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Abstracts

Editorial note

(EN) means that the talk is presented in English, (PL)—in Polish.

Gamma Graphs of Trees

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Every dominating set of the smallest possible cardinality is called γ-set. We consider a graph $\gamma.G$, whose vertices correspond to γ-sets of G, and two γ-sets S, S' are adjacent in $\gamma.G$ if there exist such [adjacent] vertices $u, v \in V(G)$ that $S = S' \setminus \{u\} \cup \{v\}$ and $u \neq v$.

The results presented in this talk refer to some questions of Fricke et al. [1] about gamma graphs of trees. We will show that $\Delta(T(\gamma)) = \mathcal{O}(n)$ for any tree. We will also present a special class of graphs, for which all gamma graphs are isomorphic to n-dimensional cubes.

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Trends in the History of Infinity

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There are several competing trends in the history of the mathematics of infinity. These include Cantor's theory of infinite sets and (the much earlier) Euler's arithmetic of infinite numbers.

Cantor also developed an arithmetic of infinities. However, it hardly mimics the arithmetic of real or rational numbers. On the other hand, Euler's infinite numbers from the very beginning belong to a structure known today as an ordered field.

We argue that John Conway's *On numbers and games* provides a uniform perspective that allows one to compare these two trends. Arguably, the perspective of ordered fields provides a more general and consistent account of infinity.

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A Deontic Logic for Normative Dilemmas

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Standard deontic logic does not tolerate normative conflicts. If we assume that one can ought to do A and ought to do B, but cannot do them both, we get back contradiction within deontic logic. Philosophers who deny that there could be genuine moral dilemmas treat this fact as a proof that dilemmas are logically impossible. By the same time, the advocates of the possibility of moral dilemmas propose to reject or restrict standard deontic principles. What consequences does it have for the resulting logic? Some of them are too strong, because they contain the theorem of normative triviality or "deontic explosion", which says that if there is any case of normative conflict, then everything is obligatory. On the other hand, some of them are too weak, since they are not able to validate more important deontic inferences (especially Smith Argument).

Lou Goble introduces three criteria of adequacy that any deontic logic should meet if it is to accommodate normative conflicts successfully. First, I am about to present these conditions and then I will introduce a new logic of ought that fully meets all of them.

Goodbye, Dedekind

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Key words: antichains of subsets, the covering relation, boolean lattices.

Two enumerations of freely generated distributive lattices are presented. Both are based on a partition of the very lattices on boolean blocks. Computer visualization is presented up to eight generators. Some new results are obtained.

Logic and Method in Gassendi and in School of Port-Royal

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Seventeenth century logic strive to renounce Scholastics along with Aristotle's influence. With logic's aid philosophy was to ultimately settle all metaphysical disputes and empower Man to rule the Nature. The task was to re-analyze and re-formulate the Scholastic logic so it will fit the aims of the century. That was the reason for the need of a new manual of logic which would provide a presentation of a new method. I would like to point to two such manuals: first written by Pierre Gassendi and second co-authored by Nicole and Arnauld.

Authors of *Port-Royal Logic* believe that a proper presentation of a method is pivotal and they criticize other fields of logic that were traditionally included. In fact, one could find chapters devoted to syllogistics, sentential calculus and even modal logic but they are accompanied by remarks depreciating formal logic. The whole fourth part of their work concerns the method, where they use methodological principles of Descartes and Pascal's rules of proof. The main principle for them is states that what is perceived in a clear and distinct way has to be true. As for the remaining methods, Arnauld and Nicole don't consider them as valid and merely touch upon them. Their scientific method is most strongly inspired by Pascal's rule proof than the Cartesian method. Examples they provide come almost exclusively from mathematics. The method they described can be labelled as the aprioric-deductive. It would seem that logic manuals at that time should look alike.

