinguishes two types of sequent calculi for modal logics—semantic and syntactic. Labelled sequent calculus belongs to the semantic approach, and is obtained by adding relational atomic formulae: w_iRw_j denoting accessibility relation between possible worlds, as well by labelling each formula in a sequent (as described in [3]). The advantage of using labelled calculi for proof search in normal modal logics is that they manage to accurately reflect Kripke's possible worlds semantics using relatively simple means of extending the language. They can also be used to construct countermodels in said systems. In my talk I would like to present an algorithm for building minimal prooftrees in labelled sequent calculus for Propositional Modal Logic K ([1]). The implementation was written in Haskell ([2]), adhering to principles set out by functional programming paradigm.

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$\{0;1\}$ *and* $[0;1]$*: From Classical Logic to Fuzzy Quantum Logic*

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The starting point of the unsharp approach to quantum mechanics (QM) ([2]) is deeply connected with a general problem that naturally arises in the framework of Hilbert space quantum theory. Let us consider an event-state system $(\Pi(\mathcal{H}), \mathcal{S}(\mathcal{H}))$, where $\Pi(\mathcal{H})$ is the set of **projections**, while $\mathcal{S}(\mathcal{H})$ is the set of all **density operators** of the Hilbert space $\mathcal{H}$ (associated to the physical system under investigation). Do the sets $\Pi(\mathcal{H})$ and $\mathcal{S}(\mathcal{H})$ correspond to an *optimal* possible choice of adequate mathematical representatives for the intuitive notions of *event* and of *state*, respectively? Once $\Pi(\mathcal{H})$ is fixed, Gleason's Theorem guarantees that $\mathcal{S}(\mathcal{H})$ corresponds to an *optimal* notion of state: for, any probability measure defined on $\Pi(\mathcal{H})$ is determined by a density operator of $\mathcal{H}$ (provided the dimension of $\mathcal{H}$ is greater than or equal to 3). On the contrary, $\Pi(\mathcal{H})$ does not represent the largest set of operators assigned a probability-value since there are bounded linear operators E of $\mathcal{H}$ that are not projections and that satisfy the *Born's rule*: for any density operator ρ, $\mathrm{Tr}(\rho E) \in [0,1]$. In the unsharp approach to QM, the notion of *quantum event* is liberalized and the set $\Pi(\mathcal{H})$ is replaced by the set of all *effects* of $\mathcal{H}$ (denoted by $\mathcal{E}(\mathcal{H})$), where an effect of $\mathcal{H}$ is a bounded linear operator E that satisfies the following condition, for any density operator $\rho : \mathrm{Tr}(\rho E) \in [0,1]$. Clearly, $\mathcal{E}(\mathcal{H})$ properly includes $\Pi(\mathcal{H})$.

The set $\mathcal{E}(\mathcal{H})$ can be naturally structured ([1], [2]) as a *Brouwer-Zadeh poset* (BZ-poset) $\langle \mathcal{E}(\mathcal{H}), \leq, {}', {}^{\sim}, \mathbb{O}, \mathbb{I}\rangle$, where

(i) $E \leq F$ iff for any density operator $\rho \in \mathcal{S}(\mathcal{H}) : \mathrm{Tr}(\rho E) \leq \mathrm{Tr}(\rho F)$;

(ii) $E' = \mathbb{I} - E$ (where $-$ is the standard operator difference);

(iii) $E^{\sim} = P_{Ker(E)}$, where $P_{Ker(E)}$ is the projection associated to the kernel of E;

(iv) $\mathbb{O}$ and $\mathbb{I}$ are the null and the identity projections, respectively.

The BZ-poset $\mathcal{E}(\mathcal{H})$ turns out to be properly fuzzy since the noncontradiction principle is violated ($E \wedge E' \neq \mathbb{O}$). Further, the BZ-poset $\mathcal{E}(\mathcal{H})$ fails to be a lattice ([2]). In a quite neglected paper, however, Olson ([4]) proved that $\mathcal{E}(\mathcal{H})$ can be equipped with a natural partial order $\leq_s$ (called *spectral order*) in such a way that $\langle \mathcal{E}(\mathcal{H}), \leq_s\rangle$ turns out to be a *complete lattice*. In this talk, we will present the algebraic properties of the structure $\langle \mathcal{E}(\mathcal{H}), \leq_s, {}', {}^{\sim}, \mathbb{O}, \mathbb{I}\rangle$

and we will introduce a new class of BZ-lattices (called *BZ^*-lattices*) that represents a quite faithful abstraction of the concrete model based on $\mathcal{E}(\mathcal{H})$ (see also [3]). Interestingly enough, in the framework of BZ*-lattices different abstract notions of "unsharpness" collapse into the one and the same concept, similarly to what happens in the concrete BZ*-lattices of all effects ([5, 6]). We will finally present the structure theory of PBZ*-lattices and we provide an initial description of the lattice of PBZ*-varieties.

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Different Aspects of Tolerance Relations

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The idea of tolerance relations seen as a formalization of the intuitive notion of resemblance was discerned in the late works of Henri Poincaré. In 1962 Eric Christopher Zeeman formally introduced the notion of a tolerance as a relation that is reflexive and symmetric, but not necessarily transitive. Studying models of visual perceptions, E. C. Zeeman found it useful to axiomatize the notion of similarity and formalized the notion of tolerance spaces. The idea of "being within tolerance" or of "closeness" or "resemblance" is universal enough to appear, quite naturally, in almost any setting. It is particularly natural in practical applications: real-life problems, more often than not, deal with approximate input data and require only viable results with a tolerable level of exactness. Therefore, the topic became popular among researches from different areas such as linguistics, information theory, humanities, social sciences, but also logic and mathematics.

As a natural generalization of congruences, tolerances appeared to be a very useful tool, especially in universal algebra. In an algebraic structure $\mathcal{A} = (A, F)$ by a tolerances we mean only those reflexive and symmetric relations which are compatible with the operations of $\mathcal{A}$. However, there many other ways of looking at this notion, for example as special subalgebras of the algebra $\mathcal{A}^2$, as homomorphic images of congruences or as some types of covering systems. In this talk we discuss different approaches to the notion of tolerance in algebraic structures.

