XXI Conference

Applications of Logic in Philosophy and the Foundations of Mathematics

Szklarska Poręba

Poland

9–13 May 2016

XXI Conference
Applications of Logic in Philosophy and the Foundations of Mathematics

---

The Conference is organized by:

Department of Logic and Methodology of Sciences, University of Wrocław

Institute of Mathematics, University of Silesia

Institute of Mathematics and Informatics, University of Opole

Under the auspices of:

Polish Association for Logic and Philosophy of Science

Edited by:

Krzysztof Siemieńczuk
Bartłomiej Skowron
Marcin Selinger

First published 2016

Publisher:
Department of Logic and Methodology of Sciences, University of Wrocław, Wrocław

ISBN 978-83-940690-2-5

Contents

Abstracts

Editorial note

(EN) means that the talk is presented in English, (PL)—in Polish.

Formal Concept Analysis (FCA) with Evaluative Content

Tomasz Albiński (EN)
Adam Mickiewicz University, Poznań
Institute of Philosophy
Poland
albinski@amu.edu.pl

I present an explanation for direct and indirect evaluations in the so-called statements of value. I use the concept of *modified values language*, i.e., language that consists of utterances expressing values with different force. My analysis will be based on FCA.

There are many ways of modifying the expressed judgments: using the appropriate grammatical forms (e.g. diminutives), intonation, using particular contexts, etc. I analyze one specific form of manipulation: the modification of the concept meaning in sentences that express direct/indirect evaluations.

For the purposes of my analysis, I use the following exemplification: the language used by the diplomats of The US Department of State in the official announcements and interviews describing the events in the Ukraine in the first half of 2014. This language provides us with a very good example of various indirect evaluations, in which the values expressed have a mitigating tone, such as, for example, "*situation*", "*conflict*" and "*escalation*" instead of "*war*".

I assume that statements such as "*There is now a war*", "*There is now a conflict*", "*There is now a situation*" express different evaluations about the same events described. FCA is used to define the formal context of military conflicts and to distinguish the formal concepts of "*war*", "*conflict*", etc.

In conclusion, I show that value modifications results from various modifications in the structure of a formal concept.

Haskell Implementation of Abductive Question-Answer System (AQAS)[1]

Szymon Chlebowski & Andrzej Gajda (EN)
Adam Mickiewicz University, Poznań
Institute of Psychology
Poland
szymon.chlebowski@amu.edu.pl, andrzej.m.gajda@gmail.com

Abduction can be considered as a kind of reasoning with question processing involved. At the beginning, when it is impossible to obtain an information ϕ from the knowledge base Γ, the initial question arises: what should be added to the knowledge base Γ to derive ϕ? The initial question posed in such a way could be hard to answer, so it is further processed by means of a set of rules. As a result we obtain a question which is *minimal* in a well-defined sense and is easy to answer. We prove that good answers to this minimal question are good answers to the initial question as well (they are abductive hypotheses fulfilling certain criteria). Answers to the minimal question are generated by two abductive rules. By means of the first abductive rule partial hypotheses (partial answers) of the form of a negated atom are created, while in the case of the second abductive rule partial hypotheses of the form of implication are generated (with information from the knowledge base and information from the unexplained phenomenon). Answer to the initial question is a conjunction of answers to the minimal questions. This is the way the process is carried out in accordance with the Erotetic Decomposition Principle. Wiśniewski's method of Socratic Proofs developed on the grounds of Wiśniewski's Inferential Erotetic Logic is being used as a main mechanism of decomposition [3]. It should be noted that during the transformation the information encoded by the knowledge base is being kept separate from the information represented by the formula ϕ.

This was the first part of the Abductive Question-Answer System (AQAS). Along with the question transformation, two other tools are used in order to implement criteria of *being good* for the hypothesis: Hintikka sets and dual Hintikka sets. There is one restriction for each of the two abductive rules, which involve Hintikka sets, and which concerns the consistency of the hypothesis with the knowledge base. Similarly, there is one restriction for each of the two abductive rules, which involve dual Hintikka sets, and which concerns the significance of the hypothesis (the hypothesis is significant when ϕ can not be derived from the hypothesis alone). This is the second part of the AQAS, which is responsible for generation of only *good* hypotheses. Therefore, our approach is a one phase mechanism, unlike the vast majority of the concepts of formalisation of the abductive procedures where a huge number of hypotheses

[1] This work was supported by funds of the National Science Centre, Poland (DEC-2013/10/E/HS1/00172 and DEC-2012/04/A/HS1/00715).

is generated at first and then the evaluation over those hypotheses leaves only desired ones (e.g. [2]).

AQAS is already formalised for classical propositional logic (CPL). Our presentation will concern the Haskell [1] implementation of this formalisation. As a spinoff we have obtained an elementary theorem prover for hypersequent calculus for CPL.

References

[1] G. Hutton. *Programming in Haskell.* Cambridge University Press, 2007.

[2] M. Komosiński, A. Kupś, D. Leszczyńska-Jasion, and M. Urbański. Identifying efficient abductive hypotheses using multi-criteria dominance relation. *ACM Transactions on Computational Logic*, 15(4), 2014.

[3] A. Wiśniewski. Questions, *Inferences and Scenarios*, volume 46 of *Studies in Logic. Logic and Cognitive Systems.* College Publications, London, 2013.

Automated Generation of Erotetic Search Scenarios: Classification, Optimisation and Knowledge Extraction[2]

Szymon Chlebowski, Maciej Komosiński & Adam Kupś (EN)
Adam Mickiewicz University, Poznań
Institute of Psychology
Poland
szymon.chlebowski@amu.edu.pl, maciej.komosinski@cs.put.poznan.pl,
adamku@amu.edu.pl

Erotetic search scenarios (*e-scenarios*) are formal constructs defined on the grounds of Inferential Erotetic Logic [3,4]. Given a question Q and some assumed knowledge X an e-scenario for the question Q relative to the knowledge base X represents a strategy of answering the initial question Q by means of precisely defined decomposition rules. We introduce operations which enable processing e-scenarios that fail to satisfy certain criteria into another e-scenarios satisfying them. This brings us close to the notion of optimal e-scenario.

Computations performed on evaluated e-scenarios concern search for the most interesting scenarios by applying multi-criteria dominance relation. Furthermore, to explore the relations between the criteria, discordance analysis is performed. Final stage of our work concerns employment of Apriori algorithm which is used for exploration of rules relating values of the different criteria. As it will be shown, thanks to this procedure general prepositions about erotetic search scenarios may be discovered. The presented work is a continuation of applying of multi-criteria dominance analysis and discordance analysis in the

[2]This work was supported by funds of the National Science Centre, Poland (DEC-2012/04/A/HS1/00715).

domain of philosophical logic, which started with the work on finding efficient abductive hypotheses [1,2].

References

[1] M. Komosinski, A. Kupś, D. Leszczyńska-Jasion, and M. Urbański. Identifying efficient abductive hypotheses using multi-criteria dominance relation. *ACM Transactions on Computational Logic*, 15(4), 2014.

[2] M. Komosinski, A. Kupś, and M. Urbański. Multi-criteria evaluation of abductive hypotheses: towards efficient optimization in proof theory. In *Proceedings of the 18th International Conference on Soft Computing*, pages 320–325, Brno, Czech Republic, 2012.

[3] A. Wiśniewski. Erotetic search scenarios. *Synthese*, 134(3):389–427, 2003.

[4] A. Wiśniewski. Questions, *Inferences and Scenarios*, volume 46 of *Studies in Logic. Logic and Cognitive Systems.* College Publications, London, 2013.

Probabilistic Interpretations of Predicates

Janusz Czelakowski (EN)
University of Opole
Insitute of Mathematics and Informatics
Poland
jczel@math.uni.opole.pl

In classical logic, every m-ary predicate is interpreted as an m-argument two-valued relation defined on a non-empty universe. In probability theory, m-ary predicates are interpreted as probability measures on the m-th power of a probability space. m-ary probabilistic predicates are equivalently semantically characterized as m-dimensional cumulative distribution functions defined on $\mathbb{R}^m$. The talk is mainly concerned with probabilistic interpretations of unary predicates (attributes) in the algebra of cumulative distribution functions defined on $\mathbb{R}$. This algebra, enriched with two constants, forms a bounded De Morgan algebra. Logical systems based on the algebra of cumulative distributions are defined and their basic properties are isolated. Comparisons with the infinitely-valued Łukasiewicz logic are also discussed.

References

[1] Billingsley, P., *Probability and Measure*, John Wiley & Sons, Inc., New York 1995.

[2] Billingsley, P., *Convergence of Probability Measures*, John Wiley & Sons, Inc., New York 1999.

[3] Davey, B.A. and Priestley, H.A., *Introduction to Lattices and Order*, Cambridge University Press, Cambridge 2002.

[4] Dunn, J.M. and Hardegree, G.M., *Algebraic Methods in Philosophical Logic* (Oxford Logic Guides, Oxford Science Publications, Volume 41), New York 2001, Oxford University Press.

[5] Ganter, B., Stumme, G., Wille, R. (eds.), *Formal Concept Analysis: Foundations and Applications*, Lecture Notes in Artificial Intelligence, No. 3626, Springer-Verlag, Berlin 2005.

[6] Hájek, P., *Metamathematics of Fuzzy Logics*, Kluwer, Dordrecht 1998.

[7] Wójcicki, R., *Theory of Logical Calculi. Basic Theory of Consequence Operations*, Kluwer, Dordrecht 1988.

