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Editorial note

(EN) means that the talk is presented in English, (PL)—in Polish.

On a Problem with the Definition of Counterfactuals

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Counterfactuals are conditional sentences that differ from other conditionals: both those built by material implications and those built with strict implications. It can be said that the material implication is too weak and strict too strong for counterfactuals. The place for the counterfactual conditional is somewhere in between. The problem is where exactly it should be placed. Attempts to resolve this problem result in different solutions; it is assumed that the validity of the counterfactuals depends not on the relation between the antecedent and the consequent of the conditional, but rather on certain conjunctions of the antecedent with, for example: the laws of nature, the description of the world, the state of knowledge. Depending on the solutions adopted, we face new challenges, both philosophical and logical. It should be also mentioned that some logicians (eg. D. Edgington) claim that formulating truth conditions for counterfactuals is a worthless work. So the very first problem is how to define the counterfactuals. Comparison of different definitions (Lewis, Stalnaker, Goodman, Bennett, Kvart, Adams, Gibbard and others) will not be enough to propose a new unproblematic definition but rather allow to indicate the impact of philosophical consideration on formal solutions.

First-Order Refutability Properties[1]

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In his book [1] Fitting introduced the notion of a *consistency property* for first-order logic, which is closely related to the notion of a Hintikka set [2]. Consistency property is a family of sets of first-order formulas meeting certain conditions. The central observation called *model existence theorem* states that every element of a consistency property has a Herbrand model. From a proof-theoretical point of view, the notion of a consistency property enables an uniform completeness proof for various proof systems such as tableaux method, sequent calculus, and resolution, but is not so natural when applied to dual systems, such as dual resolution. In my talk I will introduce notions of a *dual Hintikka set* and *refutability property* together with *counter model existence theorem*, which states that every member of a refutability property can be falsified in a Herbrand model. I will use this concept to prove completeness of a dual erotetic calculi (based on Inferential Erotetic Logic [3] and reolution) for first-order logic.

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Logic and Implication

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Algebraic logic has developed a rich theory of non-classical propositional logics. Virtually all prominent logical systems studied in the literature have a reasonable notion of implication (that is, satisfying the following minimal requirements: reflexivity, transitivity, modus ponens, and symmetrized congruence). This has motivated the introduction of weakly p-implicational logics [5] as an alternative presentation of protoalgebraic logics [11] that highlights and exploits the role of implications.

In the first part of the talk we remain on the propositional level, starting with motivations and basic definitions. After presenting the core theory, we focus on areas which are not much accented in the 'traditional' Abstract Algebraic Logic [11, 12]: infinitary logics [7, 17]; the use of generalized disjunction connective [7, 9], and the completeness w.r.t. special classes of matrices [10], most prominently those linearly ordered by the implication [5, 6]. We illustrate all our notions and results with a wealth of examples coming mainly from substructural and fuzzy logics [3, 6].

In the second part, we focus on predicate logics. Classical predicate logic interprets n-ary predicates as mappings from the n-th power of a given domain into the two-valued Boolean algebra **2**. The idea of replacing **2** by a more general structure is very natural and was shown to lead to very interesting results: prime examples are the Boolean-valued or Heyting-valued models of set theory (or even other models proposed e.g. by Takeuti–Titani [21, 22], Titani [23], Hájek–Haniková [14]). There exists a stream of research, by Mostowski [18], Rasiowa–Sikorski [20], Rasiowa [19], Horn [15, 16], and Hájek [13] to give just a few names, studying logics where predicates can take values in lattices (with additional operators). We present a general framework [8] for studying predicate logics where the mentioned class of lattices is the equivalent algebraic semantics of a propositional finitary finitely implicational logic (i.e., algebraizable logic in the sense of Blok and Pigozzi [1]). We conclude by presenting some recent results on Skolem and Herbrand theorems for some of these logics [2, 4].

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Performability of Actions

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Action theory may be regarded as a theoretical basis of AI, because it provides in a logically coherent way the principles of performing actions by agents irrespective of the fact they are humans or robots. But, more importantly, action theory provides a formal ontology mainly based on set-theoretic constructs. This ontology isolates various types of actions as structured entities: atomic, sequential, compound, ordered, situational actions etc. This ontology is a solid and non-removable foundation of any rational activity.

In the talk the theory of performability of actions based on relational models and formal constructs borrowed from formal lingusistics is presented in the form of a coherent and strict logical system $\models$. This system is semantically defined in terms of its intended models in which the role of actions of various types (atomic, sequential and compound ones) is accentuated. Since the consequence relation $\models$ is not finitary, other semantic variants of $\models$ are defined. The focus in on the system $\models_f$ of performability of finite compound actions. An adequate axiom system for $\models_f$ is defined. The strong completeness theorem is the central result. The role of the canonical model in the proof of the completeness theorem is highlighted.

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Key words: binary relation, frame, model, atomic action, sequential action, compound action, performability of actions.

AMS Subject Classification: 03B50, 03B60, 03B80.

Unification in Predicate (Modal) Logic. Part II

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Unification in logic was introduced by S. Ghilardi for propositional (intuitionistic and modal) logics, and applied to e.g. admissible rules.

A *unifier for* a formula A in a logic L is a substitution σ such that $\vdash_L \sigma(A)$. Given two unifiers σ and τ, σ is *more general than* τ in L iff $\vdash_L \theta\sigma(x) \leftrightarrow \tau(x)$, for some θ. There are four unification types: $1, \omega, \infty, 0$ depending on the number of maximal unifiers. Unification in L is *filtering* if for every two unifiers there exists a unifier that is more general than both of them (in L).

The above definitions from propositionla logic are now reformulated for predicate (modal) logics. We use a notion of substitutions for predicate formulas (endomorphisms modulo bound variables) by W.A. Pogorzelski and T. Prucnal but with an important modification: we allow introducing by substitutions new free individual variables. For a propositional modal logic L, the least predicate modal logic corresponding to L is denoted by $\mathsf{Q\text{-}L}$.
Let $\Box^+ A = A \wedge \Box A$, $\Diamond^+ A = A \vee \Diamond A$ and $2^+ \ : \Diamond^+\Box^+ A \to \Box^+\Diamond^+ A$.

THEOREM 1. For every modal predicate logic L extending $\mathsf{Q\text{-}K4}$:
L has filtering unification iff $\mathsf{Q\text{-}K4.2^+} \subseteq \mathsf{L}$.

COROLLARY. For every modal predicate logic $\mathsf{L} \supseteq \mathsf{Q\text{-}S4}$:
L has filtering unification iff $\mathsf{Q\text{-}S4.2} \subseteq \mathsf{L}$; moreover, if the unification type of L is 1 then $\mathsf{Q\text{-}S4.2} \subseteq \mathsf{L}$.

In contrast to the results in propositional modal logics we have:

THEOREM 2. The unification type of Q–S4.2 and of Q–S4.3 is 0.

THEOREM 3. The unification type of Q–S4 is either 0 or ∞.

Key words: unifiers, unification types, filtering unification, admissible rules, passive rules, modal predicate logic.

Consequence Operations in the Lattices $\mathcal{P}(I)^L$ and $\mathcal{P}_R(I)^L$

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Fuzzy attitude to closure operator is a mathematical attempt to render reasoning in which certainly (uncertaintly) is measured in linearly ordered set. In our speech we present a different approach, in which operations are defined on the set $\mathcal{P}(I)^L$ where I is some set of agents and L is some propositional language. A fuzzy set $X \in \mathcal{P}(I)^L$ can be treated as a function that associates with every formula α of the language L the set $X(\alpha)$ of agents which state that α is true. Therefore, we will consider operations $c : \mathcal{P}(I)^L \longrightarrow \mathcal{P}(I)^L$ fulfilling $x \leq c(x)$, $x \leq y \Rightarrow c(x) \leq c(y)$, $c(c(x)) \leq c(x)$, or more compactly, $x \leq c(y)$ iff $c(x) \leq c(y)$.

Moreover, we provide some generalisation of such the operations by considering the closures on the set $\mathcal{P}_R(I)^L = \{J \subseteq \mathcal{P}(I) : \forall_{i \in I}(\overleftarrow{R}(\{i\}) \subseteq J \Rightarrow i \in J)\}$ for some $R \subseteq I \times I$. Following the Fitting's idea (see [1, 2]) the value of the sentence φ is the set of agents who agree that φ is true. If agents are independent, then all the values $\mathcal{P}(I)$ are possible. Otherwise one agent, say k, may depend on the agent i, that is if i maintains that "φ", then k must agree with him/her. That is, if an agent i is obliged to maintain that α when all the agents he/she depends on maintain that α. Such a relation determines a set of allowed true values, that is the set of all $x \subseteq I$ fulfilling $i \in x$, whenever $\overleftarrow{R}(\{i\}) \subseteq x$ (where $\overleftarrow{R}(\{i\})$ stands for all $j \in I$ for which Rji). These sets are allowed in the sense that it is not possible for some sentence α to be accepted by all agents dominating i and not to be accepted by i.