Dynamic Logic of Propositional Assignments as a Framework for Knowledge Representation

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Dynamic Logic of Propositional Assignments (DL-PA) is an interesting variant of PDL whose atomic programs are assignments of propositional variables [1]. Its mathematical properties differ from PDL: satisfiability and model checking are both PSPACE-complete. These results follow from the close relation of DL-PA with quantified boolean formulas, coming with expressivity and succinctness results. DL-PA is a powerful framework for knowledge representation, encompassing reasoning about actions and plans [4], update and revision operations [3], judgment aggregation [5], and abstract argumentation frameworks and their modification [2].

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The Dynamic Logic of Policies and Contingent Planning

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In classical deterministic planning, solutions to planning tasks are simply sequences of actions. This is not sufficient for non-deterministic environments: in so-called contingent planning, the action to be performed may depend on the non-deterministic outcomes of preceding actions. Semantically, contingent plans are modelled as policies, alias strategies, that map states to actions [3]. A natural question is whether policies can be specified as programs in the syntax of Propositional Dynamic Logic (PDL). However, it can be shown that none of the standard PDL modalities directly captures contingent planning.

We add a modality to PDL that had previously only been introduced for sequential programs [4], simplifying the extension of [1]. We show that the new modality correctly captures policies. More precisely, we show how a policy solution to a planning task gives rise to a program solution expressed via the new modality, and vice versa. We also provide an axiomatisation.

We finally discuss an epistemic extension that captures the notion of implicitly coordinated plans as recently proposed in [2].

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Completing the Circle—Cut Admissibility for Carnielli-Style n-Sequent Calculus

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The earliest sequent and tableau calculi for many-valued logics were based on the application of n-sided sequents or n-labelled formulae for each n-valued logic. This intuitively natural approach was independently proposed by many logicians in many variants based on two interpretations: verificationist and falsificationist. Although in the setting of two-valued logic a choice of interpretation has no effect on the shape of rules, in case of $n > 2$ values we obtain significantly different calculi. Verificationist interpretation was commonly used by more proof-theoretically oriented logicians and usually formulated by means of n-sequent calculi (e.g. Rousseau, Takahashi). A general cut elimination theorem for such kind of calculi was provided by Baaz, Fermüller and Zach. Falsificationist interpretation was preferred by logicians focusing on proof-search and formulated usually by means of labelled tableux (e.g. Surma, Suchoń, Carnielli). To the best of our knowledge no constructive proof of cut elimination was provided for the latter kind of calculi. In this talk we present a modified structured sequent calculi which may serve as an uniform framework for both approaches and allow for better comparison of their features. We also show how to provide cut admissibility results for calculi based on falsificationist interpretation.

Even Logical Truths Are Falsifiable

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A special group of sentences, namely logical true sentences like $p \vee \neg p$ or $\neg(p \wedge \neg p)$, are interesting for most philosophers because of—among other things —their infallibility. Moreover, it seems that their truth value is so obvious that it is not necessary to justify them. These properties lead some philosophers to use them as trustworthy sources to construct philosophical theories or even as direct justifications of philosophical theories. But are they really infallible or are they really self-evident?

In this paper, I want to answer both of these questions with *no*. For the infallibility-part, I will argue that just in the case that a sentence is analytic, necessary or a priori, it makes sense to speak about its infallibility. In other words, if a sentence is neither analytic, nor necessary, nor a priori, then it is not infallible. With some examples, I will show that a logical true sentence like the Law of Excluded Middle—as we use it in philosophy—has none of these properties and therefore is not infallible.

In the second part—the justifiability-part—I will argue that there is a direct connection between sentences in need of justification and falsifiable sentences. Since logical truths are neither analytic, nor necessary, nor a priori sentences and therefore falsifiable, they are not exempt from justifications either. In other words, their truth value is not always assessable, is context dependent, and often cannot be determined by rational and/or transcendental methods alone. Thus, logical truths need justification.

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Admissible Rules and Their Complexity[1]

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A rule is admissible in a given logic if the set of tautologies of the logic is closed under the rule, or equivalently, if the addition of the rule to the logic does not create any new tautologies. The concept also has a natural generalization to multiple-conclusion rules. Algebraically speaking, admissible rules of a nicely algebraizable logic correspond to quasi-identities (or clauses, in the multiple-conclusion case) valid in free algebras of the corresponding variety.

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Admissibility is closely related to unification: equational unification over the equational theory of the corresponding variety can be stated in terms of the logic, namely that a unifier of a formula is a substitution that turns it into a tautology. This makes it a special case of inadmissibility (for rules with inconsistent conclusions). Unification can be generalized to the disunification problem, in which case it encompasses inadmissibility of multiple-conclusion rules.

It is standard in unification literature to work in the expansion of the given equational theory by free constants. We may do this in the logical setting as well, leading to admissibility with a new kind of atoms—variously called parameters, constants, coefficients, or metavariables—that are required to be left intact by substitutions.

In this talk, we are going to investigate admissibility with parameters in transitive modal logics (extensions of **K4**). We will be primarily interested in logics satisfying suitable frame extension properties (cluster-extensible logics), but we will also look at other logics, in particular logics of finite depth and width.

We shall be interested for example in semantic descriptions of admissible rules, constructions of complete sets of unifiers, and axiomatization of admissible rules by means of bases. We will pay special attention to algorithmic complexity questions, such as what is the computational complexity of admissibility in various logics; as we will see, they are intimately connected to structural properties of the logics.

The talk is mostly based on [1, 2].

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Barbershop Paradox and Connexive Logic

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In 1894 Lewis Carroll published in Mind the paper "A Logical Paradox". `http://fair-use.org/mind/1894/07/notes/a-logical-paradox`, see also: Storrs

McCall, A History of Connexivity, vol. 11 Gabbay, Handbook of the History of Logic. A history of its central concepts 415–451.

Uncle Joe and Uncle Jim are going to a barbershop run by Allen, Brown and Carr, and uncle Jim hopes that Carr will be in to shave him. Uncle Joe says he can prove Carr will be in by an argument having as premisses two hypotheticals. First, if Carr is out, then if Allen is out, Brown must be in (since otherwise there'd be nobody to mind the shop). Secondly if Allen is out Brown is out (since Allen, after a recent bout of fever, always takes Brown with him). Taking A to stand for Allen is out, B for Brown is out, etc.