Defeasible Inferences for Free Choice Permission with Substructural Logics

Huimin Dong (EN)
Department of Philosophy, University of Bayreuth, Germany
ellutung@gmail.com

Norbert Gratzl (EN)
Munich Center for Mathematical Philosophy, Ludwig-Maximilians Universität, Germany
norbertgratzl@gmx.net

This talk aims to accommodate the notion of free choice permission firstly defined by von Wright [7] into defeasible reasoning, such that the inference of free choice permission is only licensed in the *normal* cases. As von Wright mentioned, "one could then admit that *normally* the permission '$P(p \vee q)$' goes together with the conjunction of permissions '$Pp \wedge Pq$', but that this is not logical *neccessity* [8, p.21–p.22]." One example fails the feasible inferences is the "vegetarian free lunch" given by Hansson [5]. Though it is permitted to order a vegetarian lunch, it is not permitted to order it and not pay for it. Free choice permission deserves logicians' attention in a defeasible perspective.

The semantic core of free choice permission suggested in this talk is a followed-up of the accounts on free choice permission called "open interpretation" [3,6], "open reading" [1,2] and "free choice permission as the sufficient condition of rationality" in [4]: An action is free-choice permitted iff all executions of this action will lead to legal states. Adopting this semantic core, the inferences of free choice permission hence will depend on the execution-relations between actions.

We suggest this semantic core because of two reasons. First it figures out that the counter-intuitive *feasible condition* of free choice permission: *All executions of a conjunctive action A and B are the executions of its conjunct action A.* Yet the execution of ordering a lunch and not paying for it is not a *normal* execution of ordering a lunch. This condition is derived by applying the *weakening* rule, and works as a premise in the *feasible* inferences of free choice permission. The feasible condition and the weakening rule hence should be failed in the *defeasible* inferences of free choice permission, otherwise "it is permitted to do A then it is permitted to do A and B" for arbitrary action B arise as the consequences in the inferences. Second it sorts out various frame conditions of the execution-relation corresponding to related defeasible inference rules. Two kinds of rules will be discussed. The mingle rule is a defeasible rule related to particular defeasible inferences, and the non-monotonic rules called cautious monotony and rational monotony are rules that related to more general defeasible reasoning cases. For instance, the mingle rule is mainly used to settle the resource sensitive inferences: "If it is permitted to eat one cookie then it is permitted to eat more than one," when the situation is not resource sensitive to the cookies. The non-monotonic rules are not only used in the resource sensitive inferences, but also the non-monotonic inferences like "it is permitted to order a lunch then it is permitted to order a lunch and pay for it."

At last, we will present substructural logics for the defeasible inferences of free choice permission. Substructural logics are logics that able to be weaker than the classical logics, especially the classical logics that contain the weakening rule. We therefore are able to go for substructural logics to figure out the frame conditions correspond to the rules used in the defeasible inferences of free choice permission.

References

[1] Albert J.J. Anglberger, Huimin Dong, and Olivier Roy. Open reading without free choice. In Fabrizio Cariani, Davide Grossi, Joke Meheus, and Xavier Parent, editors, *Deontic Logic and Normative Systems*, volume 8554 of *Lecture Notes in Computer Science*, pages 19–32. Springer International Publishing, 2014.

[2] Albert J.J. Anglberger, Nobert Gratzl, and Olivier Roy. Obligation, free choice, and the logic of weakest permissions. *The Review of Symbolic Logic*, 8:807–827, 12 2015.

[3] Jan Broersen. Action negation and alternative reductions for dynamic deontic logics. *Journal of applied logic*, 2(1):153–168, 2004.

[4] Huimin Dong and Olivier Roy. Three deontic logics for rational agency in games. *Studies in Logic*, 8(4):7–31, 2015.

[5] Sven Ove Hansson. The varieties of permissions. In Dov Gabbay, John Horty, Xavier Parent, Ron van der Meyden, and Leon van der Torre, editors, *Handbook of Deontic Logic and Normative Systems*, volume 1. College Publication, 2013.

[6] Robert Trypuz and Piotr Kulicki. Towards metalogical systematisation of deontic action logics based on boolean algebra. In *Deontic Logic in Computer Science*, pages 132–147. Springer, 2010.

[7] Georg Henrik von Wright. *Norm and Action – A Logical Enquiry*. Routledge, 1963.

[8] Georg Henrik von Wright. An essay in deontic logic and the general theory of action, 1968.

Projective Unification and Structural Completeness in Superintuitionistic Predicate Logics. Part I

Wojciech Dzik (EN)
Silesian University, Katowice
Institute of Mathematics
Poland
wojciech.dzik@us.edu.pl

Piotr Wojtylak (EN)
University of Opole, Opole
Institute of Mathematics and Computer Science
Poland
piotr.wojtylak@gmail.com

Key words: unifiers, projective unifiers, admissible rules, passive rules, almost structural completeness, superintuitionistic predicate logics.

Projective unifiers were introduced by S. Ghilardi in propositional (intuitionistic and modal) logics, and applied to admissibility of rules.

A *unifier for* a formula φ in a logic L is a substitution σ such that $\vdash_L \sigma(\varphi)$. A unifier σ for φ is a *projective unifier* for φ in L if, for each $x \in \mathrm{Var}(\varphi)$, $\varphi \vdash_L \sigma(x) \leftrightarrow x.$. L has projective unification if every unifiable formula has a projective unifier in L.

The above definitions from propositionla logic are formulated for superintuitionistic predicate logics. We use substitutions for atomic formulas (endomorphisms modulo bounded variables) by W.A. Pogorzelski and T. Prucnal. Let $\mathbf{Q}-\mathbf{LC}$ be the Gödel-Dummett predicate logic, i.e. intuitionistic predicate logic $\mathbf{Q}-\mathbf{INT}$ plus $(A \to B) \vee (B \to A)$ and let $\mathbf{IP}$ be
(IP) $(A \to \exists_x B(x)) \to \exists_x (A \to B(x))$, (Independence of Premises)

THEOREM. For every superintuitionistic predicate logic L,
L have projective unification iff $\mathbf{IP}.\mathbf{Q}-\mathbf{LC} \subseteq L$.

COROLLARY. If $\mathbf{IP}.\mathbf{Q}-\mathbf{LC} \subseteq L$, then L is almost structurally complete, i.e. every admissible rule with unifiable premises is derivable, or, every admissible rule is either passive or derivable.
Moreover an explicit basis for all passive rules and a criterion for non-uniafibility of formulas is provided.

Counting Some Closure Operations

Szymon Frankowski (EN)
University of Łódź
Department of Logic
Poland
frankowski@filozof.uni.lodz.pl

Closure operators are considered in various areas of mathematics, logic and computer science. As a basis one can take the following conditions for such a closure (where x, y are subsets of $\{1, 2, \ldots, n\}$):

(i) $x \subseteq c(x)$;

(ii) if $x \subseteq y$, then $c(x) \subseteq c(y)$;

(iii) $c(c(x)) \subseteq c(x)$;

(iv) $c(x \cup y) = c(x) \cup c(y)$;

(v) $c(0) = 0$;

(vi) if $c(x) = c(y)$, then $c(x \cup y) = c(x)$.

It is well known that the number of topological closures on $\mathcal{P}(\{1, 2, \ldots, n\})$ (Boolean algebra with n-atoms) is equal to the number of quasi-orders on n-element set. The natural correspondence between these two classes establishes an isomorphism from T_0 topologies into partial orders on a set having n-elements. In [1] some other types of operators on $\mathcal{P}(\{1, 2, \ldots, n\})$ have been counted.

We are going to provide few theorems relating to the numbers of closure like operators. The most important results of the presentation concerns comparing the number of ordinary monotonic operations with the number of the so called *p-closure operations* (i.e. operations fulfilling (i) and (ii)) and p-closure operators to the number of ordinary closure operators (i.e. fulfilling (i),(ii),(iii)) when the number of atoms tends to infinity.

References

[1] Malinowski, J., 'On the number of quasi modal algebras', *Bulletin of the Section of Logic* 14/3:99–102, 1985.

Modal Companion of Intuitionistic Control Logic

Anna Glenszczyk (EN)
University of Silesia, Katowice
Institute of Mathematics
Poland
anna.glenszczyk@us.edu.pl

Intuitionistic Control Logic (ICL) has been lately introduced by Ch. Liang and D. Miller. This logic arises from Intuitionistic Propositional Logic (IPL) by extending the language of IPL by additional new constant for falsum. The new constant requires a simple but significant modification of intuitionistic logic both proof-theoretically and semantically. Intuitionistic Control Logic has natural deduction proof system *NJC* which is sound and complete with respect to the Kripke semantics. A Kripke model for ICL, called r-model, is based on a rooted Kripke frame with a partial ordering relation on the set of possible worlds. The only difference between forcing rules in r-models and those of regular Kripke models for intuitionistic logic is in regard to the additional constant. All worlds properly above the root force that constant, but not the root itself.

There is a well-known embedding of Intuitionistic Propositional Logic into modal logic S4 by means of Gödel-Tarski translation. As ICL is an extension of IPC, we would like to present a translation of ICL into a (second order) modal logic and prove an appropriate analogon of the acclaimed Tarski Theorem.

References

[1] A. Glenszczyk, *Negational Fragment of Intuitionistic Control Logic*, Studia Logica, vol. 103, issue 6, pp 1101–1121, Springer, 2015.

[2] C. Liang, D. Miller, *An intuitionistic Control Logic*, to appear.

[3] C. Liang, D. Miller, *Unifying classical and intuitionistic logics for computational control*, Proceedings of LICS, 2013.

Tautology Elimination Rule

Andrzej Indrzejczak (EN)
University of Łódź
Department of Logic
Poland
indrzej@filozof.uni.lodz.pl

Tautology elimination rule (TER) was applied originally in the framework of Davis Putnam procedure and is still effectively used in automated theorem proving. Recently its applications in sequent calculi were considered by e.g. Lyaletsky, Brighton, Tourlakis. In this setting the rule has the form: $\top, \Gamma \Rightarrow \Delta \vdash \Gamma \Rightarrow \Delta$. It is provably equivalent to cut rule and in some versions of sequent calculi the proof of its admissibility may be simpler than proofs of cut admissibility. The natural question is if we can prove admissibility of TER for cases where proofs of admissibility of cut fail. Some version of sequent calculus for S5 which is known to be cut-free (by semantical proof) is examined. It is shown that we can prove for it admissibility of TER although the proof of cut admissibility fails to hold.