However, Gassendi's book is different. An easily discernible difference could be seen in author's extensive commentary on ancient philosophy, especially on Epicure, and his ongoing references to seventeenth century naturalists. Gassendi is overwhelmed by Galileo's work and Copernican astronomy. In his method he manipulates the examples so that they don't look plain but present a real value for science. That is the same way that Galileo has adopted. Synthesis and analysis are the two main tools, which, in different ordering, can be used for discovery, assessment or presentation. Gassendi introduces in *Logic* a methodology which can serve a purpose in scientific endeavor. Empirical data which are perceived by senses can be properly, that is logically, analyzed. Gassendi's method could be labelled as the hypothetical-deductive but in fact is strongly empirical. Critical comparison of those seventeenth century manuals may shed more light on logic in that era. It is strictly connected to the purpose, which it serves and, though it is effectively still an Aristotelian organon, it is well suited for different uses.

Wroński's Sum of Modular Lattices

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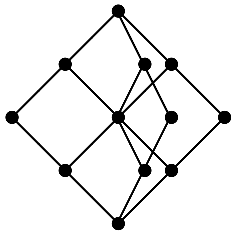
A. Wroński in [3] introduced the operation $\oplus$ of gluing of lattices. The finite lattice L is said to be a *sum* of A and B, in symbols $L = A \oplus B$, if A and B are proper sublattices of L, such that $A \cup B = L$, and moreover, $A \cap B$ is a filter of A, and $A \cap B$ is an ideal of B. If $\mathbb{A}$ is a given class of finite lattices, then $[\![\mathbb{A}]\!]$ stands for the closure of $\mathbb{A}$ with respect to the sum operation.

Let $\mathbb{B}$, $\mathbb{C}$, and $\mathbb{D}$ be the classes of all finite Boolean lattices, all finite complemented lattices, and all finite distributive lattices, respectively. J. Kotas and P. Wojtylak in [2] proved that

Theorem. $[\![\mathbb{B}]\!] = \mathbb{D}$.

Since $\mathbb{B} = \mathbb{D} \cap \mathbb{C}$, the Theorem says $[\![\mathbb{D} \cap \mathbb{C}]\!] = \mathbb{D}$. We know from [1] that complemented lattices are $\oplus$-irreducible, thus the class $\mathbb{D} \cap \mathbb{C}$ is the smallest basis for $\mathbb{D}$.

Let $\mathbb{M}$ be the class of all finite modular lattices. Unfortunately, the analogous equality $[\![\mathbb{M} \cap \mathbb{C}]\!] = \mathbb{M}$ is not true (see the figure below).



We consider the class $\mathbb{M}^*$ of these finite modular lattices which does not contain large diamonds. In the talk, we are asking about the basis of $\mathbb{M}^*$.

Conjecture. $[\![\mathbb{M}^* \cap \mathbb{C}]\!] = \mathbb{M}^*$.

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Definite Descriptions and Proof Theory

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This talk is concerned with two topics which usually do not come together. Definite descriptions are ubiquitous in natural languages and provide a very popular topic for the philosophy of language and logic. Since the publication of B. Russell famous paper "On Denoting", many researchers provided deep and detailed studies of this phenomenon. One can mention here for example: Frege, Hilbert, Bernays, Carnap, Quine, Rosser and Hintikka - just a few eminent scholars from the earliest stage of investigation. In the second half of XX century a lot of new proposals were added which were developed on the basis of several nonclassical logics. Yet, despite the long history and variety of proposed solutions we can hardly say that some approaches may be treated as obvious or commonly acceptable. In fact, proper definite descriptions having a unique designatum, are rather not problematic, in contrast to those which fail to designate, called improper (or unfulfilled) definite descriptions. The famous Russellian "the present King of France", is of this kind but even innocent-looking "the son of Jack" may be problematic in case Jack has no son, or more than one.

In the first part we survey the most important and interesting theories of definite descriptions with focus on their advantages and disadvantages. In the context of classical logic we will focus on the well known reductionist approach of Russell and the chosen object theory of Frege and its formalization provided by Kalish and Montague. The former shows how to get rid with definite descriptions (and individual names in general) and is one of the most popular solution, however at the costs of many drawbacks of different kind. The latter is one of the four approaches sketched by Frege which treat descriptions as genuine names. It is formally convenient but has its own disadvantages. Next, we describe some of the theories developed in the framework of free logic by Lambert, Scott, van Fraasen and others. In general, free logic is much better tool for developing a satisfactory theory of definite descriptions but some of them are too weak. We finish this part of the presentation with three theories developed on the ground of modal logic by Thomason and Garson, Goldblatt, Fitting and Mendelsohn. It seems that relational semantics with varying domains and nonrigid terms offers even better framework for definite descriptions, yet the presented approaches are significantly different in many respects.