Thus we have: (i) If C then (if A then not-B); (ii) If A then B, and these two premises, according to Uncle Joe, imply not-C, because from (i) at least one of them must always be present to mind the shop, and whenever Allen leaves he always takes Brown with him. Now, suppose that Carr is out. In that case then if Allen is out then Brown must be in, in order to tend the shop.

But we know that this isn't true—we've been told that whenever Allen is out then Brown is out. The result is, of course, paradoxical, because under the stated conditions Carr can perfectly well be out when the other two are in, or even when Allen alone is in. The question is, at what point is Uncle Joe's argument fallacious?

Solution is that the two hypotheticals *If A then B* and *If A then not-B* are not incompatible: they may in fact both be true when A is false, as is the case in classical two-valued logic. Hence we cannot infer not-C by modus tollens. The thought underlying this solution is that If *A then not-B* does not properly negate *If A then B*. Burks and Copi disagree, however. When interpreted as causal implications rather than as material implications, the two hypotheticals above are in their opinion incompatible, and this is in general true of causal implication.

The aim of the presentation is to present Lewis Carrol paper and show its relation to connexive logic based the following Aristotle's and Boethian these.

(A1) $\sim (A \Rightarrow \sim A)$

(A2) $\sim (\sim A \Rightarrow A)$

(B1) $(A \Rightarrow B) \Rightarrow \sim (A \Rightarrow \sim B)$

(B2) $(A \Rightarrow \sim B) \Rightarrow \sim (A \Rightarrow B)$.

The World of Ideas modulo Topological Ontology

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Let P be any non-empty set and $\{\emptyset, \mathsf{P}, \ldots\}$ any family included in 2^{P} with distinguished sets $\emptyset$ and P. So, we consider any families of power set of P. Each family of the form $\{\emptyset, \mathsf{P}, \ldots\}$ shall be called an *idea E* and any set of ideas *Platonian world of ideas*. Treating some idea as the subbasis S of the smallest topology (topological space) generated by S, let us call this topological space $(\mathsf{P}, \mathcal{T}_{\mathsf{S}})$.

It turns out that the topological space can be used for interpreting or modelling:

(a) the possible world in the sense of Wittgenstein (and later Wolniewicz) and

(b) substances—monads in the sense of Leibniz.

I will therefore show the relationship between the ideas themselves and between the ideas and the real world (possible world). What will be important will be certain statements concerning the hierarchy of ideas (e.g. the Porphyry tree indicating the relations between ideas—e.g. species and kinds), which are of a purely topological nature. For example, we will show the theorems:

Proposition 1: If the root of a tree generates an inconnected topological space, then each element of the tree is a sub-base generating an inconnected space.

Proposition 2: If the root of a tree generates T_2 space, then each element of the tree is a sub-base that generates T_2 space.

All necessary definitions of basic concepts (philosophical and topological ones) will be given during the lecture.

On Multiple-Conclusion Consequence Relation for Classical Logic[2]

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We know that there are close relations between such proof systems as sequent calculi ([1]) and analytic tableaux ([6]); the relations can be viewed in historical perspective, but, more importantly, in terms of translations between the systems, and finally ... in terms of the underlying intuition that derivations in the proof systems formalize the same reasoning.

This can be said about sequent calculi and analytic tableaux on the one hand, but also about sequent calculi (with multisuccedent sequents) and the Rasiowa-Sikorski systems (presented for the first time in [4], see also [3]). If analytic tableaux occupy the left side of a sequent, R-S systems occupy the right side, whereas both-sided multisuccedent sequents generalize both. To sum up, all the three stories can be told in terms of sequents.

It is also well known that we can interpret all the three cases: the "left-sided" sequents (with empty succedent), the "right-sided" sequents (empty antecedent) and both-sided sequents semantically in terms of multiple-conclusion entailment relation (see [5]). Under this interpretation, a valid left-sided sequent expresses the fact that the formulas in its antecedent form an inconsistent set (the conjunction of the formulas is a contradiction). A valid right-sided sequent expresses the fact that the formulas form a "safeset" (a notion introduced in [7], in the finite classical case amounts to the fact that the disjunction of the formulas is a valid formula). "Ordinary" sequents are treated as expressing entailment, if there is one formula in the succedent. All the cases are generalized by the relation of multiple-conclusion entailment.

What about a proof-theoretical interpretation? If $\mathbb{P}$ is a proof system, then the left side of a sequent may correspond to inconsistency defined on the grounds of $\mathbb{P}$. The case of a both-sided single succedent sequent is equally easy to interpret: in terms of derivability in $\mathbb{P}$. What about the right side? Furthermore, what about the generalization of all this cases, one that would fit the semantic relation of multiple-conclusion entailment?

In my talk I present a proposal of an answer to these questions described in [2, Chapter 3], that is, a proof-theoretical settlement of the multiple-conclusion entailment relation. Roughly speaking, the meaning of the relation is the following: Y "follows from" X, iff Y presents a space of possibilities such that whatever can be derived from the whole Y **by cases**, can be "just" derived

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from X. In other words, Y presents the cases to be considered in proofs by cases from X.

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The $\to$-Decomposition Property. II

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This is a continuation of my and K. Siemieńczuk talk given at the XX$^{\text{th}}$ Conference *Applications of Logic in Philosophy and the Foundations of Mathematics.* In [1] we consider $\to$-irreducible elements in finite Heyting lattices. An element a is called *$\to$-irreducible* if $a = x \to y$ implies $a = x$ or $a = y$.

Theorem. ([1]) An element a of a finite Heyting lattice L is $\to$-irreducible iff a is the least element in some maximal Boolean interval of L (MBI for short).

The set of all $\to$-irreducible elements of L is denoted by $S(L)$. If $a, b, c \in S(L)$, then we can combine these elements to obtain implication polynomials $a \to b$, $(a \to b) \to c$, $((a \to b) \to c) \to a$, etc. The question is: which elements of L can be generate in this way? More formally, let $[\![S(L)]\!]$ stand for the closure of $S(L)$ with respect to the operation $\to$.

Definition. We say that lattice L has the *$\to$-decomposition property* if $L = [\![S(L)]\!]$.

Problem. Which lattices have the $\to$-decomposition property?

In the talk we present a partial solution of the preceding problem. We consider so-called *n-regular* (each MBI is isomorphic to n-dimensional Boolean lattice B_n) and *well-glued* (if A and B are adjacent MBIs, then $A \cap B \cong B_{n-1}$) lattices. Moreover, we define some special n-regular and well-glued lattices $n \otimes B_n$ (see Figure below).