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An Extension of Fixpoint Characterization for Grounded Definite and Grounded Acceptable General Logic Programs[2]

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The formalisation of logic programs is conveyed in the first order logic [1]. There are definite, general (sometimes called normal) and extended logic programs. In my presentation I will focus on definite and general logic programs which are sets of definite and general Horn clauses, respectively. The scheme of a Horn clause can be roughly described as follows:

$$head \leftarrow body$$

where the *head* is a single predicate and the *body* consists of predicates. In case of general logic programs predicates in the body of a clause can be preceded by negation interpreted as negation by finite failure. The declarative semantics for logic programs is given by the usual semantics of formulas in the first order logic [1]. However, in my presentation I would like to focus only on grounded logic programs, where the language is fixed and contains only constants. In that case interpretations and models can be defined as sets of predicates without variables (called atoms), which are mapped to *true* by a valuation.

Emden and Kowalski [2] proposed an interpretation of logic programs using fixpoint characterisation. They defined the immediate consequence operator which „provides the link between the declarative and procedural semantics of [logic programs]" [1, p. 37]. The immediate consequence operator takes as argument an interpretation and returns heads of all those Horn clauses form a logic program that all atoms from the bodies are mapped to *true* (i.e., are elements of that interpretation). Emden and Kowalski proved that fixpoints of immediate consequence operator are models for logic programs [1, 2].

My goal is to define a modification of a given definite or general logic program and to prove that by means of the immediate consequence operator used on modified logic program it can be established if a Horn clause is a logical consequence of the original logic program. In my presentation I will show potential applications of such solution.

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Investigations of Intuitionistic Control Logic and Its Fragments

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Intuitionistic Control Logic (ICL) introduced by Ch. Liang and D. Miller arises from Intuitionistic Propositional Logic (IPL) by extending the language of IPL by additional new constant for falsum. Having two different falsum constants enables to define two distinct negations: an ordinary intuitionistic negation and a new negation defined using additional falsum, which bears some characteristics of classical negation. It results in ICL being fully capable of typing programming language control constructs (such as *call/cc*) while maintaining intuitionistic implication as a genuine connective.

The new constant requires a simple but significant modification of intuitionistic logic both proof-theoretically and semantically. Intuitionistic Control Logic has natural deduction proof system which is sound and complete with respect to the Kripke semantics.

In our talk we would like to present properties of monadic fragments of ICL, starting with the analysis of the number of distinct operators that can be defined by sequences of both negations. We will also present a translation of ICL into second order propositional modal logic.

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Some Combinatorial Aspects of Lattice Tolerances

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A tolerance relation on a lattice L is a reflexive and symmetric relation compatible with the operations of L. It is clear that every congruence of a lattice L is a tolerance on L. The tolerances of a lattice L, ordered by inclusion, form an algebraic lattice denoted by $\mathrm{Tol}(L)$.

Every tolerance can be represented by the covering system of blocks. A block of a tolerance T of a lattice L is a maximal subset X of L such that every two element from X are in the relation T. Blocks of T are convex sublattices by [1] and [3] and they form a lattice called the factor lattice of L modulo T ([2]). In case of finite lattices blocks of a tolerance $T \in \mathrm{Tol}(L)$ are intervals of L.

We concentrate on finite lattices only and, in particular, on quantitative aspects of their tolerances, with a special emphasis on enumerating congruences as well as tolerances and their blocks in the case of finite chains.

We construct a recursive bijection between all tolerances on the finite chain $\mathcal{C}_n$ and all Dyck paths of length $2n$, which are enumerated by Catalan numbers. Then, using the theory of bivariate generating functions, we can enumerate tolerances with a prescribed number of blocks. Moreover, by means of symbolic methods in enumerative combinatorics, we are able to recursively enumerate tolerances of any degree.

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The Impossibility of the Logical Definition of the Concept of the "Existence" in Post-Avicenna Tradition: Fakhruddîn-Er-Râzî and His Approach to This Problem

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The nature of concepts in system of Ibn Sînâ (Avicenna, d. 1037) is classified epistemologically. In this context, according to Avicenna, the concept of existence is the clearest one and therefore can not be defined logically. For, the definition in logic is made to clear the ambiguity. For this reason, the logical definition of the concept of existence can not be made. Fakhruddîn Er-Râzî (d. 1209) follows Avicenna significantly in the period of subsequent *Kelâm* of Avicenna tradition. After the science of logic enters to Islamic *Kelâm*, The discipline of *Kelâm* is divided into two periods: The Preceding *Kelâm* and Subsequent *Kelâm*. In this way, *Kelâm* argumantation entered under the effect of Aristotelian logic. The logic, in the period of subsequent Kelam, is the measure of science expressing the truth. It occured around the circles of science in the disciplines of Philosophy and Kelâm that the knowledge that is not controlled and filtered by logic can not be accepted. Therefore, Er-Râzî discusses in his famous philosophical texts investigating, interpreting and analyizing Avicenna tradition whether the concept of existence is defined logically or not. Er-Râzî expresses that the concept of existence can not be defined in three ways: In the first way, Er-Râzî discusses whether the concept of existence can be defined by itself. Because in the point of logical view, the something that is defined by itself brings us to the totology. However, the totology is vicious circle and does not clear the ambuigty. In the second way, Er-Razi discusses whether the concept of existence is defined with its inner particulars or not. Consequently, Er-Râzî asserts that this way will bring us to nonesense. In third way, Er-Râzî discusses finaly whether the concept of existence is defined by the outer particulars or not. Er- Râzî says that this explanation about the nature of existence will bring us to the description. Thus the description can not give the real nature of the existence to us. By all these

justifications, ıt occurs that the existence is self-evident in the sense that it does not need to be defined logically in the system of Avicenna and Er-Râzî, the famous follower and interpreter of post Avicenna tradition.

Key words: Existence, Logic, Definition, Ibn Sînâ, Er-Râzî.

The Critique of Suhrawardî Al-Maktûl (d. 1191) to Aristotelian Logic: The Nature of Logical Definition

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There are a lot of critiques on Aristotalian logic and tradition in the heritage of Islamic knowledge and civilization. By entering of foreign sciences to Islamic World in its classical period in the middle ages, some scholars seriously reacted that foreign sceinces, mostly social ones, especially philosophical thoughts dont suit to the nature of Islamic argumantation. For this reason, Suhrawardi Al-Maktûl asserted that the method of Meşşâi (Peripatetics) like Fârâbî and Ibn Sînâ, won't give the exact knowledge of the nature of things to us by the logical argumantation. Especially Suhrawardî motivates that the abstraction of the ıntellectus, taking forms from matter to construct the concepts, does not fit to the nature of things and is not adequate to arrive at the truth. Instead of this method, Suhrawardî advises the Ishrâqî method which has mystical core, to take the exact knowledge. By this way, the wiser that Suhrawardî characterizes as the one who knows the truth, has the essential nature of the things. This essential nature can not be defined by logical terms and can not be taken by logical argumantation. According to Suhrawardî's point of view, Al-Kashf that is the mystical style of knowledge, gives us the essential truth about the internal and external World. In this system, the nature can not open itself to human mind by the logical way and the rational studies. Because the nature consists of the facts. The rationalist describes the external side of the nature; and every rational ones explain and define the nature according to their point's of view. For Suhrawardî, the objective reality/truth can be arrived by the mystical way. In this method, the essential nature of things open itself to human soul. But this opening is possible so long as the soul is clarified from bad dispositions and temperaments.

Key words: Logical Definition, Intellectus, Al-Kashf, Aristotle and Suhrawardî

Dual Erotetic Version of System KE

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Comparisons of different proof-systems and possible translations between them are topics from the mainstream of proof theory (see [7], [8]). Especially, the issues of complexity of proof search have gained a lot of attention during the last decades ([3], [9], [2]). Tableau system **KE** is an interesting and important example of proof system which has been introduced as an improvement—*int.al.* in terms of complexity—of another system, that of analytic tableaux, in this case. **KE** has been developed by D'Agostino and Mondadori in the early nineties ([3], [4], [5]), in order to solve certain "anomalies" which follow from the absence of cut. In our talk we present a version of this system described, first, in the erotetic format, and second, in a dual account.

By "erotetic" format we mean the method of Socratic proofs (see [10], [11]), developed as a formal method transforming questions concerning, *int.al.*, validity. Originally, the main motivation was a formal reconstruction of erotetic (*i.e.* using questions) reasoning. The erotetic rules transform questions into questions. The construction is equipped with two-level semantics: on the level of declaratives we have usual transmission of validity, and on the level of questions we have transmission of erotetic implication. But the proof-theoretic skeleton of the method is an object of interest on its own—the erotetic rules act on sequences of sequents, which are not, however, hypersequents as understood in [1], since the sequences "act" rather like conjunctions of sequents. However, without cutting off the original motivation, we focus on the proof-theoretic aspects of the method of Socratic proofs. The formal result is a sound and complete calculus with a form of cut, which is a form of the **KE** system, and which—as **KE**—may be restricted to analytic applications of cut without disturbing completeness. Moreover, the rules act on the right side of turnstile, so the resulting system is dual with respect to the original account (exactly in the sense examined in [7]).

Finally, the completeness result has been obtained by adjusting the technique of consistency properties to the "sequence of sequents" dual format. We think that the technique may serve as a general technique for proving completeness of calculi founded on the erotetic (*i.e.* sequence-of-sequents) format.