The second part will be devoted to presentation of proof theory for definite descriptions. In fact, a modern proof-theoretic apparatus was not applied in this field so far. We hope to show that the application of techniques taken from structural proof theory may shed a new light on the good and bad sides of different approaches to definite descriptions. Sequent calculi for two different theories of definite descriptions will be examined. The first is equivalent to Kalish and Montague version of Fregean theory developed in the setting of classical logic. The second, equivalent to Thomason and Garson's theory, is for modal system with rigid and nonrigid terms based on free logic. We focus on proof theoretic features and problems with their application to description-operator as additional constant. For both theories we prove cut elimination theorem, discuss some of its properties, and — in the latter — some extensions by extra rules. We also discuss problems which makes some other theories of definite descriptions more complicated to deal with in proof theory.

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Compactness in Metatheory of Tableau Systems

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In the presentation we would like to concentrate on a problem of compactness of tableau systems. It is an important part of the tableau metatheorem which says that: (a) semantic consequence, (b) tableau consequence, and (c) an existence of a closed tableau, are equivalent, under some conditions.

The problem od compactness property of tableau system we examine in a very general approach which is an attempt of realization of some program of formalization of a notion of tableau proof that is inspired by Melivin Fitting who said that standard tableau notions are instances of certain abstract notions [D'Agostino M., Gabbay D., Haehnle R., Posegga J., 1999, p. 5].

In fact no abstract and general notions were delivered ever. So, we try to change it, with delivering some abstract notions. We show that for compactness some properties of tableau rules are sufficient.

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Solution to the Fitch Paradox Based on De Re Knowability Modality

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The Fitch Paradox, also known as the 'knowability paradox' has been introduced in [2]. It states that the assumption that each truth is knowable, leads to the conclusion that each truth is in fact known.

Since the Fitch's reasoning has a deep philosophical meaning (it is a serious argument against antirealism), some solutions have been proposed. Most of those solutions are based on changing the logical basis (paraconsistent/paracomplete logic instead of classical logic [1], [4]) or restricting the knowability principle to a certain class of formulas [3], [5], [6] e.g. satisfiable formulas.

Our approach is different. We postulate the change in logical expression of the knowability modality which has been traditionally expressed as the iteration of alethic and epistemic modality: $\Diamond K$. Te string of both modalities we read: *it is possible that it is known that …* which is a typical *de dicto* use of the diamond. Instead we introduce the single modality read as: *it is knowable that …* which in fact is intended to correspond to *de re* understood modality: *it is possibly known that …*

We provide semantics for the whole logical investigations with the *de re* epistemic modality and finally show that in the logics determined by the newly defined classes of models the paradox doesn't hold.

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Boolean Connexive Logics

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In this paper we define a new type of connexive logics which we call Boolean connexive logics. The Boolean operations: negation, conjunction and disjunction behave in a classical, Boolean way in such logics. We determine these logics through application of the relating semantics. In the final section of the paper, we present a tableau approach to the discussed logics.

The connexive logic is based on the theses set forth by Aristotle and Boethius, which only use negation and implication connectives. What is more, these theses are contradictive to the classical logic. Therefore, in the connexive logic we must interpret at least one of these connectives in a non-classical manner.

(A1) $\sim (A \Rightarrow \sim A)$

(A2) $\sim(\sim A \Rightarrow A)$

(B1) $(A \Rightarrow B) \Rightarrow \sim (A \Rightarrow \sim B)$

(B2) $(A \Rightarrow \sim B) \Rightarrow \sim (A \Rightarrow B)$.

In this study we shall only consider such connexive logics where the negation, conjunction and alternative have equal meanings with those in the classical logic. Thus each logic of this type we shall refer to as *Boolean connexive logic* since they preserve the meanings of the basic Boolean connectives.