Forcing, Exotic Smoothness, and Physics

Paweł Klimasara, Krzysztof Bielas, Jerzy Król (EN)
Silesian University, Katowice, Institute of Physics, Poland
p.klimasara@gmail.com, krzysiek.bielas@gmail.com, iriking@wp.pl

Torsten Asselmeyer-Maluga (EN)
German Aerospace Center (DLR), Berlin, Germany
torsten.asselmeyer-maluga@dlr.de

Nowadays physics is based on two fundamental theories. Quantum mechanics (QM) grasps the laws of nature in micro-scale whilst general relativity (GR) serves as a description of spacetime in macroscale. Unfortunately, we do not have a consistent relation between them. For example, the predictions of particle physics (based on QM) of so called cosmological constant (CC) differ from the macroscopic cosmological observations (related to GR) by more than 100 orders of magnitude! We propose and investigate the formal description of micro- to macroscale shift using the tools of mathematical logic and smoothness structures in dimension 4. In [1] we related such shift to random forcing. The discrepancy of CC prediction in particle physics with observations comes from vastly overestimated contributions (to CC) from zero-modes of quantum fields. In our approach such contributions completely vanish [2]. Nevertheless, the CC cannot be zero to fit the experimental data. Realistic value of CC (along with other cosmological parameters) can be obtained if we use exotic smooth $\mathbb{R}^4$ (homeomorphic but not diffeomorphic to the standard $\mathbb{R}^4$) as a manifold describing the spacetime in GR ([3]). Random forcing, here emerging from QM, is based on measure algebra (Borel subsets of $\mathbb{R}$ modulo null sets). Under Continuum Hypothesis, one obtains the duality between measure and category (the Sierpiński-Erdös theorem). Unambiguous mapping of null sets into meager ones (1^{st} Baire category sets) allows us to switch measure algebra to Cohen algebra (Borel subsets of $\mathbb{R}$ modulo meager sets). We are considering the change of ZFC models underlying the formalism of QM and hence we face the varying structure of the real line. Transition into Cohen forcing allows interpreting real numbers from a model and its Cohen extension as absolute subtrees of the binary tree (Cantor space). These trees are spanning nontrivial Casson handles of smooth exotic 4-manifolds (especially $\mathbb{R}^4$). Accordingly, ZFC based QM and exotic smooth spacetime constitute complementary description of the micro- and macroscale relation. We propose them as the foundations of the forcing-based cosmological model and discuss the consequences of such description of the Universe. Furthermore, connecting abstract approach to real numbers with parameters in reality can raise new questions and possibilities in the related areas of the philosophy of science.

References

[1] P. Klimasara, J. Król: *Remarks on mathematical foundations of quantum mechanics*, Acta Phys. Pol. B, 46, No7, 1309 (2015).

[2] K. Bielas, P. Klimasara, J. Król: *The structure of the real line in quantum mechanics and cosmology*, Acta Phys. Pol. B, 46, No11, 2375 (2015).

[3] T. Asselmeyer-Maluga, J. Król: *Inflation and Topological Phase Transition Driven by Exotic Smoothness*, Advances in High Energy Physics Vol. 2014, 867460 (2014).

[4] J. Król: *Model and Set-Theoretic Aspects of Exotic Smoothness Structures on* $\mathbb{R}^4$, will appear in "At the Frontier of Spacetime" (edited by T. Asselmeyer-Maluga, Springer 2016).

[5] T. Asselmeyer-Maluga, C. H. Brans: *Exotic Smoothness and Physics* (World Scientific, Singapore, 2007).

Yes/No Formulae and Describing Theories of Intuitionistic Kripke Models

Małgorzata Kruszelnicka (EN)
University of Silesia, Katowice
Institute of Mathematics
Poland
malgorzata.kruszelnicka@us.edu.pl

The notion of logical equivalence still remains one of the most interesting subjects of investigation. In many logical systems the question that arises is whether the theory of a considered structure can be described by means of a single formula.

In the talk we consider Kripke semantics for intuitionistic first-order logic, and discuss the aforementioned problem. Since intuitionistic connectives differ significantly from the classical ones, one may expect a more complex representation. Thus, we will deal with two kinds of formulas that will describe a positive and a negative information of a node, respectively.

For an arbitrary node α of a Kripke model $\mathcal{K}$ we construct so-called *Yes/No Formulae* that describe the theory of α. We establish the relationship between Yes/No Formulae and the notion of logical equivalence of Kripke models. Furthermore, we focus on properties of Yes/No Formulae and their applications in describing theories of particular Kripke models.

Philosophical Theories and Their Logical Applications on the Example of the Logic of Agency of Anselm of Canterbury

Marek Lechniak & Andrzej Stefańczyk (PL)
John Paul II Catholic University of Lublin, Faculty of Philosophy, Poland
marek.lechniak@kul.pl, astefanczyk@kul.pl

Philosophical Fragments – Lambeth of St. Anselm of Canterbury are a kind of dictionary that explains the meaning of certain terms, such as: facere, velle, posse, necesse, debere. They include a discussion, conducted on the intersection of logic and ethics, of such deontic concepts as "obligation" and "goodness". Through the explication of meanings Anselm attempts to create a conceptual apparatus for a rational proofs of the main tenets of the Christian doctrine and, even more broadly, for the exegesis of the Scripture. In addition this new apparatus allows him to examine some purely philosophical topics, including free will, causation and the relationship between human freedom and divine foreknowledge. Recently attempts have been made (by D. Walton at the level of syntactic and by S. Uckelman at the level of semantics of the neighborhood) to reconstruct the logic of agency implicit in the Lambeth Fragments.

The paper will briefly introduce the main topics discussed in the Philosophical Fragments. Next we will outline and analyze the attempts to formalize its main claims by means of the system of modal logic (logic of agency), namely by the weak, classical modal logic, which is an extension of the modal system E. On the basis of this analysis we will then impose some requirements on the relations between a system of formal logic and a philosophical theory, requirements which should help to make these relations more fruitful for the history of philosophy as well as logic.

Characterization of Modularity by Means of Cover-Preserving Sublattices

Marcin Łazarz (EN)
University of Wrocław
Department of Logic and Methodology of Sciences
Poland
lazarzmarcin@poczta.onet.pl

A lattice L is called *upper continuous* if it is complete and for every element $x \in L$ and every chain $C \subseteq L$ holds

$$\text{(UC)} \qquad x \wedge \bigvee C = \bigvee \{x \wedge c : c \in C\}.$$

A lattice L is called *strongly atomic* if

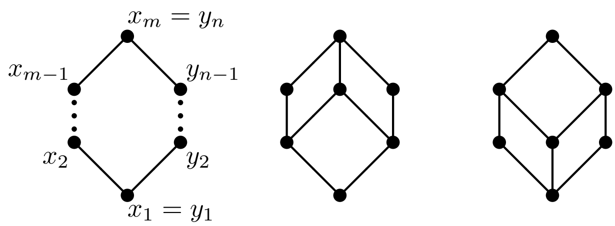
$$\text{(SA)} \qquad (\forall x, y \in L)\big(x < y \Rightarrow (\exists z \in L)(x \prec z \leq y)\big).$$

A well known result of P. Crawley and R. P. Dilworth (see [1], 3.6) states that an upper continuous and strongly atomic lattice L is modular iff L is both *upper* and *lower semimodular*, i.e. satisfies the following conditions:

$$\text{(Sm)} \qquad (\forall x, y \in L)(x \wedge y \prec x \Rightarrow y \prec x \vee y),$$

$$\text{(Sm}^*\text{)} \qquad (\forall x, y \in L)(y \prec x \vee y \Rightarrow x \wedge y \prec x).$$

A sublattice K of a lattice L is said to be *cover-preserving*, if $x \prec y$ in K implies $x \prec y$ in L, for all $x, y \in L$.



$N_{m,n}$ $\qquad$ S_7 $\qquad$ S_7^*

J. Jakubík in 1975 proved the following characterization of modularity for locally finite lattices (i.e. lattices where every bounded chain is finite):

Theorem 1 ([2]). *Let L be a lattice of locally finite length. Then L is modular if and only if it contains no cover-preserving sublattice isomorphic to S_7 nor to S_7^* nor to $N_{m,n}$ $(m \geq 4,\ n \geq 3)$.*

A straightforward consequence of Theorem 1 is an earlier result of F. Šik:

Theorem 2 ([3]). *An upper semimodular lattice of locally finite length is modular if and only if it contains no cover-preserving sublattice isomorphic to S_7.*

In the talk we present a sketch of the proof of Theorem 2 extended to the class of upper continuous and strongly atomic lattices. Moreover, constructing an appropriate counterexample we show that it is impossible to give an analogous extension of Theorem 1. Finally, we discuss some consequences.

References

[1] P. Crawley, R. P. Dilworth, **Algebraic Theory of Lattices**, Prentice-Hall, Inc., Englewood Cliffs, New Jersey 1973.

[2] J. Jakubík, Modular Lattice of Locally Finite Length, **Acta Sci. Math.**, 37 (1975), 79–82.

[3] F. Šik, A Note on the Modular Lattices of Locally Finite Length, (unpublished manuscript cited in [2]).