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Elimination of Cut in Non-Commutative Hypersequent Calculus for Temporal Logic

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A non-commutative hypersequent calculus (HC) for some temporal logics of linear frames including **Kt4.3** and its extensions for dense and serial flow of time was introduced in [3]. The system was proved to be cut-free HC formalization of respective temporal logics of linear frames satisfying subformula-property. Completeness of this system was proved by means of Schütte/Hintikka-style semantical argument using models built from saturated hypersequents. However, from the standpoint of proof theory it is profitable to have also purely syntactical, constructive proof of admissibility/elimination of cut rule. The problem of finding such a proof for non-commutative hypersequents is significantly harder than for hypersequents being multisets or sets of sequents. Most of the techniques applied in such proofs for hypersequent calculi are of no use in the present setting. Fortunately, a combination of a technique applied by Avron [1] with general strategy proposed in Metcalfe, Olivetti and Gabbay [4] and modified in Ciabattoni, Metcalfe and Montagna [2] works for suitably defined cut rule in non-commutative hypersequents. In the presentation we provide a sketch of fully syntactical, constructive proof of cut elimination along these lines for slightly modified variant of HC from [3].

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Advances in the Theory of the Leibniz Hierarchy

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Universal algebra and abstract algebraic logics are two theories that study, respectively, arbitrary algebraic structures and arbitrary substitution-invariant consequence relations (sometimes called deductive systems). The interplay between the two theories can be hardly overestimated. On the one hand, techniques from universal algebra have been fruitfully applied to the study of propositional logics in the framework of abstract algebraic logic. On the other hand, any class of algebras K is naturally associated with a substitution-invariant equational consequence $\vDash_{\mathsf{K}}$ (representing the validity of generalized quasi-equations in K), which is amenable to the techniques of abstract algebraic logic. The fact that universal algebra and abstract algebraic logic pursue two tightly connected paths is nicely reflected in the fact that one of the main achievements of both theories is a taxonomy in which, respectively, varieties and deductive systems are classified. In universal algebra, this taxonomy is called *Maltsev hierarchy*, while in abstract algebraic logic it is known as *Leibniz hierarchy*.

The goal of this contribution is to show that this analogy between the Maltsev and Leibniz hierarchies can be made mathematically precise, in a such way that the traditional Maltsev hierarchy coincides with the restriction of a suitable *finite companion* of the Leibniz hierarchy formulated for two-deductive systems. To this end, we need to solve a fundamental asymmetry between the theories of the Maltsev and Leibniz hierarchy: while there is a precise definition of what the Maltsev hierarchy is [2, 3, 4], no such agreement exists for the case of the Leibniz hierarchy. To introduce a precise definition of a *Leibniz class* of logics, we describe a preorder Log of all logics (in any language) ordered under a suitable notion of interpretability. In this setting, Leibniz classes can be characterized in several different ways. One of them describes Leibniz classes as the complete filters of Log. The fact that Leibniz classes can be identified with complete filters of Log rises the question of understanding which of the classical Leibniz classes determines a meet-irreducible or prime filter (cf. [1]). This is a completely new direction of research in abstract algebraic logic. Nevertheless, we were able to obtain some promising results: for example, it turns out that, in the setting of logics with theorems, the class of equivalential logics is meet-reducible, while (under the assumption of Vopěnka's Principle) the classes of truth-equational and assertional logics are prime.

Getting back to the similarities between the Leibniz and the Maltsev hierarchies, we prove that the Maltsev hierarchy coincides with the restriction to equational consequences relative to varieties of a suitable *finite companion* of the Leibniz hierarchy (formulated for arbitrary two-deductive systems). Thus the logical theory of the Leibniz hierarchy may be seen as a generalization of the algebraic theory of Maltsev classes. Moreover, in our opinion, this perspective shows that the conceptual taxonomies, which lie at the heart of modern abstract algebraic logic and universal algebra, have a common root.

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Bayesian Propositional Logic

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We define and investigate from a logical point of view a family of consequence relations defined in probabilistic terms. We call them *relations of supporting*, and write: $\approx_w$, where w is a probability function on a Boolean language. $A \approx_w B$ iff the fact that A is the case does not decrease a probability of being B the case. Finally, we examine the intersection of $\approx_w$, for all w, and give some formal properties of it.

Concepts in Ontology Defined by Lindenbaum's Lattice

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In the paper I analyse the notion of concepts in ontology. By representing concepts in Lindenbaum's lattice I try to show how we can define relations between denotations of names (including denotations of the so-called apparent names which was considered by Kotarbiński in his reistic ontology) and how we can define concepts in quasi-Wolniewicz lattices.

Computational Complexity of Abductive Procedures Based on Two Methods: Analytic Tableaux and Synthetic Tableaux

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Abductive reasoning aims at an explanation of surprising phenomena in the best possible way (see [3]). According to the explanatory-deductive model of abduction, which we shall follow in our presentation, the essence of abductive reasoning is filling the explanatory gap between premises and conclusion by deductive methods. A formal model of abduction such understood is defined by the choice of: basic logic, proof method, an algorithm for generating abductive hypotheses (which is based on the proof method), and an implementation of criteria of the hypotheses evaluation.

In our research we have focused on the component of the model which determines computational complexity of the algorithm for generating abductive hypotheses, that is, the proof method. We consider two abductive procedures based on two different proof methods—the method of analytic tableaux ([4], [1]) and the method of synthetic tableaux ([6], [5]). We define two classes of abductive problems: "fat problems" (the idea stems from the phenomenon of the so-called "fat formulas", see classic [2]) and "lean problems". The first class of problems constitutes a computational challenge for the abductive procedure based on the method of analytic tableaux; complexity of the solution, understood as the number of branches of a tableau, is $\mathcal{O}(n!)$ (where n is for the number of distinct variables used to express the abductive problem), whereas the method of synthetic tableaux produces a solution with exponential complexity (with respect to n). On the other hand, lean abductive problems have linear solutions when analytic tableaux are employed, and exponential solutions when the so-called canonical synthetic tableaux are used. However, as we will show, in the second case one may gain a lot by optimising the use of synthetic tableaux.

At the end we propose an optimised abductive procedure which adapts proof mechanism to the structure of abductive problem.

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Some Extensions of Relating Logic $\mathsf{RF}_{\mathsf{imp}}$ *as a Logic of Causal Implication*

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In the paper we discuss standard properties and some of well known problems of a formal theory of causal implication, widely discussed for instance in [3]. We introduce, however, a new solution based on relating logics (in short: **RL**), which can be considered as a new type of intensional logics that capture various non-extensional relationships between propositions in a specific way. In a syntax a phenomenon of being related is captured on the level of inetnsional connectives. In order to determine truth conditions of such connectives a binary relation R (*relating relation*) imposed on a set of formulas is introduced, so $\mathrm{R} \subseteq \mathsf{For} \times \mathsf{For}$. Such relation, especially, enables to consider problem of causality on the formal ground.

A brief introduction to **RL** can be found in [3], as well as some tableau approach to the smallest relating logic RF. Some important and fundamental notions were introduced in [1] and some philosophical motivation in [5]. We will discuss briefly a syntax and semantics of **RL** together with the smallest relating logic RF but rather in Hilbert-style formulation.

Starting with RF as a basic relating logic we modify it in a such way to catch different formal properties of causality. For this purpose we cut a language of RF to formulas with Boolean functors and only one relating functor—*relating implication*. A logic received in this way is an implicational fragment of logic RF and is denoted by $\mathsf{RF}_{\mathsf{imp}}$.

In the main part of the paper we show how $\mathsf{RF}_{\mathsf{imp}}$ can be modified by putting some constraints on relating relation in order to get demanded properties of relating implication—interpreted as a causal one. For instance we take into consideration a constraints which enables to fulfil conditions given by Urchs in [4]. And so depending on philosophical concepts of causality we can semantically make this implication transitive or not transitive, asymmetric or to get other properties introduced by some theory of causality. In this way we can receive logics of a causal implication which are correct on the ground of some philosophical concepts of causality. We will consider a following extensions of $\mathsf{RF}_{\mathsf{imp}}$ received by proper constraints of relating relation $\mathrm{R} \subseteq \mathsf{For} \times \mathsf{For}$:

$$
\begin{array}{lcc}
\forall_{\psi,\chi,\psi \,\in\, \mathsf{For}}(\mathrm{R}(\psi, \chi \wedge \psi) \Longrightarrow \mathrm{R}(\psi, \chi)) & \longmapsto & \mathsf{RF}^{1}_{\mathsf{imp}} \\
\forall_{\psi,\chi \,\in\, \mathsf{For}}(\mathrm{R}(\psi, \chi) \Longrightarrow \widetilde{\mathrm{R}}(\psi, \neg\chi)) & \longmapsto & \mathsf{RF}^{2}_{\mathsf{imp}} \\
\forall_{\psi \,\in\, \mathsf{For}}\widetilde{\mathrm{R}}(\psi, \psi) & \longmapsto & \mathsf{RF}^{\mathsf{ar}}_{\mathsf{imp}} \\
\forall_{\psi,\chi \,\in\, \mathsf{For}}(\mathrm{R}(\psi, \chi) \Longrightarrow \widetilde{\mathrm{R}}(\chi, \psi)) & \longmapsto & \mathsf{RF}^{\mathsf{as}}_{\mathsf{imp}} \\
\forall_{\psi,\chi,\psi \,\in\, \mathsf{For}}((\mathrm{R}(\psi, \chi) \;\&\; \mathrm{R}(\chi, \psi)) \Longrightarrow \mathrm{R}(\psi, \psi)) & \longmapsto & \mathsf{RF}^{\mathsf{t}}_{\mathsf{imp}}
\end{array}
$$

It is easy to combine those logics in addition to get a further extensions. A figure 1 presents an order among logics that we received from $\mathsf{RF}_{\mathsf{imp}}$. For each of a constraint we consider a formula that is specifically connected with it, so we make the first step toward some axiomatizations of the introduced logics.