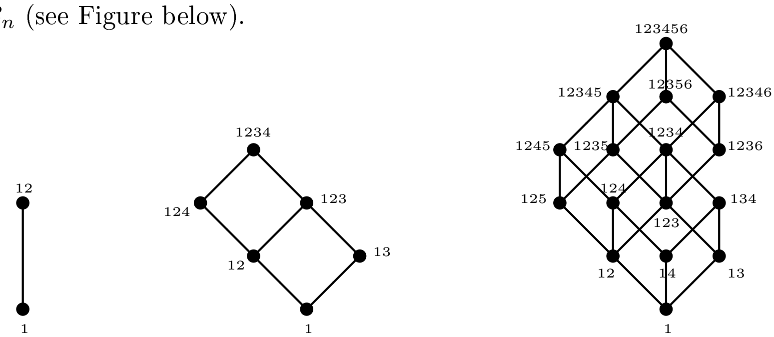

Figure 1: Lattices $1 \otimes B_1$, $2 \otimes B_2$ and $3 \otimes B_3$.

Our result is the following:

Theorem. If L is n-regular and well-glued lattice, then L has the $\rightarrow$-decomposition property iff $n \otimes B_n$ is a covering sublattice of L.

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Logic and Existence

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The lecture deals with several problems concerning notion of existence and related ontological notions of existential dependence, possibility and necessity.

The issue is, that we all sometimes assert or reject propositions like *electrons exist*, *minds exist* or *Pegasus exists*. Sometimes we may be confused as regards the meaning of the term ‘exist(s)’. Therefore we should to establish the meaning of this term, as it occurs in the formula ‘x exist(s)’ where ‘x’ is a variable for which any noun-expression can be substituted.

Following some ideas of Eugenia Ginsberg-Blaustein, we assume that the concepts of existence, existential dependence, possibility and necessity could be defined by means of the concepts of state of affairs and subject of the state of affairs taken as primitive.

The Connective ‘tudzież’

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In the paper I ask the question whether the connective “tudzież”, which is Polish for “as well as”, may be a suitable equivalent to one of the functors in logic. I consider two types of examples, from normative and non-normative sources.

It turns out that, in the former case, the connective “tudzież” is the equivalent to the functor of conjunction, whereas in the latter, to the functor of conjunction, the functor of inclusive disjunction and the functor of exclusive disjunction.

When a Closure System Is a Complete and Atomic Boolean Algebra?

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An answer for the title question is: *a closure system over a nonempty set A forms a complete and atomic Boolean algebra iff the corresponding closure operation is Boolean with respect to a subset of A*, in the following sense.

Definition. Let A be a set and Z_0 any its subset. A closure operation $C : \wp(A) \longrightarrow \wp(A)$ will be said to be *Boolean with respect to* Z_0 iff for any $X \subseteq A$,

(1) $C(X) \subseteq C(C(X) \cap Z_0)$,
(2) $X \subseteq Z_0 \;\Rightarrow\; C(X) \cap Z_0 \subseteq X$.

For a Boolean closure operation $C : \wp(A) \longrightarrow \wp(A)$ with respect to a set $Z_0 \subseteq A$, the closure system $(Cl(C), \subseteq)$ (of all its closed elements) forms a complete and atomic Boolean algebra $(Cl(C), \cap, \vee, ', C(\emptyset), A)$ in which for any $T_1, T_2, T \in Cl(C)$, $T_1 \vee T_2 = C(T_1 \cup T_2), T' = C(Z_0 - T)$. The atoms are of the form: $C(\{a\}), a \in Z_0$.

A proof of the answer goes via the following facts:

Fact 0. *A Boolean algebra is complete and atomic iff it is isomorphic to a field of all subsets of a set.*

PROPOSITION 1. *For any Boolean closure operation $C : \wp(A) \longrightarrow \wp(A)$ with respect to a $Z_0 \subseteq A$, the restriction $C \restriction \wp(Z_0)$ is an isomorphism of the complete lattices $(\wp(Z_0), \subseteq)$, $(Cl(C), \subseteq)$, where $Cl(C) = \{X \subseteq A : C(X) = X\}$. The inverse isomorphism is the mapping $F : (Cl(C), \subseteq) \longrightarrow (\wp(Z_0), \subseteq)$ defined by $F(T) = T \cap Z_0$, for each $T \in Cl(C)$.*

PROPOSITION 2. *Any closure operation $C : \wp(A) \longrightarrow \wp(A)$ whose family of all closed elements forms a complete lattice $(Cl(C), \subseteq)$ isomorphic to a complete lattice $(\wp(B), \subseteq)$ of all subsets of a set B, is Boolean with respect to some set $Z_0 \subseteq A$.*

Both propositions are proved independently on Fact 0. Moreover, the essential part of Fact 0: *Every CABA is isomorphic to a powerset algebra*, follows from Proposition 1, the well-known theorem:

a complete and atomic Boolean algebra $\mathcal{A} = (A, \wedge, \vee, -, 0, 1)$ is isomorphic to the Boolean algebra of all principal ideals of $\mathcal{A}$: $(\{(x] : x \in A\}, \cap, \bar{\vee}, ', \{0\}, A)$, where $(x_1]\bar{\vee}(x_2] = (x_1 \vee x_2]$ and $(x]' = (-x]$.

and the proposition:

the closure operation C, corresponding to the closure system of all principal ideals of a CABA $\mathcal{A}$, is Boolean with respect to the set of all atoms of $\mathcal{A}$.

The Undecidability of Profiniteness

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A topological algebra is *profinite* if it is isomorphic to an inverse limit of finite algebras endowed with the discrete topology. Their topologies are always Boolean, i.e., Hausdorff, compact and totally disconnected. However, not all Boolean topological algebras are profinite. As an example, one may take any Boolean topological algebras which algebraic reduct is subdirectly irreducible (has a least nontrivial congruence).

Let $\mathcal{V}$ be a variety (an equationally defined class of algebras). We consider the class $\mathcal{V}_{Bt}$ of Boolean topological algebras with the algebraic reducts in $\mathcal{V}$, and the class $\mathcal{V}_{Bc}$ of profinite algebras with the algebraic reducts in $\mathcal{V}$. The class $\mathcal{V}_{Bc}$ is called the *Boolean core* of $\mathcal{V}$.