On the Philosophical Notion of the Universe in Leśniewski's Mereology. Supplement to Sobociński's Remarks

Marcin Łyczak (EN)
Cardinal Stefan Wyszyński University, Warsaw
Institute of Philosophy
Poland
marcinlyczak@o2.pl

The class of all objects was called by Stanisław Leśniewski 'universe' and he proved its existence and uniqueness in frame of his early mereology. In works devoted to mereology Lesniewski did not give any explicit philosophical interpretation of the introduced notion. However, its philosophical connotations were considered by B. Sobociński in his unpublished correspondence with I. M. Bocheński. The point of our lecture is to present ideas proposed by Sobociński and in particular to reconstruct proofs for theorems formulated by him in the considered letter. We are going to show that the notion of the universe has different meanings in original mereology, atomistic mereology, mereology with existential theorems and on the ground of axiomatic set theory enriched by notions of part and mereological sum. It comes out that in frame of the considered theories the universe is an object with the characterization which does not bring any philosophically interesting vision of 'totum reale'.

References

[1] Leśniewski, S. (1916), "Podstawy ogólnej teorii mnogości I", Prace Polskiego Koła Naukowego w Moskwie, Sekcya matematyczno-przyrodnicza, No. 2. Moskwa.

[2] Correspondence between Sobociński and Bocheński. Copies of manuscripts.

[3] M. Porwolik, M. Łyczak, K. Świętorzecka (2016), , in print.

Truth and Existence, a Comment to the Correspondence Theory of Truth

Marek Magdziak (PL)
University of Wrocław
Department of Logic and Methodology of Sciences
Poland
mmagdziak@tlen.pl

The two oldest theories of truth in western philosophy, those of Plato and Aristotle are both correspondence theories. The correspondence theory of truth is a theory which connects the opposition between the true one and the false one with the opposition between the existing one and the fictional one. According to Plato and Aristotle, *to say of what is that it is not, or of what is not that it is, is false, while to say of what is that it is, and of what is not that is not, is true* (Plato, Cratylus 385 b 2, Sophist 263 b, Aristotle, Metaphysics 1011 b 25). The lecture provides a tentative formal logical analysis of the concept of truth by Plato, Aristotle and Polish philosophers Twardowski, Czeżowski, Ajdukiewicz and Kotarbiński. It is stressed that the formulas of by Plato, Aristotle, Twardowski, Czeżowski, Ajdukiewicz and Kotarbiński suggest at least three different definitions of truth.

The Connective "ewentualnie" in Polish

Elżbieta Magner (PL)
University of Wrocław
Department of Logic and Methodology of Sciences
Poland
dr.em@wp.pl

The Polish word "ewentualnie" is most frequently viewed as an adverb as well as a connective (equivalent to "possibly" in English). In this paper I will deal with the connective form of the word "ewentualnie".

I will be looking for an answer to the question whether the connective form of "ewentualnie" can be a natural language equivalent to one of the functors in logic.

A Few Notes on the Connective "względnie" and the Connective "ewentualnie" in Polish

Elżbieta Magner (PL)
University of Wrocław
Department of Logic and Methodology of Sciences
Poland
dr.em@wp.pl

The Polish words "względnie" and "ewentualnie" in their connective meaning (equivalent to "or" and "possibly" in English) are often viewed as synonymous. Looking for natural language equivalents to various logical functors I focused on these two connectives. In this paper I offer some thoughts regarding this issue.

Monotone Operations Designated by Matrices with Mappings

Marek Nowak (EN)
University of Łódź
Department of Logic
Poland
marnowak@filozof.uni.lodz.pl

The notion of a generalized matrix $((A, o_1, \ldots, o_n), \mathcal{B})$ $(\mathcal{B} \subseteq \wp(A))$ for a given propositional language $(S, s_1, \ldots, s_n)$ is enriched by a new parameter which is any mapping $f : \wp(A) \longrightarrow \wp(A)$. Such a new structure $((A, o_1, \ldots, o_n), \mathcal{B}, f)$ will be called a *matrix with a mapping, an m-matrix* in short. In general any m-matrix induces a monotone operation $M : \wp(A) \longrightarrow \wp(A)$, in the following way. For any $a \in A$ and $E \subseteq A$,

$a \in M(E)$ iff for every $D \in \mathcal{B}$ $(E \subseteq D \Rightarrow a \in f(D))$, that is,

$M(E) = \bigcap\{f(D) : D \in \mathcal{B} \ \&\ E \subseteq D\}$.

Such an operation M will be called *designated by an m-matrix*. It can be translated on propositional language by projective generation:

$F(X) = \bigcap\{\overleftarrow{h}(M(\overrightarrow{h}(X))) : h \in Hom\}$, for any $X \subseteq S$,

where Hom is the class of all homomorphisms from the language to the algebra of m-matrix. In case f is the identity function of $\wp(A)$, the mapping F is the ordinary consequence operation designated on the propositional language by the generalized matrix $((A, o_1, \ldots, o_n), \mathcal{B})$.

Next we generalize the notions to the following ones. By an m-matrix we mean a structure $(A, \leq, B, f)$, where $(A, \leq)$ is any complete lattice, $B \subseteq A$ and $f : A \longrightarrow A$ is any mapping. We shall consider the bundle of all such m-matrices having the same lattice $(A, \leq)$ and different in the parameters B, f. In this way the following mappings $\underline{M}, \overline{M} : A^A \times \wp(A) \longrightarrow A^A$ are considered. For any map $f : A \longrightarrow A$ and any $B \subseteq A$, the operations $\underline{M}(f, B), \overline{M}(f, B)$ from A to A are designated by the m-matrix $(A, \leq, B, f)$, i.e., defined for any $a \in A$ by

$\underline{M}(f, B)(a) = \inf \overrightarrow{f}(B \cap [a)) = \inf\{f(x) : x \in B \ \&\ a \leq x\}$,

$\overline{M}(f, B)(a) = \sup \overrightarrow{f}(B \cap (a]) = \sup\{f(x) : x \in B \ \&\ x \leq a\}$.

When $B = A$ is fixed such the operations $\underline{M}$, $\overline{M}$ have already been considered in [1].

Given fixed $B \subseteq A$, the mappings $\underline{M}_B$, $\overline{M}_B : A^A \longrightarrow Mon(A)$, defined for any $f \in A^A$ by $\underline{M}_B(f) = \underline{M}(f, B)$ and $\overline{M}_B(f) = \overline{M}(f, B)$ are considered. When restricted to the class $Mon(A)$ of monotone mappings from A to A they are closure and interior operators, respectively.

On the other hand, given fixed any monotone map $f : A \longrightarrow A$ one may consider the functions $\underline{M}_f, \overline{M}_f : \wp(A) \longrightarrow Mon(A)$ defined for any $B \subseteq A$ by $\underline{M}_f(B) = \underline{M}(f, B)$ and $\overline{M}_f(B) = \overline{M}(f, B)$. The pairs $(\phi_f, \underline{M}_f)$, $(\psi_f, \overline{M}_f)$

form an antimonotone and monotone Galois connections for the complete lattices $(Mon(A), \leq)$, $(\wp(A), \subseteq)$, respectively. Here, for any monotone map α from A to A, $\phi_f(\alpha) = \{x \in A : \alpha(x) \leq f(x)\}$ and $\psi_f(\alpha) = \{x \in A : f(x) \leq \alpha(x)\}$.

References

[1] Cousot P., Cousot R., *A constructive characterization of the lattices of all retractions, preclosure, quasi-closure and closure operators on a complete lattice*, Portugaliae Mathematica 38(1979) 185–198.

Deontic Action Logic Using Substructural Systems

Clayton Peterson (EN)
Munich Center for Mathematical Philosophy Ludwig-Maximilians, Universität Munchen, Germany
clayton.peterson@outlook.com

Piotr Kulicki (EN)
Faculty of Philosophy, John Paul II Catholic University of Lublin, Poland
kulicki@kul.pl

This talk extends the results presented in [2,3] and explores how new paradoxes arise in various substructural logics used to model conditional obligations. Our investigation starts from the comparison that can be made between monoidal logics and Lambek's [1] analysis of substructural logics, who distinguished between four different ways to introduce a (multiplicative) disjunction. While Lambek's analysis resulted in four variants of substructural logics, namely BL1, BL1(a), BL1(b) and BL2, we show that these systems are insufficient to model conditional obligations insofar as either they lack relevant desirable properties, such as some of de Morgan's dualities or the law of excluded middle, or they satisfy logical principles that yield new paradoxes. To answer these concerns, we propose an intermediate system that is stronger than BL1 but weaker than BL1(a), BL1(b) and BL2.

References

[1] Joachim Lambek, From categorial grammar to bilinear logic, Substructural Logics (P. Schroeder-Heister and K. Došen, eds.), Studies in Logic and Computation, vol. 2, Oxford Science Publications, 1993, pp. 207–237.
[2] Clayton Peterson, Contrary-to-duty reasoning: A categorical approach, Logica Universalis 9 (2015), no. 1, 47–92.
[3] Clayton Peterson, The categorical imperative: Category theory as a foundation for deontic logic, Journal of Applied Logic 12 (2014), no. 4, 417–461.

A Few Notes on λ-Computation and TIL-Construction

Ivo Pezlar (EN)
Masaryk University
Department of Philosophy, Faculty of Arts,
Czech Republic
ivo.pezlar@gmail.com

Lambda calculus (abbr. LC) originally developed by (Church, 1932) can be regarded as the most universal tool for expressing computations (Turing, 1937). Its fundamental computation rule, so called β-reduction, is defined in terms of substitution and it embodies the idea of function application. Consequently, each application of β-reduction rule can be regarded as a single computational step.

There is, however, at least one formal system utilizing lambda calculus in which these correspondences (roughly put, computational step $\approx$ β-reduction $\approx$ function application) do not hold. It is called tranparent intensional logic (abbr. TIL) and it was developed by (Tichý, 1988). I will, however, focus on one of its later variants found in (Duží, Jespersen, Materna, 2010).

In the present talk I will examine this deviation from standard lambda calculus and explore the outcomes it entails for the corresponding system.