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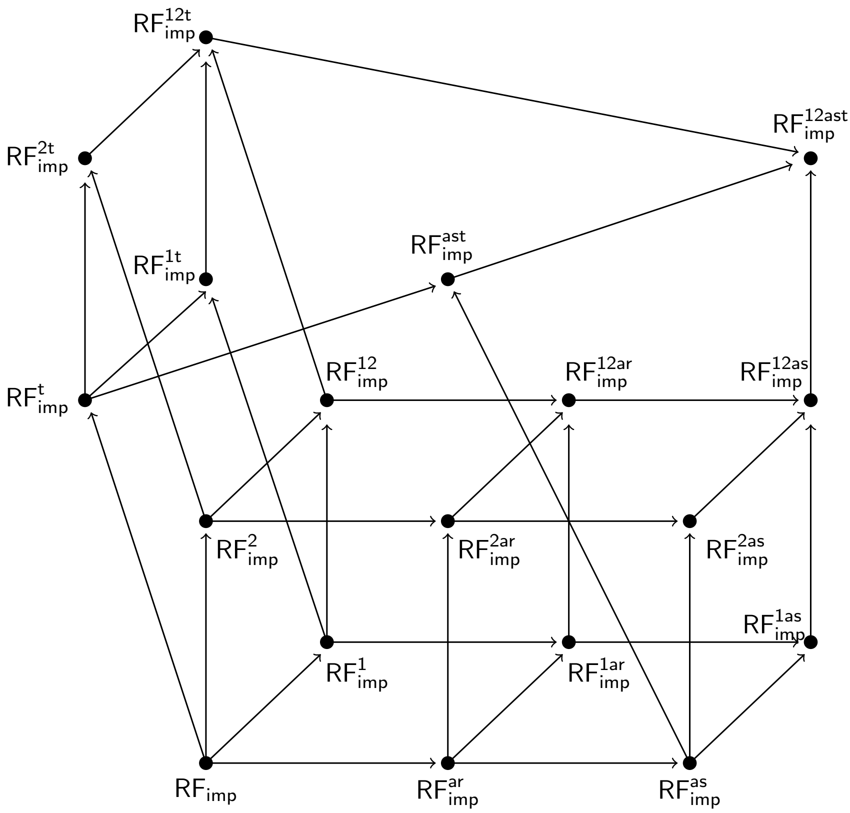

Figure 1: Some extensions of RF_{imp}

Obligation, Prohibition and Permission in the Action Language nC+

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We base our paper on M. Sergot and R. Craven's works on logic of action and agency. The authors in [1] proposed an interesting logical framework, named Action Language nC+, that has enough expressive power to describe labelled transition systems (LTSs) taking into account both the properties of transitions and states that are connected by the transitions. The authors introduced a two-sorted language: one sort contains propositional formulas expressing properties of states, the other one expresses properties of events and actions. There are also operators connecting the two sorts, e.g. "$\Box\varphi$" that is true at a state s when every transition from the state s satisfies the transition formula φ and "$0{:}F$" that is true at transition τ when its initial state satisfies the state formula F.

M. Sergot in [2] has shown that the language can be easily extended to capture many interesting aspects of intentional and unwitting agency and also some normative aspects of the agents' behavior. Normative aspects of LTSs are captured through coloring of states and transitions. So there are green (permitted, acceptable, legal) transitions and green (permitted, acceptable, legal) states. Transitions and states that are not green are red.

In our paper we present and discuss some extensions of Sergot-Craven works. We aim at a more expanded representation of deontic notions of obligation, prohibition and permission, and their mutual relations.

Firstly, we show how the results from deontic action logic, especially those concerning the notions of obligation from [4] can be transferred into nC+ ground. We pay special attention to the analysis of how different concepts of obligation and permission can be accommodated in nC+.

Then, we study the relations between norms established on transition and those established on states. We transfer some results presented in [3] into the nC+ framework. Since the nC+ language is significantly stronger then the one used in [3], the way of expressing them is more natural and simpler.

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Subdirect Representation in Abstract Algebraic Logic

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In this talk we focus on some aspects of the algebraic approach to infinitary propositional logics. By finitarity, we mean that the logic has Hilbert-style presentation where all the rules have only finitely many premises. Although the majority of logics studied in the literature are finitary, there are prominent natural examples of infinitary ones like the infinitary Łukasiewicz logic $Ł_\infty$ of the standard $[0,1]$ chain or, analogously, the infinitary product logic Π_∞.

In [1] we proposed a new hierarchy of infinitary logics based on their completeness properties. Every finitary logic is well-known to be complete w.r.t. the class of all its relatively subdirectly irreducible models and hence also w.r.t. *finitely* relatively subdirectly irreducible models. However, not even the latter is true for infinitary logics in general. We studied an intermediate (syntactical) notion between finitarity and RFSI-completeness, namely the fact that every theory of the logic is the intersection of finitely-$\cap$-irreducible theories. This property is called the *intersection prime extension property* (IPEP) and has already proven important in the study of generalized implication and disjunctive connectives. The hierarchy also includes a natural stronger extension property that refers to $\cap$-irreducible theories, the CIPEP, which is shown to hold in both Π_∞ and $Ł_\infty$.

A natural matricial semantics for a propositional logic L is that of its reduced models, denoted as $\mathbf{MOD}^*(\mathrm{L})$. For a finitary logic L, we know that each member of $\mathbf{MOD}^*(\mathrm{L})$ can be represented as a subdirect product of subdirectly irreducible members. This property can be seen as a generalization to matrices of the well-known Birkhoff's representation theorem.

In the talk we will discuss transferred versions of the aforementioned syntactical properties: L has the τ-IPEP whenever for each matrix model $\langle \boldsymbol{A}, F\rangle$ the filter F is the intersection of a collection of finitely-$\cap$-irreducible L-filters on the algebra $\boldsymbol{A}$, and analogously for τ-CIPEP with $\cap$-irreducible L-filters. Then we can prove the following characterization theorem:

Theorem 1. *For any logic* L *the following are equivalent*

1. L *is protoalgebraic and has the* τ-CIPEP.

2. L *is protoalgebraic and the* τ-CIPEP *holds on any free algebra* $\boldsymbol{Fm}_{\mathrm{L}}(\kappa)$.
3. *Each member of* $\mathbf{MOD}^*(\mathrm{L})$ *is a subdirect product of subdirectly irreducible members.*

An analogous theorem can be proved for τ-IPEP using finitely subdirectly irreducible models. Moreover this theorem naturally extends to a characterization of subdirectly representable generalized quasivarieties. As for the examples, we will show that Π_∞ has the CIPEP, but not the τ-IPEP, whereas $Ł_\infty$ enjoys the subdirect representation property.

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Antistructural Completeness in Propositional Logics

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In this contribution, we shall investigate the notion of an antistructural completion $\alpha\mathcal{L}$ of a propositional logic $\mathcal{L}$, which is in a natural sense dual to the well-known notion of a structural completion of a logic, and provide several equivalent characterizations of such completions under some mild conditions on the logic in question.

Recall that the *structural completion* of a logic $\mathcal{L}$ is the largest logic $\sigma\mathcal{L}$ which has the same theorems as $\mathcal{L}$ (see [1]). A logic $\mathcal{L}$ is then called *structurally complete* if $\sigma\mathcal{L} = \mathcal{L}$. The logic $\sigma\mathcal{L}$ exists for each $\mathcal{L}$ and it has a simple description: $\Gamma \vdash_{\sigma\mathcal{L}} \varphi$ if and only if the rule $\Gamma \vdash \varphi$ is *admissible*, that is, for each substitution σ we have $\emptyset \vdash_{\mathcal{L}} \sigma\varphi$ whenever $\emptyset \vdash_{\mathcal{L}} \sigma\gamma$ for each $\gamma \in \Gamma$.

Antistructural completions involve the same notions, but with respect to antitheorems rather than theorems. Here some clarification is in order: an *antitheorem* of $\mathcal{L}$ is a set of formulas Γ such that no valuation into a model of $\mathcal{L}$ designates each $\gamma \in \Gamma$. Equivalently, Γ is an antitheorem of $\mathcal{L}$ (symbolically,

$\Gamma \vdash_{\mathcal{L}} \emptyset$) if $\sigma\Gamma \vdash_{\mathcal{L}} \mathrm{Fm}_{\mathcal{L}}$ for each substitution σ, where $\mathrm{Fm}_{\mathcal{L}}$ is the set of all formulas of $\mathcal{L}$. A set of formulas Γ is an antitheorem of $\mathcal{L}$ if $\Gamma \vdash_{\mathcal{L}} \mathrm{Fm}$ *provided that $\mathcal{L}$ has an antitheorem* (or provided that Γ is finite). It may happen, however, that a logic has no antitheorems, e.g. the positive fragment of classical or intuitionistic logic.

The *antistructural completion* of a logic $\mathcal{L}$ is defined as the largest logic $\alpha\mathcal{L}$ (whenever it exists) which has the same antitheorems as $\mathcal{L}$. Naturally, a logic $\mathcal{L}$ is then *antistructurally complete* if $\alpha\mathcal{L} = \mathcal{L}$. For example, Glivenko's theorem essentially states that classical logic is the antistructural completion of intuitionistic logic. Our aim will be to generalize this Glivenko-like connection between $\mathcal{IL}$ and $\mathcal{CL}$ to a wider setting.