A general problem is the axiomatizations of $\mathcal{V}_{Bc}$ relative to $\mathcal{V}_{Bt}$, see [1]. Or, more precisely, when we can define $\mathcal{V}_{Bc}$ relative to $\mathcal{V}_{Bt}$ without referring to topology. (In [2] a general scheme for axiomatizations of even more general classes of topological algebras with the use of topology was given.)

The basic question is simply when $\mathcal{V}_{Bc} = \mathcal{V}_{Bt}$? If it is the case we say that $\mathcal{V}$ is *standard*. It appears that it is true for many varieties of classical algebras like varieties of groups, rings, semigroups, distributive lattices or Heyting algebras. Still, already in [4, Section VI.2.6] Johnstone speculated that it may be hard to give a simple condition for varieties which is both necessary and sufficient for standardness. Confirming this speculation, Jackson proved in [3] that there is no algorithm which decides if a given finite set of identities defines a standard variety. We proved a similar fact, but for finitely generated varieties [5].

Theorem. *There is no algorithm which decides if a given finite algebra of a finite type generates a standard variety.*

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Dual Logic of Rational Agent

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In their recent paper [1], Kubyshkina and Zaitsev have developed a four-valued logic **LRA** (**L**ogic of **R**ational **A**gent) which truth-functionally represents the epistemic state of an agent. In this report, we introduce the logic **DLRA** (**D**ual **LRA**) which differs from **LRA** with respect to the set of designated values. One of the features of these logics is the existence of two negations (epistemic and ontological ones) in their language. Due to these negations one may formalize knowing or ignoring something. Besides, using relational semantics, Kubyshkina and Zaitsev present necessity and possibility operators for **LRA**. In the virtue of the standard definition of the possibility and the use of two negations (as well as their combination), we introduce two new possibility operators for **LRA**. After that we modify all these modal operators to be suitable for **DLRA**. Then we present natural deduction systems for all the extensions of the negation fragment of **DLRA** by truth-functional n-ary operators. As a particular case, we formalize **DLRA** itself. Last, but not least, we introduce the natural deduction system for a **K**-style modal logic which propositional basis may be any of the n-ary extensions of the negation fragment of **DLRA**.

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Archetypal Rules and Intermediate Logics

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a joint work with:
Lloyd Humberstone

The notion of archetypal rule was introduced and investigated by Lloyd Humberstone in the case of classical and intuitionistic propositional logic, cf. [1]. Informally, we say that a rule r is archetypal for a logic L if, up to provability in L, r is derivable, not invertible and for any other derivable rule s there is a substitution such that the premisses of s are the instances of premisses of r and the conclusion of s is the instance of the conclusion of r. The problem of semantic characterization of archetypal rules in classical propositional logic was solved recently in [3]. In this talk we present some results concerning archetypal rules in the context of intermediate logics.

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Empirical Investigation of the Liar Paradox. Neuroimaging Evidence That the Human Brain Perceives the Liar Sentence as False

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The virtual entailment principle devised by Jean Buridan in the XIV$^{\text{th}}$ century proposes that every sentence of the natural language implicitly asserts its own truth. Accepting this principle renders the Liar paradox as a simple falsehood, since the content of the Liar sentence begins to contradict the hidden implied principle. Because of that, Buridan postulated that humans should perceive the Liar sentence the same way as any other false sentence. Given that this solution entails accepting strong claims about laguage use, it was criticised for making *ad hoc* claims without evidence. However, modern psychophysiological techniques made it possible to test if human brain really reacts to the Liar sentence like to false sentences. An experiment was conducted to examine brain activity when viewing true sentences, false sentences and self-referential sentences (including the Liar and the Truthteller). The results showed that the human brain processes the Liar sentence identically to false sentences, whereas the Truthteller sentence identically to true sentences. This provides evdience for the Buridan's predictions derived from the *virtual entailment principle* and supports the notion that we think with the logic of truth—a logic for which the truth is a designated value of its adequate semantics. We show that the conclusions of non-Fregean logics regarding the Liar paradox coincide with the human comprehension of language. In non-Fregean logics the Liar sentence turns out to be contradictory, which means that the sentence is false and its negation is true. Perception of sentences from the perspective of their content rather than reducing them to logical values, on one hand, solves the Liar's antinomy, and on the other hand, it is consistent with our cognition.

A Framework for Evaluation of Structured Arguments

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I present a framework for evaluating the strength of structured arguments and counterarguments, based on the kinds of diagrams used in informal logic. The evaluation in this approach is bottom up, general and abstract, making it easier to compare specific models of evaluation (e.g. [1, 4, 3, 2]) and to formulate a consistent methodology. For the generalized model abstracts not only from the particular set of values that represent argument strength, but also from particular algorithms that transform the acceptability of premises into the acceptability of conclusions.

The applied, underlying model of argument structure represents serial as well as linked, convergent and divergent arguments, but also additional, dialectic elements such as rebutting, undercutting and undermining defeaters. These dialectic extensions of the standard diagramming method enable us to display arguments as aggregated with their counterarguments in the same diagram while evaluating the effectiveness of attack ([2]).

The set of values can be any (non-empty) set containing at least two elements that are assigned to the sentences of a given language by a partial function. This evaluation function represents an audience. Elements of a distinct (non-empty) proper subset of this set of values are assigned to audience-accepted sentences. I discuss reasonable ways of ordering the set of values. Another set, namely the set of argument weights is introduced, which can be any (non-empty) set containing at least two elements assigned to the strengths of direct inferences, regardless of the premises' actual values, but pertaining only to the relevance of the premises to the conclusion. In this set some elements are again distinguished that correspond to valid inferences.

In the evaluation process, the (bottom) values of the first premises combine with the weights of the component inferences in an appropriate order, corresponding to the structure of the examined whole. Using algorithms that are introduced abstractly as operations on both values and weights, we obtain the (upper) value of the final conclusion. This value is per definition the strength of the whole argument in question.