For example: (1) In TIL function application is done via so called Composition construction. (Duží, Jespersen, Materna, 2010) state: "($\mathbf{Q_1}$) *Composition* $[X\,Y_1...Y_m]$ *v-constructs* the value [...] of [a function] f [v-constructed by X] on the tuple-argument $\langle \mathrm{B}_1, ..., \mathrm{B}_m \rangle$ [v-constructed by $Y_1...Y_m$]." (p. 45) Yet they also claim that: "($\mathbf{Q_2}$) In TIL, β-reduction is the rule for computing the value of a [...] function v-constructed by $[\lambda x_i\, Y]$ at an argument v-constructed by D_i." (p. 269)

Let's suppose that $[\lambda x_i\, Y]$ is Closure construction $[\lambda x\, [{}^0 Succ\, x]]$ that v-constructs successor function $Succ$ and that D_i is Trivialization construction ${}^0 0$ that v-constructs number 0.[3] If we want to apply this function $Succ$ to argument 0, we have to form Composition construction (see $\mathbf{Q_1}$) $[[\lambda x\, [{}^0 Succ\, x]]\, {}^0 0]$. By the quote $\mathbf{Q_2}$, β-reducing this construction should compute the value of the successor function at argument 0. This is, however, not the case, because the above Composition is β-reducible to $[{}^0 Succ\, {}^0 0]$. which is in TIL just another Composition construction and not the value of function $Succ$ at argument 0. Therefore, the above specification of β-reduction for TIL in $\mathbf{Q_2}$ is incorrect.

(2) In TIL constructions represent computations.[4] Hence, when carrying out β-reduction, e.g., $[[\lambda x\, x]\, {}^0 1] \twoheadrightarrow_\beta^{\mathrm{TIL}} {}^0 1$ we are just transforming one computation into another (both of which v-construct number 1). No evaluation actually

[3]The superscript '^{0}' represents in TIL so called Trivialization construction that "calls in" objects, e.g., ${}^0 0$ can be roughly understood as instruction "take number 0".

[4](Tichý, 1988) repeatedly likens constructions to calculations (see e.g., p. 7, p. 12, p. 20, p. 31, p. 82, p. 222, p. 281). The more general term *computation*, however, can be no doubt used as well, considering Tichý "calculates" also truth values, individuals, etc.

takes place, because ${}^0 1$ is not the resulting value of $[[\lambda x\, x]\, {}^0 1]$. However, in standard lambda calculus the β-reduction $\lambda x.x\, 1 \twoheadrightarrow^{\mathrm{LC}}_{\beta} 1$ would be considered as a computation yielding a value. We are evaluating the term $\lambda x.x\, 1$ to a simpler one 1, which is not possible to evaluate any further. Hence, the notion of computation native to lambda calculus is very different from the notion of construction from TIL.

From the above considerations, it follows that TIL implicitly relies on two distinct notions of computations: (A.) syntactic computation (e.g., the step from $[[\lambda x\, x]\, {}^0 1]$ to ${}^0 1$) and (B.) semantic (constructional) computation (e.g., the step from $[[\lambda x\, x]\, {}^0 1]$ to 1).

Schematically:
$$\begin{tikzcd}[[\lambda x\, x]\, {}^0 1] \arrow[rr, "\beta"] \arrow[dr, "v"'] & & {}^0 1 \arrow[dl, "v"] \\ & 1 & \end{tikzcd}$$
where β represents the λ/syntactic computational step ($[[\lambda x\, x]\, {}^0 1]$ is β-reducible to ${}^0 1$) and v the TIL/semantic constructional steps (both $[[\lambda x\, x]\, {}^0 1]$ and ${}^0 1$ v-construct 1).

References

[1] Church, A. (1932). A Set of Postulates for the Foundation of Logic. *Annals of Mathematics*, 33(2), 346–366.

[2] Duží, M., Jespersen, B., Materna, P. (2010). *Procedural Semantics for Hyperintensional Logic: Foundations and Applications of Transparent Intensional Logic*. Netherlands: Springer.

[3] Tichý, P. (1988). *The Foundations of Frege's Logic*. Berlin, New York: de Gruyter.

[4] Turing, A. (1937). Computability and λ-definability. *Journal of Symbolic Logic*, 4, 153–163.

American Postulate Theorists and Their Influence on the Foundations of Mathematics

Jerzy Pogonowski (EN)
Adam Mickiewicz University, Poznań
Department of Applied Logic
Poland
pogon@amu.edu.pl

We are going to discuss some results obtained by a few American mathematicians in the first three decades of the XXth century and published in the *Transactions of the American Mathematical Society*. The works in question are devoted to the foundations of mathematics and their authors are called *American Postulate Theorists*. The prominent among them are: Eliakim H. Moore, Oswald Veblen, Edward Huntington, Leonard Dickson. They have proposed several collections of postulates characterizing fundamental mathematical structures: groups, rings, fields, real numbers, complex numbers, systems of geometry (Euclidean and non-Euclidean, projective, etc.). They were inspired by the (earlier or contemporary) works by Peano, Dedekind, Pasch and Hilbert but they have developed their own style of dealing with foundational problems. Similarly to Hilbert, they have always tried to show that the postulates are mutually independent. Worth noticing is their choice of primitive notions, first of all in the case of systems of geometry. However, the most important in their approach are efforts to obtain uniqueness of the domains characterized by the postulates. Veblen has introduced the concept of categoricity and Huntington used the term sufficiency for the situation when a model is unique up to isomorphism. Both of them have articulated very interesting remarks concerning different forms of completeness, including a modest suspicion that syntactic and semantic aspects of completeness may not coincide. Notice that a precise metalogical notion of completeness was not well established at that time. Finally, they have also commented on Hilbert's axiom of completeness in geometry. This last topic is of special interest to us because the work on this paper is being sponsored by the National Science Foundation research grant nr 2015/17/B/HS1/02232 *Extremal axioms: logical, mathematical and cognitive aspects*.

References

[1] Awodey, S. Reck, E.H. 2002. Completeness and Categoricity. Part I: Nineteenth-century Axiomatics to Twentieth-century Metalogic. *History and Philosophy of Logic* **23**, 1–30.

[2] Corcoran, J. 1981. From Categoricity to Completeness. *History and Philosophy of Logic* **2**, 113–119.

[3] Scanlan, M. 1991. Who were the American Postulate Theorists? *The Journal of Symbolic Logic* Volume **56**, Number 3, 981–102.

Classically Archetypal Rules

Tomasz Połacik (EN)
University of Silesia, Katowice
Institute of Mathematics
Poland
polacik@math.us.edu.pl

Joint work with
Lloyd Humberstone, Monash University, Australia

We consider a propositional logic L with the consequence relation $\vdash_{\mathsf{L}}$. We say that an inference rule is archetypal for L if it is derivable and implicitly subsumes every other derivable rule which is derivable in L. So, archetypal rules are, in a sense, the most general ones for the given logic. More formally, a derivable rule $A_1, \ldots, A_n/B$ is archetypal for L if for any other derivable rule

$C_1, \ldots, C_n/D$ there is a substitution σ such that for all $1 \leq i \leq n$, we have $\vdash_{\mathsf{L}} \sigma(A_i) \leftrightarrow C_i$ and $\vdash_{\mathsf{L}} \sigma(B) \leftrightarrow D$.

In this talk we are concerned with the consequence relation of classical propositional logic and provide a complete and informative characterization of the set of archetypal rules in this case. To this end, we introduce the notion of exactly valid rule, and show that the class of classically archetypal rules coincides with that of exactly valid rules. The description of the notion of archetypal rule in the case of classical propositional logic gives us some insight into the problem of archetypal rules for intuitionistic logic and intermediate logic. However, in theses cases the problem is still open.

References

[1] Lloyd Humberstone, *Archetypal Forms of Inference*, Synthese, 141(1), 45–76, 2004.

[2] Tomasz Połacik, *The Unique Intermediate Logic Whose Every Rule is Archetypal*, *The Logic Journal of the IGPL*, 13(3), 269–275, 2005.

[3] Tomasz Połacik, *Archetypal Rules and Intermediate Logics*, pp. 227–237 in: M. Peliš and V. Punčochař (eds.), *The Logica Yearbook 2011*, College Publications, London 2012.

[4] Tomasz Połacik, Lloyd Humberstone, *Classically Archetypal Rules.* Submitted.

Conditional Definitions in Zdzisław Augustynek's Axiomatic Approaches to Genidentity

Marek Porwolik (EN)
Cardinal Stefan Wyszyński University, Warsaw
Institute of Philosophy
Poland
m.porwolik@uksw.edu.pl

The existence of objects in time or, generally speaking, the existence of objects subject to changes can be described with the use of the notion of genetic identity (*genidentity*). Zdzisław Augustynek devoted a number of his works (1981, 1984, 1996, 1997a, b) to this issue. Valuable commentaries to these works were provided by Mariusz Gryganiec (2005a, b, 2007, 2011a, b). Augustynek tried to specify this notion using axiomatic definitions expressed in the language of the algebra of sets. In this way configurations of axioms were created, which he himself called systems. He presented three systems of this kind. In particular axioms, besides the term *genidentity* (G), also the following notions are used: *logical identity* (I), *quasi-simultaneity* (R), *quasi-collocation* (L), and *causality* (H). They represent binary relations whose field is the set of events S. In the axioms, also symbols of the complements of these relations are used. They include: *genetic difference* (G^*), *logical difference* (I^*), *time separation* (R^*), *space separation* (L^*), and *complement of the relations H* ($H*$).

In one of his works (1997a), Augustynek posed a number of questions regarding the possibility to formulate conditional definitions of certain type, which might refer to the notions included in his axioms. He did not answer all of these questions. We are going to do this and complete his research in this way. Apart from that, our aim is to analyze the problem of reducing the above-mentioned systems to conditional definitions containing the necessary condition and the sufficient condition of a selected notion from these systems. At the same time, we are going to prove that Augustynek's systems can be reduced to certain conditional definitions (that they are equivalent to them), including the ones containing two conditions of genidentity: the sufficient condition and the necessary condition.