For this purpose, the following notion is the natural counterpart of admissibility. A rule $\Gamma \vdash \varphi$ will be called *antiadmissible* in $\mathcal{L}$ if for each substitution σ and each Δ we have:

$$\sigma\Gamma, \Delta \vdash_{\mathcal{L}} \emptyset \text{ whenever } \sigma\varphi, \Delta \vdash_{\mathcal{L}} \emptyset$$

Lemma. *The antiadmissible rules of each logic form a reflexive monotone structural relation which is closed under finitary cuts (but not necessarily under arbitrary cuts).*

However, unlike the admissible rules, the antiadmissible rules need not define a logic and the antistructural completion need not exist. Our main result now provides a sufficient condition for the existence of $\alpha\mathcal{L}$ and several equivalent descriptions of this logic.

Theorem. *Let $\mathcal{L}$ be a finitary logic with an antitheorem. Then $\alpha\mathcal{L}$ exists and the following are equivalent: (i) $\Gamma \vdash_{\alpha\mathcal{L}} \varphi$, (ii) $\Gamma \vdash \varphi$ is antiadmissible in $\mathcal{L}$, (iii) $\Gamma \vdash \varphi$ is valid in all $\mathcal{L}$-models $\langle \boldsymbol{Fm}, \Gamma \rangle$ where Γ is a maximal consistent theory. If $\mathcal{L}$ is moreover protoalgebraic, then these are equivalent to (iv) $\sigma\varphi, \Delta \vdash_{\mathcal{L}} \emptyset$ implies $\sigma\Gamma, \Delta \vdash_{\mathcal{L}} \emptyset$ for each Δ and each invertible substitution σ, and (v) $\Gamma \vdash \varphi$ is valid in all (reduced) κ-generated $\mathcal{L}$-simple matrices for $\kappa = |\operatorname{Var}_{\mathcal{L}}|$.*

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Can We Apply AGM Belief Revision Theory to Analyzing Collision of Norms in a Legal Code?

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One of main paradigmas in belief change theory is the so called AGM theory and one of its roots is a concept of change of legal code. P. Alchourron and D. Makinson, two of three founders of AGM-paradigm (A and M in AGM abbreviation) analyzed some kinds of change in legal code. A norm is represented in AGM by a sentence, and a derogation, a basic notion of change in a legal code, in AGM is represented by operation of belief contraction. So, instead the theory of norms change we have the theory of belief-change.

In my presentation I'll apply basic concepts of AGM to analyze of some kinds of collision of norms in a legal code. I'll analyze how the assumptions of AGM-paradigm have an influence on the possibility of adequate collision characteristics of norms in this theory.

Synthetic Tableaux with Unrestricted Cut for First-Order Logic

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I shall present the Method of Synthetic Tableaux for First-Order Logic. The calculus is an extension of the method for Classical Propositional Logic presented in [5], [7], but the inspiration for the first-order version comes from [1] and [4]. The method has been explored for some cases of propositional logics (see [6], [8]), but an extension to the first-order level, as well as an extension to modal logics, substituted a challenging task.

The closest relative of the Method Of Synthetic Tableaux seems to be the calculus **KI**, which is an "inversion" of **KE**. (See [1], [4] for **KI** and [3], [2] for **KE**.) However, the calculus for First-Order Logic, which I present here, differs substantially from the version of **KI** for First-Order Logic.

The system of Synthetic Tableaux for First-Order Logic is equipped with the so-called rule of the *Principle of Bivalence*, which is a form of cut. This rule is not eliminable from the system. Moreover, the version considered here is not restricted to analytic applications. Possible restrictions of the system will be examined in the future.

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Covering Sublattices and Semimodularity

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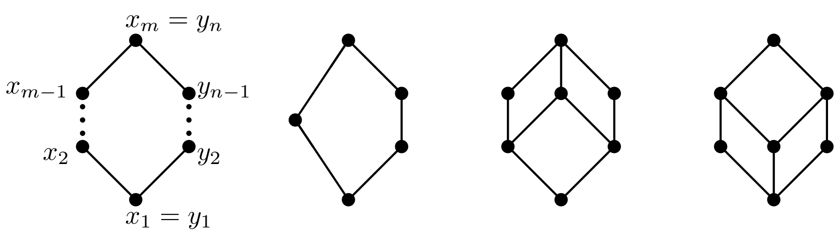
A sublattice K of a lattice L is said to be *covering*, if $x \prec y$ in K implies $x \prec y$ in L, for all $x, y \in L$. J. Jakubík in 1975 strengthened a Dedekind's classical result on modularity.

Theorem ([1]). *A discrete lattice L is modular if and only if L does not contain a covering sublattice isomorphic to some of the following lattices: S_7, S_7^*, $C_{m,n}$ $(m \geqslant 4,\ n \geqslant 3)$.*



$C_{m,n}$ $N_5 = C_{4,3}$ S_7 S_7^*

In the talk we partition the above result giving a Jakubík-style characterizations of so called Birkhoff's conditions. Next, we make an attempt to shift these results to the larger class of lattices, namely the class of upper continuous and strongly atomic lattices. Finally we announce some open problems.

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Where Logic Meets Linguistics. Inferential Layer of Annotation for the Erotetic Reasoning Corpus

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The Erotetic Reasoning Corpus (ERC, [11]) is a data set for research on natural question processing. Intuitively, we are dealing with question processing in a situation when a question is not followed by an answer but with a new question or a strategy of reducing it into auxiliary questions. This phenomenon is studied within such theoretical frameworks as Inferential Erotetic Logic (IEL) [9], [10], [5]; inquisitive semantics [3]; or KoS [1].

The corpus consists of the language data collected in the previous studies on the question processing phenomenon. These are: Erotetic Reasoning Test [7], QuestGen ([4], [6]) and Mind Maze [8]. All the data is in Polish, however the tag-set used for the annotation allows for the data analysis for English-speaking researchers.

Tagging schema for the ERC has three layers:

1. *Structural* layer—representing the structure of tasks used for ERC. Here we distinguish elements like: instructions, justifications, different types of questions and declaratives.
2. *Pragmatic* layer—representing various events that may occur in the dialogue, like e.g. long pauses. It also contains tags that allow expression of certain events related to the types of tasks used (like e.g. when forbidden question is used).
3. *Inferential* layer—which allows for normative erotetic concepts to be identified.

The inferential layer plays an important role in ERC making our data set unique. The tags used here stem from the erotetic logic ideas and concepts. Our logical framework of choice here is IEL. This logic focuses on inferences whose premises and/or conclusion are questions (erotetic inferences). IEL offers some straightforward tools for modelling erotetic inferences. What is especially important from our perspective is that IEL not only gives semantical analysis of erotetic inferences but also proposes certain criteria of their validity (the most essential notion in our case is that of erotetic implication; canonical [10], falsificationist [2] and weak one [7]).

The whole ERC consists of 402 files (133.732 words) and it is available *via* its web-site[3] (along with the documentation and useful tools). The corpus is distributed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License. In our opinion the ERC's potential scope of use is wide and reaches far beyond studies of the normative logical concepts vs. real erotetic reasonings. The potential applications may cover the following exemplary areas of interests:

- Linguistic studies of the way questions are formulated for different contexts.
- Research on dialogue management.
- Problem solving studies concerning strategies of handling with question decomposition.
- Studies focused on the way the question should be asked (or an initial problem/task should be formulated) in order to make the solution easier to reach.

The main aim of our talk is to present the unique inferential layer of annotation for the Erotetic Reasoning Corpus. We will describe how the tag-set stems from the IEL concepts and discuss its design and evaluation process.

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Some Logical Aspects of Ethical Evaluations

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From an ethical point of view, actions are treated as the contents of evaluative concepts. The issue is, that we all sometimes describe some actions as good or bad or as right or wrong. However, states of affairs also could be regarded as good ones or bad ones. So, states of affairs as well as actions, are the contents of evaluative concepts. The problem then arises: how are evaluations of states of affairs related to evaluations of actions? The question considers the relation between the rightness (or wrongness) of an action and the goodness or badness the state of affairs produced or destroyed by the action. Moreover, an action could be evaluated in two different ways. It could be regarded as a good or a bad one, as an action itself. But any particular performance or omitting the action could also be regarded as a good or bad one. The problem is connected with such matters as the relation between *prima facie* duties or obligations which may, in given situations, be overdriven by actual obligations and actual duties. The lecture deals with logical interconnections between several ethical evaluations. It provides a tentative logical study of evaluative concepts *it is good that...*, and *it is bad that...*, used in reference to actions and to states of affairs. It studies expressions like *it is good to perform* (*or omit*) a, *it is *bad to perform* (*or omit*) a, where a stands for an action, in connection with expressions like *it is good that* A, *it is bad that* A, where A stands for a state of affairs.

Key words: evaluation, action, state of affairs, good, evil.

The Connective "czy" in Polish

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In Polish, the word "czy" is most frequently viewed as an interrogative particle (the English word "whether" has a similar function but only in indirect questions) as well as a connective (equivalent to "or" in English).

In my article I will focus especially on the connective function of the word "czy".

I will attempt to answer the question whether this connective can be an equivalent to one of the functors in logic (if so, to which one?).