The study offers a new approach to the issue of connexivity. Rather than using for instance the semantics of possible worlds or ternary accessibility relation — as the starting basis for the definition of the connexive logic — we shall assume a certain type of intensional logic: relating logic. By combing the semantic structures for relating logics with a Boolean language we obtain several different logics. The strongest ones among them include Aristotle's and Boethius' connexive laws as their tautologies. Hence, they are connexive logics.

Further in the study we present the following issues. First, we bring back some basis issues involved in the connexive logics. Further, we present the semantics of the relating logic which we shall uses as grounds for specification of our Boolean connexive logics systems and related issues. By dint of the findings concerning relations between the Aristotle's and Boethius' theses and the conditions imposed on the relating relation, we can present a lattice of logics comprising the least Boolean connexive logic along with a natural extension. Lastly, as a decision-making procedure, we propose the tableau methods that we shall elaborate in the last section of the study.

On Lattices of Situations and Wittgenstein's Topology

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In this talk I will define the so-called Wittgenstein's topology and show that it is a counterpart of a given possible world in lattice of situations and the union of all Wittgenstein's topologies is a counterpart of a given lattice of situations. The last, lattice of situations was introduced by Wolniewicz to interpret Wittgenstein's ontology from *Tractatus Logico-Philosophicus*.

The Weak Amalgamation Property

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We will discuss the concept of weak amalgamation, presenting some recent examples and describing the role of this property in the theory of so-called generic objects.

Recall that the amalgamation property says that any two embeddings of a single structure can be combined together. A natural weakening is requiring that every structure can be enlarged to a bigger one, so that any two embeddings of this bigger structure can be combined together. It turns out that this property still has a weakening which at first glance looks somewhat technical, nevertheless it turns out to be crucial for characterizing certain 'generic' objects in terms of a natural infinite game.

The results come from two joint works with A. Krawczyk, A. Kruckman, and A. Panagiotopoulos.

An Outline of Logical Analysis of the Concept of Value

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The subject of the reading are selected aspects of ontological structures of perfect values in reference to the analyzes and comments of Henryk Elzenberg. The definitions of various types of values and the ontological premises of such definitions are discussed. The subject of attention was also logical relations between different concepts of perfect value and axiological notions of sense. An important feature of the Ezenberg analysis of the perfect values is their strict connection with the states of affairs which should be the case. According to the author, at the level of logical analysis of this issue, however, it is necessary to move away from the subject-predicate structure of the state of affairs and to treat the content of the phrase '*should be the case*' as a modality that refers directly to the situation. However, this requires a more detailed characterization of the relationship that can occur between situations (or states of affairs), ie. something that can should be the case, and objects that can be subjects of these situations and, therefore, can be valuable. The author introduces the multimodal logical calculus with propositional quantifiers and applies it to analysis of some aspects of the subject.

The Connective "czy" in Polish Viewed as a Conjunction

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When looking for the best natural language equivalents of various functors in logic I focused my attention on the Polish word "czy". It is widely viewed as synonymous with the expressions "lub", "albo" (equivalent of the inclusive and exclusive "or" in English).

In my paper I will focus on the relatively rare use of the word "czy" as a connective of conjunction.

On Modalities in the Context of Discussive Logics

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Keywords: discussive-modal operators; discussive-modal logics; a modal logic over discussive logic, axiomatisation of discussive modal logic.

In [3] there are given systems being certain extensions of the S5, M-S5 (a system considered in [6, 8] called M-counterpart of S5) and D_2 (see [4, 5]). In the first two cases, the language of the propositional part contains connectives of disjunction, negation and necessity. The discussive implication is definable there. In the case of discussive logic there are considered discussive implication, left discussive conjunction, disjunction, and negation. In this case modal connectives are definable.

Having in mind that in the context of discussive logics it is crucial for the resultative systems which definitions are used, taking also into account partially connected to this issue problems with axiomatising of D_2 (see [1, 2, 6, 7]) and remembering that D_2 is not extensional, it seems to be interesting to consider a task of giving syntactic characterisation of a modal extension of D_2 in the language with the right discussive conjunction and to discuss the issue of enriching discussive language with modal connectives, in general.