References

[1] Augustynek Z. (1981), *Genidentity*, "Dialectics and Humanism" 1, 193–202.

[2] Augustynek Z. (1984), *Identyczność genetyczna*, "Studia filozoficzne" 2(219), 31–42.

[3] Augustynek Z. (1996), *Relacje czasoprzestrzenne*, "Filozofia nauki" 4(16), 7–19.

[4] Augustynek Z. (1997a), *Wspólna podstawa czasu i przestrzeni* [in:] Czasoprzestrzeń. Eseje filozoficzne, Warszawa: WFiS UW, 51–57.

[5] Augustynek Z. (1997b), *Substancja – przyczynowość – przestrzeń – czas* [in:] Czasoprzestrzeń. Eseje filozoficzne, Warszawa: WFiS UW, 99–111.

[6] Grygianiec M. (2005a), *Variants and Criteria of Genidentity*, "Logic, Methodology and Philosophy of Science at Warsaw University" 2, 161–171.

[7] Grygianiec M. (2005b), *Genidentyczność a metafizyka persystencji*, "Filozofia nauki" 2(50), 87–102.

[8] Grygianiec M. (2007), *Identyczność i trwanie. Studium ontologiczne*, Warszawa: Semper.

[9] Grygianiec M. (2011a), *Aksjomatyczne definicje genidentyczności*, "Filozofia nauki" 1(73), 25–37.

[10] Grygianiec M. (2011b), *Trwanie w czasie* [in:] Przewodnik po metafizyce, S. T. Kołodziejczyk (ed.), Kraków: WAM, 211–276.

Combining Direct and Inferential Negation[5]

Adam Přenosil (EN)
Czech Academy of Sciences,
Institute of Computer Science, Prague
Czech Republic
adam.prenosil@gmail.com

In this contribution, we shall investigate a logic introduced in [2] which combines the direct, non-inferential negation of the Belnap–Dunn logic [1] with the inferential negation found in Johansson's minimal logic, and compare it with a

[5]This work was supported by the grant P202/12/G061 of the Czech Science Foundation.

related logic introduced by Vakarelov [3] which combines the Nelson negation with the same inferential negation.

By an inferential negation $\sim\varphi$ of a formula φ, we mean a formula of the form $\varphi \to 0$, where $\to$ is a binary connective representing implication (in our case, intuitionistic implication) and 0 is a constant representing contradiction. To assert such an inferential negation is therefore to state that assuming φ yields a contradiction. In other words, the unary operator $\sim$ allows us to formalize negation in the sense of *reductio ad contradictionem*.

By constrast, the de Morgan negation of a formula φ, denoted $-\varphi$, is a primitive notion. To assert $-\varphi$ is to deny φ, and assertion and denial are taken to be co-primitive pragmatic notions. The connection between the two negation is then the following: $\varphi, -\varphi \vdash 0$ (the same condition is used in [3]). In particular, we have $-p \vdash \sim p$ but $\sim p \nvdash -p$.

We shall introduce an algebraic and relational semantics for this logic and establish some of its basic properties, including local tabularity (local finiteness) and the deduction-detachment theorem (the EDPC property in algebraic terms). Interestingly enough, this logic turns out to be incomparable with Vakarelov's logic, even if we restrict our attention to theorems. In other words, having an inferential negation in the language is already sufficient to distinguish between the de Morgan negation and the Nelson negation (governed by the falsity conditions cI_1 and cI_4 of [4], respectively).

References

[1] Nuel D. Belnap. A useful four-valued logic. In J. Michael Dunn and George Epstein, editors, *Modern uses of multiple-valued logic*, volume 2 of *Episteme*, pages 5–37. Springer Netherlands, 1977.

[2] Adam Přenosil. Reductio ad contradictionem: an algebraic perspective. *Studia Logica*, 2016, forthcoming.

[3] Dimiter Vakarelov. Nelson's negation on the base of weaker versions of intuitionistic negation. *Studia Logica*, 80(2):393–430, 2005.

[4] Heinrich Wansing. Proofs, disproofs, and their duals. In C. Areces and R. Goldblatt, editors, *Advances in Modal Logic 7*, pages 483–505, 2010.

Deduction in Ramified Partial Type Theory: Focus on Derivations with Type Judgements

---

Jiři Raclavský (EN)
Masaryk University, Brno
Department of Philosophy, Faculty of Arts,
Czech Republic
raclavsky@phil.muni.cz

0. Church (1940) proposed a highly expressive system of *higher-order logic* usually called *typed λ-calculus*; he devised it for foundation of mathematics (and as a rectification of his earlier type-free, but inconsistent system). From early 1970s, a family of Churchian *simple theories of types* (STTs) has became increasingly popular in computer science (cf. at least Girard et al. 1989) for STT underlies *functional programming languages*. Many *theorem provers* – fulfilling famous Leibniz's programmatic goal of *calculus ratiocinator* – use a version of STT. Montague's (1974) application of STT for *analysis of natural language* is quite famous too. Tichý (2004) proposed a rivalling approach, called Transparent Intensional Logic (TIL).

1. The talk offers an outlook on Tichý's system of *natural deduction* (with sequents) of TT (but independently on TIL). Here are some exclusive features of Tichý's TT:

- Tichý (1982) generalized STT by
  - allowing, except total function, also *partial functions*
  - allowing *multi-argument functions* (he proved their irreducibility to unary ones)
- Tichý (1988) extended his STT to his TTT – we may alternatively call it *ramified partial type theory* – by
  - its *ramification* – employing thus procedures-algorithms he called *constructions*
  - *relaxing type requirements* for applications (called *compositions*)

    The system is pretty comprehensive and complicated. Basic rules for the ramified part of TTT have been eventually proposed by Kuchyňka in (Raclavský et al. 2015).

2. In a greater focus of the talk they are *type judgements*. They are generally of form

$$t : \tau,$$

where t is a term and τ a type. Their main purpose is to control terms/their semantics within derivations – we make derivations with type judgements rather than terms alone. We compare (sometimes briefly) the following approaches to derivations with type judgements:

a. their general use (proposed in Andrews 1965)

b. the use of Curry-Howard isomorphism (e.g. Sørensen, Urzyczyn 1998), leading e.g. to recently well-renowned intuitonistic type theory (Martin-Löf 1984)

c. Raclavský's (2015) approach for TTT (extending e. by a.)

d. Pezlar's (2016) approach for TTT (only partly overlaping with c.)

e. Tichý's (1982) early 'implicit approach'.

References

[1] Andrews, P. B. (1965): *A Transfinite Type Theory with Type Variables.* North-Holland.

[2] Church, A. (1940): A Formulation of the Simple Theory of Types. *The Journal of Symbolic Logic* 5: 56–68.

[3] Girard, J. Y., Taylor, P., Lafont, Y. (1989): *Proofs and Types.* Cambridge University Press.

[4] Martin-Löf, P. (1984): *Intuitionistic Type Theory.* Bibliopolis.

[5] Montague, R. (1974): *Formal Philosophy.* New Haven, London: Yale University Press.

[6] Pezlar, I. (2016): *Investigations into Transparent Intensional Logic: Rule-based Approach.* PhD dissertation. Brno: Masarykova univerzita.

[7] Raclavský, J., Kuchyňka, P., Pezlar, I. (2015): *Transparentní intenzionální logika jako characteristica universalis a calculus ratiocinator* [Transparent Intensional Logic as Characteristica Universalis and Calculus Ratiocinator]. Brno: Munipress.

[8] Sørensen, M. H., Urzyczyn, P. (1998): *Lectures on the Curry-Howard Isomorphism.* Elsevier.

[9] Tichý, P. (1982): Foundations of Partial Type Theory. *Reports on Mathematical Logic* 14: 57–72.

[10] Tichý, P. (1988): *The Foundations of Frege's Logic.* Walter de Gruyter.

[11] Tichý, P. (2004): *Pavel Tichý's Collected Papers in Logic and Philosophy.* V. Svoboda, B. Jespersen, C. Cheyne (eds.), Dunedin: University of Otago Publisher, Prague: Filosofia.

A Recursive Definition of Arguments

Marcin Selinger (EN)
University of Wrocław
Department of Logic and Methodology of Sciences
Poland
marcisel@uni.wroc.pl

By combining the classical argumentation diagrams and schemes with the classification of reasoning elaborated by Ajdukiewicz (1955) one can represent a rich variety of reasoning types with different degrees of complexity (Selinger, 2016). In this talk I will show a possible way to give a formal definition of arguments on the basis of Ajdukiewicz's approach. The proposed definition is a recursive extension of the definition given in (Selinger 2014), where I considered arguments as finite and non-empty sets of sequents, i.e. pairs of the form $\langle P, c\rangle$, where P is a finite and non-empty set of sentences, and c is a sentence of a given language.

The extended definition takes into account epistemic and heuristic status of the sentences being components of reasoning – namely, whether they are accepted actually or only potentially (epistemic status), and whether they are given at the starting point or they become apparent only in the course of reasoning (heuristic status). The aim of representing the epistemic status is to introduce suppositional reasoning, and the aim of representing the heuristic status is to distinguish spontaneous inferences and derivations from the goal driven ones.