Discrete Dualities for some Lattice-Based Algebras

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Discrete duality is a relationship between classes of algebras and classes of frames (relational systems). If Alg is a class of algebras and Frm is a class of frames, establishing a discrete duality between these two classes requires the following steps:

1. With every algebra L from Alg associate a canonical frame $\mathrm{Cf}(L)$ and show that it belongs to Frm.
2. With every frame X from Frm associate a complex algebra $\mathrm{Cm}(X)$ and show that it belongs to Alg.
3. Prove two representation theorems:

   (3a) For each algebra L in Alg there is an embedding h: $L \to \mathrm{CmCf}(L)$,

   (3b) For each frame X in Frm there is an embedding k: $X \to \mathrm{CfCm}(X)$.

In case of distributive lattices canonical frames correspond to dual spaces of algebras in the Priestley-style duality, however in case of discrete duality they are not endowed with a topology and hence may be thought of as having a discrete topology. Similarly, in case of distributive lattices complex algebras of canonical frames correspond to canonical extensions in the sense of Jonsson-Tarski.

In my talk discrete dualities for the following two lattice-based classes of algebras will be discussed: a not first order-definable class of algebras based on Boolean algebras and a non-canonical class of algebras based on distributive lattices. Furthermore, it will be shown that, in general, not necessarily distributive lattices do not admit discrete duality (theorem 3b does not necessarily hold).

A Logical Account of Subjective Rights

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We understand subjective rights as a specific situation of an agent (who has the right) in which the agent is entitled to behave in a certain way, chosen freely by him or her. Rights in a broad sense splits into privileges and freedoms. As examples of subjective rights we can mention the right to education, freedom of speech, the right to vote in public, political elections, the right to be elected, freedom of religious worship.

Sometimes exercising a right requires co-operation or help from other agents such as authorities, service providers, etc. For example if children have the right to education there must exist institutions (schools) that provide educational services.

Subjective rights are elements of complex normative systems such as legal codes. In legal theory usually only two basic types of norms are considered: the ones imposing obligations and the ones imposing prohibitions. Thus, to consider subjective rights in that main paradigm, one needs to introduce them as derivatives of the rights of the basic types.

We construct our formalization on the basis of the notion of normative positions. The notion comes from Hohfeldian theory of duties and rights. We propose a fine-grained distinction concerning normative positions that enables us to define precisely the notion of subjective norms.

We distinguish three main types of normative positions: basic, fundamental and primary. Basic normative positions are normative proprieties of a given subject which are created (constituted) by norms addressed to the subjects or by the absence of such norms.

Fundamental normative positions (the normative positions introduced by Hohfeld) are such positions whose characteristics includes a relationship between a subject of a position and another subject. Each fundamental position has its correlative – a normative position of one subject is related to a correlative normative position of another subject.

Finally, primary normative positions are complexes of normative positions and can be composed of basic, fundamental and also other primary normative positions.

We state that a normative position is realized when an agent in bound by the position actually behaves in a way the normative position indicates.

Now, we can come back to subjective rights. A subjective right is defined as a complex primary normative position which is composed of several norms and rules, or, more precisely, normative positions created by these norms and rules, which form a functional unity to secure a certain state of affairs which is due to an agent. We distinguish two types of subjective rights: effective and ineffective.

In our talk we shall present a complete list of basic and fundamental normative positions and the relations among them, provide definitions of effective and ineffective subjective rights and discuss some of their consequences.

Cognitive Accessibility of Mathematical Objects[4]

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Jaroslav Hašek wrote that it is difficult to describe non-existing animals but it is much more harder to show them to the audience. We are not going to discuss the old dilemma: is mathematics *created* or *discovered*? Rather, we will focus our attention on the *access* which we have to mathematical objects themselves. Moreover, this access will be characterized *inside* mathematics and not based on, say, philosophical considerations about perception.

In turn, John von Neumann expressed the opinion that in mathematics we are not aiming at *understanding* it but we rather *get accustomed* to it. Is this *dictum* a play with words only? We think of mathematics as a *science of*

[4]The work on this paper has been sponsored by the National Scientific Center research grant 2015/17/B/HS1/02232 *Extremal axioms: logical, mathematical and cognitive aspects.*

patterns which is also an *art of solving problems*, according to prescribed rules. The *meanings* of mathematical concepts are determined by the underlying theory. *Understanding* these concepts is obtained in the *context of transmission* of mathematical knowledge, with the help of *intuitive explanations*.

Mathematical objects can be *standard*, *exceptional* or *pathological*. Whether they are considered as *well-behaving* depends on the goals they are supposed to serve. *Domestication* of mathematical objects is a result of accumulation of knowledge about them and widening the scope of their applications.

The objects of each mathematical domain may be classified with respect to their *accessibility* for the cognitive subject. We have different cognitive access to several sorts of numbers: integers, rational, algebraic, constructible, computable, irrational, transcendental, normal, etc. numbers. There are *easy* sets (finite, Borel, constructible) and *difficult* ones (Bernstein, Cantor, Vitali, large cardinals, indecomposable continua). Functions are classified in the Baire's hierarchy. One can distinguish degrees of *computability* and *incomputability*. Some examples will be discussed in details in our talk.

We are going to emphasize the fact that *degrees of accessibility* of mathematical objects can be characterized in mathematics itself, thus without support of metaphysical assumptions. However, the accessibility in question is relativized historically and depends on the expressive power of mathematical discourse, as we will try to show.

Archetypal Rules Beyond Classical Logic

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The notion of archetypal rule was introduced by Lloyd Humberstone, cf. [1]. Informally, we say that a rule r is archetypal for a logic L if, up to provability in L, r is derivable, not invertible and for any other derivable rule s there is a substitution such that the premisses of s are the instances of premisses of r and the conclusion of s is the instance of the conclusion of r. The problem of semantic characterization of archetypal rules in classical propositional logic was solved recently in [4]. Unfortunately, the approach which was applied to classical logic cannot be applied to other logics in a direct way. In this talk we survey known results and shed some light to the general problem of archetypal rules in intermediate logics.

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Bimodal Dunn-Belnap Logic

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Many-valued modal logics provide a natural formalisation of reasoning with modal notions such as knowledge or action in contexts where the two-valued classical approach is not adequate. Such contexts typically involve reasoning with incomplete, inconsistent or graded information.

A prominent example of a (non-modal) many-valued logic designed to deal with incomplete and inconsistent information is the Dunn–Belnap four-valued logic [4, 2, 3]. Dunn and Belnap evaluate formulas of the language $\{\wedge, \vee, \neg\}$ in the four-element matrix

$$\begin{array}{ccc} & \top & \\ / & & \backslash \\ f & & t \\ \backslash & & / \\ & \bot & \end{array}$$

where t, f represent the classical truth values "true" and "false", $\top$ represents "both true and false" and $\bot$ represents "neither true nor false" ($\{t, \top\}$ are the designated values). The matrix can be seen as being ordered in two ways, namely, "left-to-right" (*truth order*) and "bottom-up" (*information order*), with $\wedge, \vee$ as meet and join of the truth order and $\neg$ as "vertical flip" mapping t to f (and vice versa), $\top$ to $\top$ and $\bot$ to $\bot$. Hence, the matrix provides a natural example of a *bilattice*. Arieli and Avron [1] extend the Dunn–Belnap logic with constants denoting the truth values, an implication connective, "horizontal flip" negation and conjunction/disjunction pertaining to the information order, i.e. they use the language $\{\wedge, \vee, t, f, \otimes, \oplus, \bot, \top, \neg, -, \supset\}$.

Several modal extensions of Dunn–Belnap and Arieli–Avron have been studied recently adding a modal operator $\Box$ to either the full Arieli–Avron language [5, 7] or to its fragment $\{\wedge, \vee, \neg, f, \supset\}$ [6]. The operator $\Box$ is interpreted in terms of the truth-order infimum (simplifying a bit, the value of $\Box\varphi$ in world

w of a Kripke model is the truth-order infimum of the values of φ in worlds w' accessible from w.)

However, a modal operator $\Box_i$ corresponding to the information-order infimum is a natural addition to consider. If worlds in a Kripke model are seen as "sources" of information, then the value of $\Box_i\varphi$ at w is the *minimal information* about φ on which all the sources agree. If accessible worlds are seen as possible outcomes of some information-modifying operation (such as adding or removing information), then the value of $\Box_i\varphi$ at w is the minimal information about φ that is guaranteed to be preserved by the operation.

It is well known that such $\Box_i$ is expressible in any language extending $\{\wedge, \vee, \neg, \perp, \Box\}$; define $\Box_i\varphi := (\perp \wedge \neg\Box\neg\varphi) \vee \Box\varphi$. We focus here on the case where $\perp$ is not available. We extend the modal language used in [6] with $\Box_i$. Our main technical result is a sound and complete axiomatization. Expressiveness of the language and applications in knowledge representation are discussed as well.

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Axiomatizations for Universal Classes

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We present a scheme for providing axiomatizations for universal classes. We use infinitary sentences there. New proofs for Birkhoff's HSP-theorem and

Mal'cev's $\mathsf{SPP_U}$-theorem are derived. In total, we present 63 theorems of this sort.