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Points, Lines and Planes in Tarski's System of Point-Free Geometry

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Tarski's system of point free-geometry, called Geometry of Solids, was sketched by him in [3]. The full development of it was given by A. Pietruszczak and R. Gruszczyński in [1]. This system of geometry is based on the classical mereology, and instead of the notion of *point*, the notion of *mereological ball* and, in general, the notion of *mereological solid* are assumed as primitive ones. In [3] Tarski showed how to construct, on the basis of these notion and notions of mereology, the primitive notions of Euclidean geometry in Pieri's approach, i.e., the notion of *point* and the notion of *equidistance* relation. In [1] the Authors showed that indeed, after certain modifications, a system of point-free geometry sketched by Tarski is isomorphic to ordinary, point-based Euclidean geometry.

The aim of the talk is to give a brief sketch of Tarski's Geometry of Solids and then a sketch of construction of primitive notions of Hilbert's axiomatization of geometry on a "point-free manner". After introducing auxiliary notions, the notion of *straight line* and *plane* will be defined and then, the relations of coincidence holding between them will be introduced. It will also be shown that there is something like a "general way" that leads to define geometrical notions such as points, lines and planes that is expressed by the same formula.

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Some Schematic Extensions of Intuitionistic Predicate Logic

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Prenex operations in classical logic are expressed as eight equivalences, that is sixteen implications. Out them, thirteen are intuitionistically logically valid, and those that are not are the following:

CD: $\forall x(\psi \vee \varphi(x)) \to \psi \vee \forall x \varphi(x)$,

S2: $(\psi \to \exists x \varphi(x)) \to \exists x(\psi \to \varphi(x))$,

S3: $(\forall x \varphi(x) \to \psi) \to \exists x(\varphi(x) \to \psi)$.

Here x is not free in ψ and free variables (parameters) are possible (the universal quantifiers that bound them are omitted). These schemas were considered in [1] (but CD is called S1 in that paper). We will investigate the relationships between logics obtained by adding one of these schemas, or one of the following four schemas:

DNS: $\forall x \neg\neg\varphi \to \neg\neg\forall x \varphi$,

C$\downarrow$: $\exists x(\exists v \varphi(v) \to \varphi(x))$,

ED: $\forall x(\forall v(\varphi(v) \to \varphi(x)) \to \varphi(x)) \to \exists x \varphi(x)$,

C$\uparrow$: $\exists x(\varphi(x) \to \forall v \varphi(v))$

to intuitionistic logic (again, parameters are possible). DNS, *double negation shift*, and CD are well known. CD is valid in all predicate Kripke structures in which all nodes have the same domain. Logic obtained by adding CD to intuitionistic logic is known as *logic of constant domains*, or also *Grzegorczyk's logic*. C$\uparrow$ and C$\downarrow$ were also considered in [1], while ED is taken from [2] where it is used to show that in some logics the quantifier $\exists$ is expressible in terms of $\forall$ and $\to$.

It appears that S2, C$\downarrow$ and ED are intuitionistically equivalent, and also S3 and C$\uparrow$ are equivalent. Thus adding one of the schemas to intuitionistic logic yields four logics that can again be called CD, DNS, S2 and S3. S3 implies both CD and DNS. No other relationships between these schemas hold. Most of the proofs are straighforward, but the fact that DNS+S2 does not imply CD seems to require a less obvious single-purpose proof.

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There Are $2^{\aleph_0}$ *Pre-Maximal Extensions of the Relevant Logic* E

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Entailment logic **E** was presented by A.R. Anderson and N.D. Belnap in 1975 [1]. However, not much is known about the structure of extension of the logic **E**; the logic **R** and **RM** and their extensions have been a subject of deep investigations [3], [5], [6], [7], [8]. What has been shown about **E** is the lack of algebraizability [2]. We also know that the logic **E** is not structurally complete [4].

In this talk we will try to characterize the structure of the lattice of the extension of the logic **E** (without constants). We devote our special attention to the upper part of the lattice of the extension of **E**. It turns out that there are $2^{\aleph_0}$ coatoms in the interval $[\mathbf{E}, \mathbf{CL}]$, where **CL** denotes the classical logic, while the interval $[\mathbf{R}, \mathbf{CL}]$ contains 3 coatoms, and the interval $[\mathbf{RM}, \mathbf{CL}]$ contains only one coatom.

The extension of the logic E is called *pre-maximal* if and only if it is coatom in the interval $[\mathbf{E}, \mathbf{CL}]$ (of course the maximal extension of the logic **E** is **CL**).

Theorem There are $2^{\aleph_0}$ pre-maximal extension of the relevant logic **E**.

We would like to present an infinite binary tree of simple, finite **E**-algebras. The nodes of this tree are algebras based on finite chains. Each branch of the tree represents an infinite denumerable **E**-algebra.

Adding more details. The structure of the binary tree in question can be described by induction.

Step 0 (level 0). The algebra from the level 0 (algebra $\mathbf{A^0}$) is based on a 12-element chain

and the operation $\to$ is defined in the table below:

$\to$	a	$\neg a_1$	$\neg a_2$	$\neg a_3$	$\neg a_4$	a_4	a_3	a_2	a_1	$\neg a$
a	a	a	a	a	a	a_4	a_3	a_2	a_1	a_1
$\neg a_1$	0	a	a	a	a	a_4	a_3	a_2	a_2	a_1
$\neg a_2$	0	0	a	a	a	a_4	a_3	a_3	a_2	a_2
$\neg a_3$	0	0	0	a	a	a_4	a_4	a_3	a_3	a_3
$\neg a_4$	0	0	0	0	a	a_4	a_4	a_4	a_4	a_4
a_4	0	0	0	0	0	a	a	a	a	a
a_3	0	0	0	0	0	0	a	a	a	a
a_2	0	0	0	0	0	0	0	a	a	a
a_1	0	0	0	0	0	0	0	0	a	a
$\neg a$	0	0	0	0	0	0	0	0	0	a

The set of designated values of this algebra is $[a) = \{x : a \leq x\}$ (it is true for all the algebras we consider), thus $\mathbf{A^0}$ is a simple algebra.