The recursive form of the definition is a consequence of the assumption that reasoning can be a premise of arguments (*cf.* Hitchcock 2007). For instance, suppositional derivation can be regarded as a premise in *ad absurdum* arguments. On the first, basic level of complexity the defined argumentation structures, i.e. basic structures, are exactly those grasped by the definition (Selinger 2014), enriched with means for representation of epistemic and heuristic status of sentences. So, the elements of sequents are any triples of the form $\langle s, e, h\rangle$, where s is a sentence, and e and h are the Boolean values denoting, respectively, the epistemic and the heuristic status of s. All the sentences involved in the basic structures are the sentences of the first degree. The structures of the $n+1$ level of complexity are obtained from the structures of the nth level by replacing any of their nth degree first premises with any basic structure. Then all the sentences belonging to these newly added structures are of the $n+1$ degree (note that the same sentence can be involved at different levels of complexity, so that in one argument it can have different degrees in various places). It should be added that since separate derivations do not have any explicitly accepted conclusion, they do not form 'arguments' in a narrower sense of this term. In this sense only those structures, in which the epistemic status of all the first degree sentences is fixed as 'actually accepted', can be regarded as correct argumentation structures.

Reasoning can support some final conclusion directly or it can be 'summarized' at first by a sentence, which then is used to support the conclusion. This sentence can be an implication (e.g. in *ad absurdum* arguments or in practical reasoning), but sometimes it must be a metalanguage sentence, which states that the conclusion of the summarized reasoning follows from its premises (e.g. in explanation). In some arguments of a more elaborate form also other metasentences are used, which comparatively evaluate many reasoning-components (e.g. in abduction), and which are indispensable in these arguments. So, eventually a question may be raised, which part of a metalanguage must be employed to express arguments of some predetermined kind.

References

[1] Ajdukiewicz, K. (1955). Klasyfikacja rozumowań (Classification of Reasoning). *Studia Logica*, 2, 278–299.

[2] Hitchcock, D. (2007). Informal Logic and the Concept of Argument. In D. Jacquette (ed.), *Philosophy of Logic*, 101–130.

[3] Selinger, M. (2014). Towards Formal Representation and Evaluation of Arguments. *Argumentation*, 28(3), 379–393.

[4] Selinger, M. (2016). Reasoning Types and Diagramming Method. In *Argumentation and Reasoned Action: Proceedings of the First European Conference on Argumentation, Lisbon, 9–12 June 2015*, vol. II, 937–950.

Non-Monotonic Operators

Daniel Skurt (EN)
Ruhr-Universität Bochum,
Department of Philosophy II,
Germany
daniel.skurt@rub.de

In his 1992 book "A structuralist theory of logic" Arnold Koslow [1] introduced a new notion of logical operators. Based on a certain kind of (implication) relation, which reflects some kind of reasoning and satisfies the usual tarskian conditions for consequence relations, he defines logical operators as elements which are standing in a certain relation with other elements. But, contrary to the usual account of logical operators, his notion is completely independent of any syntactical or semantical features of a given logical languages, even though the operators behave like in classical or, depending on the implication relation, like in intuitionistic logic. When we go non-monotonic we usually get rid of dilution (monotonicity) and cut or projection but we use the same language, i.e., $A \vee B$ is syntactically the disjunction of A and B in the monotonic and the non-monotonic environment. By doing this we sometimes accept that certain properties, like disjunction in the premises are lost. Based on this the aim of this talk is twofold: 1) based on the non-monotonic closed world assumption (cf. [2]) a new non-monotonic implication relation is established, and 2) it will be shown that syntactical or semantical features alone are not sufficient to define a non-monotonic operator. To give a more concrete example, it may turn out that $A \vee B$ is not the disjunction of A and B, but rather another element, let's say C which is not equivalent to $A \vee B$.

References

[1] Arnold Koslow. *A structuralist theory of logic.* Cambridge University Press, 2005.
[2] David Makinson. *Bridges from classical to nonmonotonic logic.* King's College, 2005.

Universal Freeness

Michał M. Stronkowski (EN)
Warsaw University of Technology
Faculty of Mathematics and Information Science
Poland
m.stronkowski@mini.pw.edu.pl

Let $\vdash$ be a (finitary, structural) consequence relation and $\mathsf{Th}(\vdash)$ be the set of its theorems. A rule r is *admissible for* $\vdash$ if $\mathsf{Th}(\vdash) = \mathsf{Th}(\vdash_r)$, where $\vdash_r$ is a least consequence relation extending $\vdash$ and containing r. When $\vdash$ is algebraizable we can characterize admissibility in algebraic terms: Let $\mathcal{Q}$ be an equivalent quasivariety for $\vdash$ and q be a quasi-identity translation of r. Then r is admissible for $\vdash$ iff it holds in all free algebra for $\mathcal{Q}$.

The property of admissibility has been recently extended to multi-conclusion consequence relations [2]: A multi-conclusion rule r is *admissible for* $\vdash$ if $\mathsf{mTh}(\vdash) = \mathsf{mTh}(\vdash_r)$, where $\mathsf{mTh}(\vdash)$ is the set of multi-theorems of $\vdash$, and $\vdash_r$ is a least multi-conclusion consequence relation extending $\vdash$ and containing r. Despite that multi-conclusion consequence relations are known for decades, they have been almost entirely neglected until recently. The situation has been changed with Jeřábek's paper [1] and his observation that multi-conclusion rules may be used for the canonical axiomatization of intermediate and modal logics. This topic was already undertaken in many papers.

Our aim is to find an algebraic counterpart for admissibility in the multi-conclusion setting. Here the main obstacle is the lack of free algebras for universal classes. Note that an algebraic counterpart of a multi-conclusion consequence relation may be given (if exists) by an universal class of algebras.

We show how to solve this problem and provide a construction which is a substitute of free algebras for universal classes.

We present one application. Recall that the Blok-Esakia theorem states that there is an isomorphism from the lattice of intermediate logics onto the lattice of normal extensions of Grzegorczyk modal logic. Jeřábek observed that it may be extended to multi-conclusion consequence relations [1]. We show that the Blok-Esakia isomorphism (also in the extended version) preserves structural and universal completeness.

References

[1] Emil Jeřábek. Canonical rules. *J. Symbolic Logic*, 74(4):1171–1205, 2009.

[2] Rosalie Iemhoff. Consequence relations and admissible rules. *J. Philos. Logic*. Appeared online.

On What Was Proved by S. Leśniewski in His Argumentation of 1927 Against Existence of Universals

---

Kordula Świętorzecka (EN)
Cardinal Stefan Wyszyński University, Warsaw
Institute of Philosophy
Poland
k.swietorzecka@uksw.edu.pl

Traditional discussion about universals came back in Lvov-Warsaw School and became one of the important polemics provided by its members. The key role in it was played by Leśniewski's nominalistic argumentations. One of them was formalized by B. Sobociński and this is actually the point of our analysis. The formalism by Sobociński reconstructed in Leśniewski's elementary ontology allows us to show the pragmatic weakeness of the considered argumentation. It comes out that the idea of Leśniewski in Lushei's version implies a thesis on undistingushibility of being a universal of some object x and being identical with it (we are speaking about identity of individuals). Specific axiomatics formulated by Sobocinski implies that every universal repesenting anything which is not contr-object is identical with him. A weakening of the main specific axiom, which consists of the restriction of attributes which may apply to universal x (x may posses any atributes except being identical with x) effectively blocks the argumentation of Leśniewski.

References

[1] Leśniewski S. (1927), O podstawach matematyki, reprint in: Pisma zebrane, J. Jadacki (ed.), vol. 1, Warszawa: Semper, 295– 337.

[2] Waragai T. (2004), Letters and Typescripts Sent to Bocheński from Sobociński: Historical Documents about Leśniewski's Refutation of Universals, "Reports of the Keio Institute of Cultural and Linguistic Studies" 36, 203–228.

[3] Świętorzecka K., Porwolik M. (2016), *O pewnym przesądzie dotyczącym uniwersaliów. Uwagi do sformalizowanego przez Bolesława Sobocińskiego argumentu na rzecz tezy o nieistnieniu powszechników podanego przez Stanisława Leśniewskiego*, in press.

The Development of Affine Logics

Adam Trybus (EN)
University of Zielona Góra
Institute of Philosophy
Poland
adam.trybus@gmail.com

This talk describes the development of logical analysis of affine geometry. We start with describing the works of Russell and Whitehead at the beginning of the last century. Since Whitehead's contribution to the development of the so-called region-based approach is well known, we focus on his contribution to the analysis of affine and projective notions. We briefly mention the importance of other figures from that period, like Leśniewski or Hilbert, in terms of foundations of geometry. We then move on to describe the groundbreaking work of Tarski but only in the context of his contribution to the treatment of affine geometry. From that point of view, the work of Lesław Szczerba is of particular importance but one should also remember the contribution made by other logicians collaborating with Tarski, like Szmielew and others. At some point, the work on foundations of affine geometry was picked up by researchers in computer science, which generated a renewed interest in the topic at the end of the past century. We describe the development of the region-based approach to affine logics from that period, focusing on the work by Bennett, Cohn and Davis. Finally, we provide a brief overview of the current state of affairs in that respect and point to some potential new avenues of research.

Three Accounts of Moral Dilemmas in Multivalued Settings

Robert Trypuz & Piotr Kulicki (EN)
The John Paul II Catholic University of Lublin
Faculty of Philosophy
Poland
trypuz@kul.pl, kulicki@kul.pl

Multivalued setting is quite natural for deontic action logic, where actions are usually treated as obligatory, neutral or forbidden. We apply the ideas of multivalued deontic logic to the phenomenon of a moral dilemma and, broader, to any situation where there are conflicting norms. We formalize three approaches towards normative conflicts. We present matrices for the systems and compare their tautologies. Finally, we present a sound and complete axiomatization of the systems.

Almost Structural Completeness for Tabular Modal Logics

Mateusz Uliński (EN)
Warsaw University of Technology
Faculty of Mathematics and Information Science
Poland
m.ulinski@mini.pw.edu.pl

A logic L is structurally complete (SC) if interference rules admissible in L are derivable in L. A logic L is almost structurally complete (ASC) if every admissible in L rule, which is not passive, is derivable in L. A modal logic is tabular if it is given by a finite algebra or frame.