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Conservativity of Classical Set Theory over Its Constructive Counterpart

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We consider the classical set theory ZF and its constructive counterpart CZF. The theory CZF differs from ZF not only in underlying logic (which in case of

CZF is intuitionistic first-order logic) but also in that the Axiom of Foundation is in CZF replaced by the classically equivalent Axiom of Set Induction.

In the talk we investigate the mutual relations between ZF and CZF. As the main result, we prove that for the class of Π_2-formulae and negations of positive formulae, ZF is conservative over some stronger version of CZF. Since CZF is not closed under the negative translation, our result cannot be proven by the well-known syntactic methods.

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Relation of Attack

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The relation of attack holds between arguments and (counter)arguments. The aim of this paper is to propose a definition that gives the necessary and sufficient conditions of a successful counterargument, relatively to a predetermined formal model of representation and evaluation of structured arguments (Selinger 2014, 2015). Such a definition can offer a link between the known from informal logic, classical approach to structured arguments and the abstract argumentation theory.

Since the predetermined model of evaluation allows infinitely many degrees of acceptability, an attack, whether successful or not, can be more or less efficient. Furthermore, an attack can be unsuccessful, even though the counterargument is acceptable, but it is too week to prevail the attacked argument. Since the evaluation model allows to represent convergent reasoning, an attack can also be partly successful, namely, when it is not aimed at each of the converging arguments.

Three are three traditionally distinguished kinds of attack: on premises, conclusion and the relationship between them. Respectively, three kinds of attackers are: undermining, rebutting and undercutting defeaters (Prakken 2010). The nature of each of them will be discussed, but a special attention will be paid to the undercutting defeaters and their relation to the so-called hybrid arguments (Vorobej 1995).

Finally, the counter-attacks and their efficiency will be considered. It will be argued that the counter-rebuttals cannot enhance the initial strength of attacked arguments, while the counters to undercutters can.

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Deciding (Active) Structural Completeness

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A rule Γ/φ is *admissible* in a logic L (or in a consequence relation $\vdash$) if for every substitution σ, whenever $\sigma(\Gamma)$ contains only theorems of L (or of $\vdash$), then $\sigma(\varphi)$ is a theorem of L (of $\vdash$). A logic L (a consequence relation $\vdash$) is *structurally complete* if every admissible in L (in $\vdash$) rule is derivable in L (in $\vdash$).

Structural completeness was introduced by Pogorzelski in [5]. Since then, this property was investigated in many contexts. Also many generalizations and variants were proposed. Among them *active structural completeness*, introduced by Dzik in [2] (under the name of *almost structural completeness*), seems to be the closest to the original one.

We study the problem of decidability of (active) structural completeness for tabular (given by a finite matrix) logics. Decidability of structural completeness for consequence relations was obtained in [1], and once more in [4] in algebraic setting. This result yields that we have an algorithm to decide whether an algebraizable logic with semantics given by a congruence distributive variety is (actively) structurally complete. We extend this to the congruence modular case.

We accomplish it using Freese's and McKenzie's result that there is an algorithm deciding whether a finite algebra generates the congruence modular variety [3] and the folowing new fact: if the variety $\mathcal{V}$ generated by a finite algebra $\mathbf{A}$ is actively structurally complete, then the number $|A|^{(|A|+1)\cdot|A|^{2\cdot|A|}}$ bounds the size of subdirectly irreducible algebras in $\mathcal{V}$.

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Metaphilosophical controversy about logic. Selected interpretations

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The purpose of my research is try to assess the possibility of objectifying the results developed in the — broadly and evaluative understood — analytical philosophy. Understood broadly and ahistorically: as the hard core, the whole philosophical tradition. Of course, in this perspective I am not original, and I follow the metaphysical idea by Professor Jerzy Perzanowski. This approach is based on a few metaphilosophical interpretation-decisions, and logic plays in them a fundamental role. The mentioned decisions include:

- Philosophy is a sensible cognitive activity (essentially: "meaning-creative") — the statement articulated in opposition to those who, in the way of some interpretations of the metaphilosophy of the Vienna Circle, refuse to make such a philosophy.
- Analytic philosophy narrowly understood — in my conviction — are condemned to a theoretical insult, which has been digesting it for decades.
- In philosophy it is possible to develop objective cognitive results, which (perhaps) must be relativized to accepted conceptual apparatus (in the sense of radical conventionalism by K. Ajdukiewicz) and (perhaps) an irremovable hermeneutic component — present in each of the articulated philosophical positions (reference to the approach developed in recent decades by prof. J. Woleński);
- Not all the theoretical positions articulated in the context of traditionally understood philosophical issues can be considered in the discussed perspective as strictly philosophical; In other words — strongly speaking freely — not everything that *passes as philosophy* are the philosophy;

- Moderate axiological cognitivism: the conviction that it is possible to reasonably discuss about values (references to the formal approach the value developed by T. Czeżowski almost a century ago, as well and as the reflections of Prof. M. Przełęcki) in opposition to widespread non-cognitivism etc.

Trying to articulate a relatively understandable vision of my speech: basic goal is to sketch a vision of the philosophy of an on-going analytical experience that lasts over a century. Therefore, it will be not systematically presentation but rather an interpretation of metaphilosophical perspective determined by the results of the authors such as: Tadeusz Czeżowski, Kazimierz Ajdukiewicz, Roman Suszko, Jerzy Perzanowski and Marian Przełęcki. This vision, in my conviction, is one of the more attractive and promising perspectives that currently appear in front of philosophy.

A vision that creates hope for overcoming the tendency towards contributing to philosophical research that has been observable in recent decades, a tendency that results in the progressive marginalization of these investigations in broadly understood contemporary culture.

Elementary Ontology with Frege's Predication Scheme: Distributive Classes and Collective Classes

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Keeping in mind certain Bolesław Sobociński's analyses related to the terms *element* and *class* as well as the distinction between distributive and collective classes, we can also distinguish between the two senses of the functor *element* — the distributive and collective one.

We start with the following schemes:

(α) $x\varepsilon element(y) \leftrightarrow \Sigma a(y\varepsilon class(a) \wedge x\varepsilon a)$

(β) $x\varepsilon class(a) \leftrightarrow x\varepsilon x \wedge \Pi z(z\varepsilon a \leftrightarrow z\varepsilon element(x))$

The distributive interpretation of these two schemes gives:

($\alpha 1$) $x\varepsilon sub(y) \leftrightarrow \Sigma a(y\varepsilon Ca \wedge x\varepsilon a)$

($\beta 1$) $x\varepsilon Ca \leftrightarrow x\varepsilon x \wedge \Pi z(z\varepsilon a \leftrightarrow z\varepsilon sub(x))$

For distributive classes (C) the functor *element* appears in the distributive sense (*sub*): in this sense *being an element is being subordinated*.

By contrast, in the collective interpretation of the schemes (α) and ($\beta 1$) the functor *element* is interpreted in the collective sense (*el*) and the functor *class* in the collective sense (K), which gives:

(α2) $x \varepsilon el(y) \leftrightarrow \Sigma a(y \varepsilon Ka \wedge x \varepsilon a)$

(β2) $x \varepsilon Ka \leftrightarrow x \varepsilon x \wedge \Pi z(z \varepsilon a \leftrightarrow z \varepsilon el(x))$

We shall treat the formulas (β1) and (β2) as definitions of classes in the distributive and collective sense respectively.

The logical connections between distributive and collective classes in the framework of elementary ontology with Frege's predication scheme are examined.

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Intuitionistic Modal Logic in Neighborhood and Bi-Relational Setting

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We present neighborhood semantics for (propositional) intuitionistic modal logic. The minimal neighborhood of a given world simulates intuitionistic aspect of our system, while the maximal one refers to its modal part. Note that we do not assume *superset axiom*. We introduce Δ — a kind of necessity operator. We say that $\Delta\varphi$ is satisfied in w *iff* φ is true in each element of maximal w-neighborhood.

Another new functor is $\rightsquigarrow$. It behaves like classical implication but in the whole maximal neighborhood of a given world. In fact, we show that it is actually useful in the theorem about n-bisimulation between our models. We propose an axiomatization of logic in question and then we prove its semantical completeness. Later, we show how to transform our neighborhood structures into bi-relational frames which coincide with certain models explored by Božić and Došen in [1]. We investigate various conditions imposed on our structures and we show that it is possible to treat maximal neighborhoods as topological spaces. Then we discuss three operators of possibility (the most intuitive is ∇).