Step 1 (level 1). We construct two new algebras (i.e. $\mathbf{A^{00}}$ and $\mathbf{A^{01}}$) based on $\mathbf{A^0}$. We add new elements $a_5, \neg a_5$ to the old ones; now we have the chain $0 < \ldots < \neg a_3 < \neg a_4 < \neg a_5 < a_5 < a_4 < a_3 < \ldots < 1$; the new chain has 14 elements. Next, we define the operation $\to$ in $\mathbf{A^{00}}$ and $\mathbf{A^{01}}$ in the following way. The values of $\to$ for elements $0, \ldots, \neg a_3$ and their negations and for $\neg a_4$ remain the same as in $\mathbf{A^0}$. In $\mathbf{A^{00}}$ we set $a_5 = \neg a_1 \to a_4$ and in $A^0 1$ we set $a_5 = \neg a_4 \to a_4$; in consequence the values for $x \to a_4$ and $\neg a_4 \to y$ must be changed in both algebras.

Step $\mathbf{n+1}$ (level $n+1$). Let us consider the algebras from the level n; we denote them by $\mathbf{A^n}$ where n stands for the 0-1 sequence of the length n with 0 as the first element. The $\mathbf{A^n}$-algebras are based on the $(12+2n)$-element chain. Each algebra $\mathbf{A^n}$ determines two algebras: ($\mathbf{A^{n0}}$ and $\mathbf{A^{n1}}$) from the level $n+1$. The algebras from the level $n+1$ are based on the $(12+2(n+1))$-element chain, in which $0 < \ldots < \neg a_n < \neg a_{n+1} < a_{n+1} < a_n < \ldots < 1$. As in the step 1, the definition of $\to$ in $\mathbf{A^{n+1}}$-algebras is based on the definition of $\to$ in $\mathbf{A^n}$. Let $\mathbf{A^n}$ be fixed. Then in $\mathbf{A^{n0}}$ we set $a_{n+1} = \neg a_1 \to a_n$, and in $\mathbf{A^{n1}}$ we set $a_{n+1} = \neg a_n \to a_n$. The values for $0, \ldots, \neg a_{n-1}$ and its negations, and for $\neg a_n$ remain the same as in the algebra $\mathbf{A^{n-1}}$, which precedes the algebra $\mathbf{A^n}$, and the values of $x \to a_n$ and $\neg a_n \to y$ must be changed.

Now, if we consider a branch of this tree, we get an infinite denumerable $\mathbf{E}$-algebra; the operation $\to$ can be reconstructed from the $\mathbf{A^n}$-algebras in this branch.

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Temporal Logics with Metric Operators

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I will present results on temporal logics with metric operators, which we have recently established in collaboration with researchers from University in London and University of Oxford: B. Cuenca Grau, S. Kikot, V. Ryzhikov, M. Kaminski, E. V. Kostylev, and M. Zakharyaschev.

Metric temporal logic (MTL) is a propositional modal logic which was originally introduced for modeling and reasoning about real-time systems [2]. The language of MTL contains modal operators $\boxplus$, $\boxminus$, $\diamondplus$, and $\diamondminus$ whose intended meaning is 'everywhere in the future', 'everywhere in the past', 'somewhere in the future', and 'somewhere in the past', respectively. Each operator is indexed with an interval, which brings metric component into the logic; for instance we can construct the following formulas:

- $\boxplus_{(3,4.1]}\varphi$ – 'φ holds everywhere more than 3 and at most 4.1 time units in the future';
- $\diamondminus_{(0,\infty)}\varphi$ – 'φ holds somewhere in the past'.

In general, it is known that if a temporal logic is interpreted over a dense order of time points and allows us to express that a propositional variable is always followed exactly one time unit later by another variable, which can be written in MTL as $\boxminus_{[0,\infty)}\boxplus_{[0,\infty)}(p \to \diamondplus_{[1,1]}q)$, then checking satisfiability of formulas in such a logic is undecidable [1]. Hence, the satisfiability problem of MTL interpreted over dense time lines is undecidable.

We will show how to obtain decidable fragments of MTL interpreted over dense time lines by implying syntactic restrictions on the use of Boolean connectives and modal operators occurring in a formula. We will classify the obtained fragments according to their computational complexity and consider their first-order extensions. The obtain results allow us to understand better what kind of formulas with metric operators make reasoning hard.

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Generalized Topological Semantics for Weak Modal Logics

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The notion of *general* or *generalized* topology can be interpreted in many ways. In this paper, we shall adhere to the concept introduced by Császár in [1]. Hence, we assume that generalized topology contains empty set and is closed with respect to arbitrary unions. Contrary to the typical definition of topology, we discard superset axiom and we do not assume that finite intersections of open sets are also open. We introduce the notion of *generalized topological model* (for non-normal modal logics) and we show that these structures are compatible with certain subclass of neighborhood models. We discuss soundness and we show benefits of reasoning in terms of *pseudo-interiors* and *pseudo-open* sets. Moreover, we compare our results with those of Soldano [2] who investigated so-called *abstractions* which are analogous to general topologies (at least in his interpretation). Finally, we show that our *isolated* worlds (i.e. these which are beyond any open set) can be considered as *impossible* worlds (which means that nothing is necessary there but everything is possible).

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The Theory of Authority

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The studies into the notion of authority have been inspired by Józef Maria Bocheński's treatise *Co to jest autorytet?* (*What is an authority?*). The elementary expression adopted here is: "*x is subordinate to the authority y in the field a*"—symbolically: $x\varepsilon saut(y,a)$. Our basis is elementary ontology enriched with specific axioms:

$$
\begin{aligned}
&x\gamma saut(y,a) \ \mid \ saut(x,a)\delta saut(y,a)\\
&x\gamma saut(y,a) \ \mid \ -y\delta saut(x,a)\\
&x\gamma saut(y,a) \ \mid \ y\gamma y\\
&saut(x,a)\Box saut(y,a) \ \mid \ x\Box y
\end{aligned}
$$

which are an interpretation of *Frege's predication scheme*. Functor Aya, which appears in the context $x\varepsilon Aya$ read as "*x is an authority for y in the field a*", is introduced by definition. The functor's special cases A_i and A_e appear in contexts $x\varepsilon A_iya$ and $x\varepsilon A_eya$, which are respectively read as: "*x is an authority only for y in the field a*" and "*x is an authority not only for y in the field a*". Moreover, the functor aut is introduced by definition, where the sentence $x\varepsilon aut(y,a)$ elementary with it is read as "*x is an authority for y in the field a*". The paper also considers the extension of this structure with list arguments and arguments taking into account the distinction between epistemic and deontic authority.

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Unification in Superintuitionistic Predicate Logics

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Talk is based on the paper, by Wojciech Dzik and Piotr Wojtylak, *Unification in Superintuitionistic Predicate Logics and its Applications*, Review of Symbolic logic 12(1) (2019), 37–61.