A class operator O, in a fixed first order language L, is *unary* if for every set $\mathcal{S}$ of structures in L we have $\mathsf{O}(\mathcal{S}) = \bigcup_{\mathbf{A}\in\mathcal{S}} \mathsf{O}(\mathbf{A})$. Let O be a unary class operator and assume that with each structure $\mathbf{A}$ in L we may associate a sentences, say in $L_{\infty,\infty}$, such that

$$\mathbf{B} \models \chi^{\mathsf{O}}_{\mathbf{A}} \quad \text{iff} \quad \mathbf{A} \in \mathsf{O}(\mathbf{B}).$$

Then we say that the sentences $\chi^{\mathsf{O}}_{\mathbf{A}}$ are *characteristic for* O.

Observation. *Let $\chi^{\mathsf{O}}_{\mathbf{A}}$ be characteristic sentences for a unary class operator O and let $\mathcal{C}$ be a O-closed class of structures. Then $\mathcal{C}$ is axiomatizable by the class of sentences $\{\neg\chi^{\mathsf{O}}_{\mathbf{A}} \mid \mathbf{A} \notin \mathcal{C}\}$.*

The above simple observation is particularly useful for classes definable by universal sentences. Indeed, then the additional assumption that the class $\mathcal{C}$ is closed under taking ultraproducts yields that $\mathcal{C}$ is definable by first order sentences of a special form. Similarly, by assuming the closure under taking direct products, we obtain another simplification of the defining sentences.

We apply the observation to HS the homomorphic image of a substructre class operator and to S the isomorphic image of a substructre class operator. In this way we obtain classical Birkhoff's and Mal'cev's theorems. By considering various types of unary class operators and defining appropriate characteristic sentences, we obtain many similar theorems.

Two Formal Models of the Concept of Temperance

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Temperance is one of the four cardinal virtues which were first formulated by Plato. Today the notion of temperance has a central position in many ethical systems, especially in ecophilosophy (environmental philosophy). Temperance, involwing balance and moderation, may give resolution to many contemporary human problems. The attempt of analysis of this concept with simple formal means can make clear some hidden assumptions and the structure of the concept of temperance. The analised concept includes both—a decision (a choice) and an action/behaviour as an effect of the decision. First model is based on Aristotle's theory of golden mean presented in his Nicomachean Ethics. The attempt of reconstruction Aristotle's idea using mathematical notion of weighted mean shows that the concept of temperance is strongly restricted to the cultural background. Therefore in extremal cases the universal concept of temperance may not exist.

Second model given by Epicurus is based on the notion of consequence. It concerns pleasure and pain only, but temperance is just recommended in using pleasures. The set of temperant decisions is built recurently. A—the set of pleasure—is modified step by step by consequence $C_R(A)$ where R—set of rules—includes modus ponens, law of identity and some simple laws of everyday provenience. U is the set of temperant choices in Epicurean sense iff $\forall p \in P \neg(p \in C_R(U))$, where P—set of pain.

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Infinitesimals and Continuity from the Perspective of the Analysis of Motion

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The talk will present different concepts regarding infinitesimals, topics related to them and concepts regarding continuity. The following issues will be addressed:

- Plato and Hoppe's opinion about Plato's definition of infinitesimals
- Galilelo, Leibniz and Newton's ideas regarding analysis of motion in physics
- Cauchy's theory of continuity and infinitesimals on the basis of a variable quantity
- du Bois-Reymond and his rates of growth of functions Stoltz and moment of a function
- Lawvere and his come back to Plato's ideas (set theory and variability)

All of these issues have been selected from the perspective of the analysis of motion. This perspective covers, for example, analysis of motion in physics, function's definition and variable quantity in mathematic. In some sense, this perspective also solves the continuous-discreete conflict.

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An Example of a Non-Recursive Modal Logic

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We present a simple construction which gives a non-recursive modal logic given by a recursive set of frames and which has only one modal operator.

The existence of such logic was claimed in [1, Theorem 16.14]. However, there is a mistake in the proof that cannot be removed. A correct such example, even extending Gödel-Löb modal logic, was given by Gorbunov in [2].

Our reasoning is much simpler than Gorbunov's proof. In particular, we do not use canonical formulas. Actually, we use only basic knowledge about frame semantics and recursive sets.

Let us briefly present the example. Let X be a recursive set of natural numbers such that $Y = \{|x - y| \colon x, y \in X\}$ is not recursive. It is easy to see that Y is recursively enumerable, so it cannot be co-recursively enumerable. Let $\mathfrak{F}_n$ be the frame depicted bellow. The point g is the only reflexive one and there is an edge from a_k to g iff $k \in X$ (in the picture we have $i, j \in X$). Then our logic is the logic of the family $\{\mathfrak{F}_n \colon n \in \mathbb{N}_+\}$ of frames.

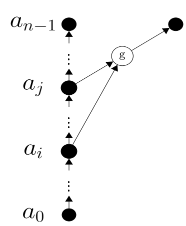

Figure 2: Frame $\mathfrak{F}_n$

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Transitivity in Some Non-Classical Modal Logics

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In general, there are several ways to expand a concrete many-valued logic with modal operators with intended semantics (for the modal operators) that generalize the classical one. Under this premise, several recent works (see eg. [2], [4, 5], [7], [8]) have generalized the model initally proposed by Fitting to different settings. Even if the new $\Box$ and $\Diamond$ operators are defined[5] in a way which generalizes the classical modalities, preserves the natural relation between the predicate and modal logics and are meaningfully still related to necessity/possibility-like notions, several important characteristics of classical modal logics are lost. Some of the most maybe surprising differences include the K axiom not necessarily being valid in the minimum modal logic over a certain propositional one, or the failure -in general- of the finite model property. These characteristics, together with the lack of finitarity and of (nice) deduction theorem in most many-valued logics, has created challenges ranging from the axiomatization of these modal logics to their understanding and applications.

In this talk, we will focus in the decidability/undecidability of some of the previous logics. The study of decision problems in fuzzy modal logic has focused on Gödel-style logics [3] and in some fuzzy description logics [1, 6]. The decidability shown in [3] can be in fact generalized to modal logics build as expansions of locally finite ones. On the other hand, we will also see that the global modal logic (understood as a deduction relation) of a large class modal logics (namely, those whose propositional algebra of evaluation is non n-contractive) is undecidable. Not only that, but the local modal logic of the transitive models over the same algebras is also undecidable. Both results are proven showing that the well known Post-correspondence problem can be reduce to the previous deductions. Surprisingly enough, the previous undecidability results hold also if we consider just finite relational models, which has as a consequence the impossibility for these logics to have a R.E. axiomatization.

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Almost Classical, Paraconsistent Logic

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Paraconsistent logics, avoiding explosion from inconsistency, try to retain as much of classical reasoning as possible. The presented, infinitary propositional logic sc rip, [10], provides a peculiar answer to this challenge in that its resolution-based rules are sound and refutationally complete for the classical semantics (of countable theories). But when used for a direct, non-refutational reasoning, weakening becomes inadmissible; along with it, Ex Falso Quodlibet disappears and the logic ceases to be explosive. sc rip allows thus to derive, in a classical manner, inconsistency of any inconsistent countable theory and, in addition, to identify its maximal consistent subtheory (in a specific sense to be explained), deriving its classical consequences, but not their negations. Only when such a subtheory happens to be empty, the full explosion ensues.

Semantics is defined using digraph kernels, [9, 4], and it coincides with the classical semantics for consistent theories. Formulated generally, semantics comprises all maximal predecessor-closed semikernels, [8, 7]. When the theory is consistent, such semikernels happen to be kernels, while for inconsistent theories they represent models of maximal consistent subtheories. Only when all statements of the theory are relevant for the appearance of inconsistency, the semantics degenerates to one such empty semikernel.

This graph-theoretic formulation of the semantics, uniform for the classical and paraconsistent logic, gives new means for expressing consistency conditions as patterns guaranteeing existence of digraph kernels. With a natural interpretation of the 'graph normal form' of the theories, [3], which underlies sc rip, they become conditions excluding semantic paradoxes. The presentation uses examples based on this observation, which originates from [5] and has been used several places for the analysis of paradoxes, [1, 2, 6, 10].

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Fragments of Halpern-Shoham Logic

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Logic of Halpern and Shoham (HS in short) [4] is a propositional interval-based temporal logic. Due to its elegant representation of relations between intervals (usually interpreted as time-intervals), the logic is deeply investigated within Artificial Intelligence, and in particular in Knowledge Representation.

$$
\begin{tikzcd}
 & \mathsf{HS}^{i,@}_{horn} \arrow[dl] \arrow[d] \arrow[dr] & \\
\mathsf{HS}^{\Diamond,i,@}_{horn} \arrow[d] & \mathsf{HS}^{i}_{horn} \arrow[dl] \arrow[d] \arrow[dr] & \mathsf{HS}^{\Box,i,@}_{horn} \arrow[d] \\
\mathsf{HS}^{\Diamond,i}_{horn} \arrow[d] & \mathsf{HS}_{horn} \arrow[dl] \arrow[dr] & \mathsf{HS}^{\Box,i}_{horn} \arrow[d] \\
\mathsf{HS}^{\Diamond}_{horn} & & \mathsf{HS}^{\Box}_{horn}
\end{tikzcd}
$$

!

Figure 3: Hasse diagram of Horn HS fragments, where an arrow indicates the relation of being a syntactical fragment.