Finally, we present classical modal systems which are also based on the difference between properties of maximal and minimal neighborhoods. We establish translation between one of them and our intuitionistic modal logic.

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Unification in Predicate Modal Logics

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This is a continuation of [1], the first paper on unification in predicate logic. Unification is a useful tool in research on propositional logic, see Ghilardi [2]. Our aim is to shift the notion from propositional to first order level. Our concerns are predicate modal logics extending $\mathsf{Q{-}K4}$. We characterize unifiable formulas in some extensions of $\mathsf{Q{-}K4}$ and give a rule basis for all passive rules there. Unifiability depends on the number of logical constants of the logic considered. We focus on extensions of $\mathsf{Q{-}K4}$ with at most four constants: $\top, \bot, \Box\bot, \Diamond\top$, see Kost [3].

Projective formulas, defined in a similar way as in propositional logic, are used to solve some questions concerning disjunction and existence property. We give a partial characterization of predicate modal logics with projective unification. Then we characterize logics with filtering unification and settle the unification type of certain logics including $\mathsf{Q{-}K4.2}$ and $\mathsf{Q{-}K4.3}$. To this aim we adopt the concept of weak existence property introduced for superintuitionstic logics by Minari [4].

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Mosaic-Based Decidability of a Logic of Order-Of-Magnitude

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Knowledge representation is one of the most vital tasks faced by the broadly understood Artificial Intelligence (AI). *Qualitative Reasoning* (QR) emerged as a field of research within AI in the early 1980s with the main objective of devising appropriate tools to formalize these aspects of human knowledge and reasoning which are of qualitative nature. In particular, formal systems proposed within QR are expected to model scenarios in which agents, having incomplete or insufficiently precise information (provided only in the approximated form or using qualitative categories), are able to draw conclusions with an acceptable degree of plausibility.

Order-of-magnitude Reasoning (OMR) is a subdiscipline of QR. It enables us to express absolute numerical values in qualitative terms. Moreover, within the OMR-paradigm it is also possible to relativize magnitudes by introducing binary relations over the set of absolute values comparing those values in various respects. Within OMR we distinguish two main approaches: Absolute Order-of-Magnitude (AOM), firstly introduced in [5], and Relative Order-of-Magnitude (ROM). The first approach consists in partitioning the real line into several qualitative categories (such as *small negative numbers*, *large positive numbers* etc.) by the so-called *landmarks*. The second one involves binary relations over a given linear order (in most cases — the real line) which name relative magnitudes of particular elements of the order, such as *neglibility* [3], *bidirectional negligibility* [3], *non-closeness* [2] or *comparability* [3].

In my talk, I take a closer look at the logic for order-of-magnitude reasoning with distance ($\mathsf{OMR_D}$, in short), first introduced by Burrieza *et al.* in [2]. We might see the distance relation as a relation that allows an agent to differentiate between two elements as belonging to two different qualitative classes. I will focus on the decidability result for $\mathsf{OMR_D}$ which was established by bo proving the following

Theorem [Finite mosaic property]

An $\mathsf{OMR_D}$-formula is satisfiable on an $\mathsf{OMR_D}$-model iff there exists an $SSB(\varphi)$ of the size at most 2^{2n+1}, where $n = \mathsf{card}(\mathsf{Sub}(\varphi) \cup \mathsf{Sub}(\Diamond\mathbb{C}))$.

I will outline the main steps of Theorem's proof.

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The Bryll Session

Recent Scholarly Activity of Professor Grzegorz Bryll

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Throughout his entire life, Professor Grzegorz Bryll was extremely prolific in terms of research and publishing. He worked in every spare moment, every possible place, till the very end of his life. He was true enthusiast, passionate and devoted educator and populariser of mathematics and mathematical logic.

The recent study of Professor Bryll, among others, was the history of mathematics and logic, including outstanding achievements of ancient Greeks in this field. His passion and commitment has inspired the whole family to examine resources and continue his research.