A *unifier* for a formula A in a predicate logic L is a substitution ε for predicate variables such that $\vdash_L \varepsilon(A)$. A formula A is said to be *projective* in L if it has a projective unifier in L, that is it has a unifier ε such that $\vdash_L A \to (B \leftrightarrow \varepsilon(B))$ for each B. We say that a logic L enjoys *projective unification* if each unifiable formula is L-projective.

Theorem 1. L *enjoys projective unification iff* $\mathsf{P.Q{-}LC} \subseteq \mathsf{L}$ *where*

$$\text{(P)} \qquad \exists_x(\exists_x B(x) \to B(x)).$$

Theorem 2. *If A is a unifiable Harrop formula and ϑ is its ground unifier, then*

$$\varepsilon(P_j(x_1,\ldots,x_k)) = \begin{cases} A \to P_j(x_1,\ldots,x_k) & \textit{if } \ \vartheta(P_j(x_1,\ldots,x_k)) = \top \\ \neg\neg A \wedge (A \to P_j(x_1,\ldots,x_k)) & \textit{if } \ \vartheta(P_j(x_1,\ldots,x_k)) = \bot \end{cases}$$

defines a projective unifier for A in any superintuitionistic predicate logic L.

It follows that the inferential rules

$$(\to\vee) \quad \frac{\neg A \to B_1 \vee B_2}{(\neg A \to B_1) \vee (\neg A \to B_2)} \qquad \text{and} \qquad (\to\exists) \quad \frac{\neg A \to \exists_x C(x)}{\exists_x(\neg A \to C(x))}$$

are not admissible in some superintuitionistic predicate logics L. The rules are admissible in $\mathsf{Q{-}INT}$ though they are not derivable there.

Truth, Grounding and Paradoxes

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The notion of metaphysical grounding has been widely discussed in the past decade. We explore how metaphysical grounding interacts with the truth predicate. In particular, the following question is investigated: given a true sentence ϕ, what grounds $Tr(\ulcorner\phi\urcorner)$?

We start with the most appealing answer to this question—the Aristotlean Principle (**AP**)—which suggests that for any true sentence ϕ, ϕ grounds $Tr(\ulcorner\phi\urcorner)$. Take grounding as a sentential operator denoted by '$<$' and let '$\phi < \psi$' stand for 'ϕ grounds ψ'. **AP** can be written as the following axiom schema:

AP $\phi \to (\phi < Tr(\ulcorner\phi\urcorner))$

Kit Fine famously shows that **AP** is inconsistent with the following principles of grounding (given that $\exists x Tr(x)$ is a theorem of our system):

$\exists$-Grounding $\phi(a) \to (\phi(a) < \exists x\phi(x))$

Transitivity $(\phi < \psi \wedge \psi < \chi) \to (\phi < \chi)$

Irreflexivity $\neg(\phi < \phi)$

While some might wish to preserve **AP** by rejecting one of the principles above, we argue that it is **AP** that should be given up. We show that **AP** is also inconsistent with the following principle, given **Transitivity** and **Irreflexivity**:

$\vee$-Grounding $\phi \to (\phi < \phi \vee \psi)$

We show this by observing that under some deviant coding, ϕ can share the same name with $Tr(\ulcorner\phi\urcorner) \vee 1 = 1$.

We then consider a natural replacement of **AP**, which the deflationists of truth might find attempting. The idea is that a true sentence and the sentence asserting its truth should be indistinguishable regarding their grounds, namely:

DP $(\psi < \phi) \leftrightarrow (\psi < Tr(\ulcorner\phi\urcorner)$

DP, as we show, faces the same problem as **AP**: it is inconsistent with the grounding principles listed above.

At the end, we propose the following regarding the question of what grounds $Tr(\ulcorner\phi\urcorner)$:

TG $(\phi < \psi) \leftrightarrow (Tr(\ulcorner\phi\urcorner) < Tr(\ulcorner\psi\urcorner))$

Namely, for any grounded ϕ, $Tr(\ulcorner\phi\urcorner)$ is grounded in $Tr(\ulcorner\psi\urcorner)$ for any ψ that grounds ϕ. We argue that **TG** fits well with the deflationary conception of truth and increases the expressive power of our language. Moreover, the formal system that includes **TG** and the grounding principles above can be proved to be consistent.

Refined Tableau Calculi for Modal Logics with Simplified Semantics

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In [2] Pietruszczak, Klonowski and Petrukhin showed that for some modal logics we can discard accessibility relation from their semantics without any loss of information about frames and models. In particular, they showed that the logics $S5$, $K45$, $KB4$, $KD45$ and some extensions of the last three are characterised by certain sub-classes of so-called *simplified frames*.

A *simplified frame* $\mathscr{F}$ is a pair $\langle W, A\rangle$, where W is a set of possible worlds and $A \subseteq W$. In a modal model $\mathscr{M} = \langle \mathscr{F}, V\rangle$ based on a simplified frame $\mathscr{F}$ the truth conditions for modal operators are defined as follows:

$$\begin{array}{lll} \mathscr{M}, w \Vdash \Diamond\varphi & \text{iff} & \text{there exists } v \in A \text{ such that } \mathscr{M}, v \Vdash \varphi \\ \mathscr{M}, w \Vdash \Box\varphi & \text{iff} & \text{for all } v \in A \text{ it holds that } \mathscr{M}, v \Vdash \varphi, \end{array}$$

while the truth conditions for Boolean connectives are the same as in ordinary Kripke models.

One can see that a simplified frame $\mathscr{F} = \langle W, A\rangle$ is equivalent to a Kripke frame $\mathscr{F}' = \langle W, R\rangle$, where $R = W \times A$, i.e., is a so-called *semi-universal relation.*

In our talk, we will show how the fact that the above-mentioned modal logics are characterised by certain classes of simplified frames can be utilised in devising tableau-based decision procedures for these logics. Strictly speaking, we will present sound, complete and terminating *labelled* tableau calculi for the logics $K45$, $KB4$, $KD45$, where no statements referring to accessibility relation occur in a derivation tree. Therefore, the announced calculi are designed in a similar fashion to the elegant and conceptually simple tableau-based decision procedure for $S5$ known from, e.g., [3]

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