Language of HS contains 12 modal operators of the form $\langle \mathsf{R} \rangle$, where R stands for one of binary relations between intervals introduced by Allen [1], namely *adjacent to*, *begins*, *during*, *ends*, *later than*, *overlaps*, and their inverses. As usual, $[\mathsf{R}]$ is a dual operator to $\langle \mathsf{R} \rangle$. A model for HS is a linear ordering of time-points (a temporal frame), in which propositional variables are interpreted by sets of intervals over this frame. Expressive power as well as decidability/computational complexity of a satisfiability problem in HS depends on the type of temporal frame. However, in most of interesting cases (including such temporal frames as $\mathbb{N}$, $\mathbb{Z}$, and $\mathbb{Q}$) HS is undecidable – the Non-Halting problem reduces to the satisfiability problem in HS [4]. As a result, the main directions of research concerning HS aims at introducing decidable fragments of HS which are still interesting from the point of view of their expressiveness.

During the talk I will present recent results on syntactical restrictions of HS language. Namely, I will introduce *Horn* HS fragments [2, 3, 5], whose formulas are given by the following abstract grammar:

$$\varphi := \lambda \mid [\mathsf{U}](\lambda \wedge \ldots \wedge \lambda \to \lambda) \mid \varphi \wedge \varphi.$$

where $[\mathsf{U}]$ is the universal modality and depending on the form of λ expressions we obtain one of the fragments presented in the lattice from Figure 3. In the cases of HS_{horn}, HS^{i}_{horn}, and $\mathsf{HS}^{i,@}_{horn}$ λ-expressions are defined by the following abstract grammars respectively:

$$
\begin{aligned}
\lambda &:= \top \mid \bot \mid p \mid \langle \mathsf{R} \rangle \lambda \mid [\mathsf{R}]\lambda; \\
\lambda &:= \top \mid \bot \mid p \mid \langle \mathsf{R} \rangle \lambda \mid [\mathsf{R}]\lambda \mid i; \\
\lambda &:= \top \mid \bot \mid p \mid \langle \mathsf{R} \rangle \lambda \mid [\mathsf{R}]\lambda \mid i \mid @_i \lambda;
\end{aligned}
$$

where p is a propositional variable, i is a nominal, i.e., an atom satisfied in exactly one interval, and $@_i$ is a satisfaction operator stating that a formula is satisfied in the interval in which i is satisfied. If $\Diamond$ ($\Box$) occurs in the upper index, then the expression of the form $\Box\lambda$ ($\Diamond\lambda$) is deleted from the grammar of λ.

I will describe main results concerning expressiveness and computational complexity of the above described fragments. I will focus on some decidable fragments, i.e., $\mathsf{HS}^{\Box}_{horn}$ over dense frames, which turned out to be tractable (more precisely, P-complete) [2], as well as $\mathsf{HS}^{\Box,i}_{horn}$ and $\mathsf{HS}^{\Box,i,@}_{horn}$ over dense frames which were proven to be NP-complete [5].

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A Brief Examination of Aristotelian Syllogisms in Kalinowski's K2 System

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Jerzy Kalinowski's two systems K1 and K2 are two of the first systems of deontic logic. Although his thought is not widely present in the contemporary literature on the deontic logic, I believe that the approach proposed by Kalinowski is worth studying and may be useful for representing deontic reasoning including types of actions and types of agents. The aim of my presentation is to provide a survey of Kalinowski's approach to formalize Aristotle's syllogisms within deontic logic.

In my talk I will briefly present Kalinowski's K1 system as a backdrop to his further developments in deontic logic. After that I will focus on syllogistic reasoning as it was presented in Kalinowski's article *Teoria zdań normatywnych* published in *Studia Logica* in 1953. I will then make a few comments about Kalinowski's recreation of the logical content of Aristotle's syllogisms and his attempt to formalize it in his K2 system. To do this I will present some examples from Kalinowski's previously mentioned paper.

Generalized Entailment Meets Interrogative Entailment

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A semantic relation between a family of sets of formulas and a set of formulas, dubbed generalized entailment, and its subrelation, called constructive generalized entailment, will be defined and examined. Entailment construed in the usual way and multiple-conclusion entailment can be viewed as special cases of generalized entailment. The concept of constructive generalized entailment, in turn, enables an explication of some notion of interrogative entailment, and coincides, at the propositional level, with inquisitive entailment. Some connections between constructive generalized entailment and Inferential Erotetic Logic will also be pointed out.

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Intuitionistic Modal Logic Based on Neighborhood Semantics without Superset Axiom

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In this paper we investigate certain systems of propositional logic defined semantically in terms of neighborhood structures. These systems have both intuitionistic and modal aspects. We continue in some sense the basic idea presented recently by Moniri and Maleki but we propose one important change: to discard *superset* axiom and to replace it by *relativized superset axiom.*

Such modification allows us to think about specific modality Δ, behaving partially like $\Box$ - but in maximal, not minimal neighborhood. Moreover, without superset axiom we are free to introduce new functors of negation and implication ($\sim$ and $\leadsto$) - depending on the notion of maximal neighborhood. We discuss hypothetical possibility operator (its semantic interpretation and connotations with Δ).

Then we study properties of functors mentioned above and compare them with modalities described e.g. by Pacuit. Also, we investigate various restrictions imposed on our models. In particular, we show that it is possible to treat maximal neighborhoods like topological spaces.

We use notions of *bounded morphism*, *bisimulation* and *n-bisimulation* (the last is probably the most significant) in our new context, thus adapting and extending theorems introduced previously by Moniri and Maleki. Finally, we analyze the concept of canonical models.

Logical Formulas and Their Existential Import

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In his dissertation dedicated to the analysis of existential import of logical formulas (1990) Karl-Heinz Krampitz presented a list of rules that allow to deduce the existential assumptions of complex formulas:

R1 All elementary predicate formulas have existential import (are *existentiell belastet*) in the sense that in order to be true they assume the existence of their arguments.

R2 If $\forall$ has existential import, then $-\forall$ does not have existential import.

R3 If $\forall$ does not have existential import, then $-\forall$ has existential import.

R4 $\forall\omega\exists$ has existential import if and only if $\forall$ and $\exists$ have existential import.

We shall formulate the theory of existential import as a base construction (**EB**) ssumptionally. It can be subsequently strengthened while analysing specific logical systems by adopting in the form of an axiom (or axioms) the elementary formula (or formulas) characteristic for a given logical system which has (which have) existential import.

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Unification in Predicate Logic. Part I

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Let us introduce unification in superintuitionistic predicate logic and apply it to solve some problems such as structural completeness and unification types. The extension of unification from propositional logics to the 1-st order level is not so immediate and it requires, in the first place, a proper definition of substitution for predicate letters.

A *unifier* for a formula A in a predicate logic L is a substitution ε for predicate variables such that $\vdash_L \varepsilon(A)$. A formula A is said to be *projective* in L if it has a projective unifier in L, that is it has a unifier ε such that $\vdash_L A \to (B \leftrightarrow \varepsilon(B))$ for each B. We say that a logic L enjoys *projective unification* if each unifiable formula is L-projective.

Theorem 1. L *enjoys projective unification iff* $\mathsf{P.Q\text{–}LC} \subseteq \mathsf{L}$ *where*

$$\exists_x(\exists_x B(x) \to B(x)). \tag{P}$$

Corollary 2. *Every superintuitionistic predicate logic extending* $\mathsf{P.Q\text{-}LC}$ *is almost structurally complete.*

We develop the theory of unification types for superintutionistic predicate logics. Standard definitions of the types: $1, \omega, \infty, 0$ are introduced and we try to follow the known results on unification types in propositional logics. However, despite some similarities, the results are different: the unification type of $\mathsf{Q\text{–}L}$ is usually different from the unification type of the propositional logic L. For instance, the unification type for Q–LC is 0, not 1. The same happens with Q–KC. For Q–INT we get ∞ or 0 instead of ω.

Corollary 3. *Unification in* P.Q–LC *and all its extensions is unitary.*

Temporalizing a Propositional Dynamic Logic for Qualitative Velocity

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In [1] Burrieza *et al.* proposed a PDL approach for qualitative velocity. Briefly speaking, program modalities in their logic represent transitions of an object parametrized with two values: qualitative velocity (such as *slow*, *moderately fast*, *fast* etc.) and qualitative orientation (such as *toward back-left*, *onward backward*). For example, what a formula

$$\psi := \langle(\mathsf{fast}, \mathsf{onward}); (\mathsf{slow}; \mathsf{leftward})\rangle\varphi$$

expresses is that if an object starts from the current state and subsequently executes to moves: fast onward and slow leftward, it reaches a state where φ holds.

A natural question arises of whether it is possible to add another parameter to this framework, namely time, thus establishing a two-dimensional PDL-temporal logic.

Temporalizing various modal logics [2, 4] (inclusive of PDL [6] or spatial logics [5]), as well as first-order logic [3], is an extensively studied problem and research conducted within this ares often leads to non-trivial results (like, e.g., unexpected blow-up of computational complexity of a product of a simple modal and temporal logic). In my talk, I would like to show that general methods used in constructing propositional temporal dynamic logic (PTDL) need to be carefully adjusted to fit our case of temporalized PDL for qualitative velocity. I will also investigate some metalogical properties of thus obtained fomralism, such as soundness and completeness of a provided axiomatization with respect to a defined frame class, as well as its decidability and computational complexity.

Key words: modal logic, temporal logic, propositional dynamic logic, product logics, qualitative reasoning

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