Historical studies discussing the various scientific accomplishments of the ancient Greeks are very vague, which often makes it difficult to understand utterly the essence of the issues. Undertaking a detailed study of the Greek achievements, it was not expected to discover such a large quantity of rich and diverse mathematical and logical problems that were in the scope of Greek interests.

Formal Research on the Logic of the Stoics

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My speech is dedicated to the memory of prof. Grzegorz Bryll. I will present our joint research on the logic of the Stoics made in the years 1990–1998.

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Frege's and Leśniewski's Concepts of Being an element *and their Connection with Russell's Paradox*

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Frege treated distributive sets as the extensions of concepts. This is why the concept of *being an element* took (for him) the following form: the expression 'object x is an element of y' is to mean: 'for some concept P: y is the extension of P and x falls under P'. And where the concepts are general names, we write: 'for some general name S: y is the extension of the name S and x is an S'. Leśniewski rejected the existence of distributive sets as well as the existence of the extensions of concepts. He considered that only collective classes (sets) exist. Subject to that «correction», the expression 'object x is an element of y' meant for him: 'for some general name S: y is a (collective) set of Ss and x is an S'. It may seem a minor difference but it brought about fundamental changes in his theory.

The concept *being a collective set of Ss* is definable in Leśniewski's theory with the help of the relation concept *is a part of* (hence the name of his theory: "mereology", or the study of parts). From that definition it follows that if a name S is empty, then there is no collective set of Ss.

On the basis of his two definitions, Leśniewski showed that that x is an element of y reduces to either x is a part of y or $x = y$. And since identity is reflexive ($x = x$), every object is its own element and every object is a set (a collective set, obviously). And there both the names 'object that is not its own element' and 'set that is not its own element' are empty. Therefore, neither of them determines a set. Thus, Leśniewski didn't in general have any problem with Russell's paradox.

In this paper, the details of the above sketch will be filled in. Furthermore, it will be shown how Russell obtained the paradox from Frege's definition of being an element and what that has to do with the current perspective on Russell's paradox (whose source — as Quine showed — is first-order logic and not set theory).

Professor's Bryll Scientific Activity

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Grzegorz Bryll died on March 28th, 2017. He was a Titular Professor and Vice-Rector at University of Opole and The Higher Pedagogical College in Opole. He was born on January 2nd, 1935 in Gostyń, small town in Greater Poland Voivodeship. He spent his childhood there, but in his teenage years he was already pursuing his dream of teaching, as a student at pedagogical highschool in nearby town, Leszno. After graduating highschool, in 1953, he enrolled into State Higher Pedagogical College in Wrocław. He continued his education even after the college was moved from Wrocław to Opole — that is how he found himself in our city and became one of the first masters of mathematics at The Higher Pedagogical College in Opole.

During early years of his scientific career he focused his research mostly on theory of matrix games and probabilistic pursuit — evasion games theory. After defending his PhD in 1968 he became a lecturer at Technical Uniwersity in Opole. He was the Dean of Faculty of Electrical Engineering between 1971 and 1975, and later, between 1975 and 1981 he was performing Vice-Rector function.

After finishing the habilitation process and receiving his post-doctoral degree in 1982, Grzegorz Bryll returned to his Alma Mater, The Higher Pedagogical College in Opole. Between 1985 and 1989 he was working in Algieria, where he was the Head of Mathematics Department at University of Blida. Later, after returning to Poland, between 1991 and 1996 he served as Vice-Rector first at The Higher Pedagogical College in Opole and later at University of Opole. In 1991 he became the Head of Department of Mathematical Logic at Institute of Mathematics and Informatics. He became a Titular Professor in 1997 and he continued working as the Head of Department of Mathematical Logic at University of Opole until his retirement in 2006.

During his scientific career professor Grzegorz Bryll promoted 4 doctoral dissertations. He is the author of 132 publications, including 10 books and numerous scripts. He was particularly interested in multi-valued logics, logical pragmatic and history of logic, especially in stoic logic. He is one of the founders of the theory of rejecting expressions.

In my talk, first I will present our joint research on the theory of rejected propositions and secondly I describe our joint research on another fields of scientific activity.

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