For tabular modal logics, there are algebraic characterizations of the SC and the ASC properties. We dualized the algebraic conditions to relational semantics.

Let L be a modal logic given by a finite modal frame $\mathfrak{F}$. Then there is a finite number k such that L is SC iff for every rooted generated subframe $\mathfrak{G}$

of $\mathfrak{F}$ there is a p-morphism from $\mathfrak{U}$ onto $\mathfrak{F}$, where $\mathfrak{U}$ is a universal frame of rank k for L. If $\mathfrak{F}$ has no dead ends, the logic L is ASC iff for every rooted generated subframe $\mathfrak{G}$ of $\mathfrak{F}$ there is a p-morphism from $\mathfrak{U}$ onto $\mathfrak{F}$ or onto a disjoint sum of $\mathfrak{F}$ and $\bullet$, where $\bullet$ is a frame consisting of one reflexive point and, as previously, $\mathfrak{U}$ is a universal frame of rank k for L.

The main advantage of this approach is the logarithmic reduction of the size of considered objects. It allowed us to write a program that checks SC and ASC properties of finite frames. There were almost 300 000 frames checked during tests. We observed that the ASC property is much more common than the SC among modal logics given by the checked frames. The results are based on my master thesis supervised by M. Stronkowski.

Representing Humans' Mental Models in Prioritized Default Logic

Przemysław Andrzej Wałęga (EN)
University of Warsaw
Institute of Philosophy
Poland
p.a.walega@gmail.com

Humans reason about time and space surprisingly accurate even if the information they possess is incomplete or imprecise. According to the so called mental model theory while reasoning, people tend to construct iconic configurations which are understood in a spirit of Wittgenstein's picture theory of meaning where "the elements of the picture are combined with one another in a definite way, [which] represents that the things [in the world] are so combined with one another". It has been noticed that when a number of various mental models may be constructed, people tend to favour some of them (called preferred models) and do not realize the remaining possibilities. Interestingly, psychological experiments confirm that the preferred models are common for most of the people.

During the presentation we focus on reasoning about relations between temporal intervals by means of the well-known relations from Allen's interval algebra [1] for which a number of psychological experiments have been performed and confirmed existence of preferred models [3,5]. We apply prioritized extension [2] of Reiter's default logic [6] in order to generate and reason about humans' preferred models. The method enables us to compute preferred models and reproduce a process of their generation. Our approach takes into account order of information given to a reasoner and as a result reflects the well known (and experimentally confirmed) effect of premisses order [4].

References

[1] Allen, J.F.: Maintaining knowledge about temporal intervals. Communications of the ACM 26(11), 832–843 (1983).
[2] Brewka, G., Eiter, T.: Prioritizing default logic. In: Intellectics and computational logic, pp. 27–45. Springer (2000).
[3] Knauff, M., Rauh, R., Schlieder, C.: Preferred mental models in qualitative spatial reasoning: A cognitive assessment of allen's calculus. In: Proceedings of the seventeenth annual conference of the Cognitive Science Society. pp. 200–205 (1995).
[4] Nejasmic, J., Krumnack, A., Bucher, L., Knauff, M.: Cognitive processes underlying the continuity effect in spatial reasoning. In: Proceedings of the 33rd annual conference of the Cognitive Science Society. pp. 1127–1132 (2011).
[5] Rauh, R., Hagen, C., Knauff, M., Kuss, T., Schlieder, C., Strube, G.: Preferred and alternative mental models in spatial reasoning. Spatial Cognition & Computation 5(2–3), 239–269 (2005)
[6] Reiter, R.: A logic for default reasoning. Artificial intelligence 13(1), 81–132 (1980).

Bi-Connexive Variants of Heyting-Brouwer Logic

Heinrich Wansing (EN)
Ruhr-University Bochum
Department of Philosophy II
Germany
Heinrich.Wansing@rub.de

In these lectures, I intend to present material from two still unpublished papers. The first lecture will be based on [2], and the second talk will be based on [8].

First talk

Systems of *connexive logic* and the *bi-intuitionistic* logic BiInt that is also known as *Heyting-Brouwer logic* have been carefully studied since the 1960s and 1970s with various philosophical and mathematical motivations, see e.g [3,7] and [1,4].

A distinctive feature of connexive logics is that they validate the so-called

Aristotle's theses: $\sim(A \to \sim A)$ and $\sim(\sim A \to A)$, and
Boethius' theses: $(A \to B) \to \sim(A \to \sim B)$ and $(A \to \sim B) \to \sim(A \to B)$.

Heyting-Brouwer logic, which is an extension of both dual-intuitionistic logic, DualInt, and intuitionistic logic, Int, was introduced by Rauszer, who proved algebraic and Kripke completeness theorems for BiInt. As was shown by Uustalu in 2003, that the original Gentzen-type sequent calculus by Rauszer does not enjoy cut-elimination, and various kinds of sequent systems for BiInt have been presented in the literature, including cut-free display sequent calculi.

In this talk we combine the two approaches and introduce the *bi-intuitionistic connexive logic* (or *connexive Heyting-Brouwer logic*), BCL, as a Gentzen-type sequent calculus. The logic BCL may be seen as an extension of the connexive logic C from [5] by the co-implication of BiInt, using a connexive understanding of negated co-implications. The logic BCL is introduced as a Gentzen-type sequent calculus, and a dual-valuation-style Kripke semantics for BCL is defined. BCL is constructed on the basis of Takeuti's cut-free Gentzen-type sequent calculus LJ′ for Int. Gentzen-type sequent calculi ICL, DCL, BL, IL and DL for *intuitionistic connexive logic*, *dual-intuitionistic connexive logic*, BiINt, Int, and DualInt, respectively, are defined as subsystems of BCL. We also present a sound and complete tableau calculus for BCL and its subsystems ICL, DCL, BL, IL, and DL using triply-signed formulas.

Second talk

The system **2C** is a connexive variant of the bi-intuitionistic logic **2Int** from [6] and contains a primitive strong negation. In both systems a relation of provability is supplemented with a certain relation of dual provability. Whereas entailment as the semantic counterpart of provability preserves support of truth from the premises to the conclusion of an inference, dual entailment as the semantic counterpart of dual provability preserves falsity from the premises to the conclusion of an inference. The strong negation that is added to the language of **2Int** to obtain the system **2C** internalizes falsification with respect to provability and it internalizes verification with respect to dual provability.

The system **2C** also emerges as an extension of the connexive propositional logic C from [5], which was obtained from Nelson's constructive paraconsistent logic **N4** by replacing the familiar falsification condition for negated implications by its connexive version.

The reason for considering *connexive* implication, $\rightarrow$, and *connexive* co-implication, $\multimap$, instead of assuming the familiar understanding of negated implications in **N4** and other logics is that one obtains a neat encoding of derivations in the $\{\rightarrow, \multimap, \sim\}$-fragment of the language under consideration by typed λ-terms built up from atomic terms of two sorts, one for proofs and one for dual proofs, using only (i) functional application, (ii) functional abstraction, and (iii) certain sort/type-shift operations that turn an encoding of a dual proof of a formula A [respectively $\sim A$] into an encoding of a proof of $\sim A$ [respectively A] and that turn an encoding of a proof of a formula A [respectively $\sim A$] into an encoding of a dual proof of $\sim A$ [respectively A]. The use of terms of two sorts makes sure that every term is uniquely typed.

References

[1] R. Goré, "Dual intuitionistic logic revisited", *Proceedings of the International Conference on Automated Reasoning with Analytic Tableaux and Related Methods* (TABLEAUX 2000), LNAI 1847, Berlin, Springer, 2000, 252–267.

[2] N. Kamide and H. Wansing, "Completeness of connexive Heyting-Brouwer logic", 2015, to appear in: *IFCoLog Journal of Logic and their Applications.*

[3] S. McCall, "A history of connexivity", in D. Gabbay et al. (eds.), *Handbook of the History of Logic. Volume 11. Logic: A History of its Central Concepts*, Amsterdam, Elsevier, 415–449.

[4] C. Rauszer, "An algebraic and Kripke-style approach to a certain extension of intuitionistic logic", *Dissertationes Mathematicae*, Polish Scientific Publishers, Warsaw, 1980, 1–67.

[5] H. Wansing, "Connexive modal logic", *Advances in Modal Logic. Vol. 5*, College Publications, London, 2005, 367–385.

[6] H. Wansing, Falsification, natural deduction and bi-intuitionistic logic, *Journal of Logic and Computation*, 26 (2016), 425–450. First published online, July 17, 2013.

[7] H. Wansing, "Connexive Logic", *The Stanford Encyclopedia of Philosophy* (Fall 2014 Edition), E. Zalta (ed.), URL = ⟨http://plato.stanford.edu/archives/fall2014/entries/logic-connexive/⟩ (First published January 6, 2006).

[8] H. Wansing, "Natural deduction for bi-connexive logic and a two-sorted typed λ-calculus", 2015, to appear in: *IFCoLog Journal of Logic and their Applications*.

A System of Natural Deduction with Wittgenstein's Operator N

Eugeniusz Wojciechowski (EN)
University of Agriculture, Cracow
Department of Philosophy of Nature
Poland
rlwojcie@cyf-kr.edu.pl

In Ludwig Wittgenstein's *Tractatus logico-philosophicus* there is the logical operation of generalised negation (5.501–5.51) in the sense of *simultaneous logical conjunction of propositional negation of proposition sequence*. Beside the standard interpretation of operator N, which is here referred to as *argument interpretation* in the following form:

$$\text{ON}\quad \mathrm{N}(\alpha_1,\alpha_2,\ldots,\alpha_n)/\mathrm{N}(\alpha_1)\quad \mathrm{N}(\alpha_1,\alpha_2,\ldots,\alpha_n)/\mathrm{N}(\alpha_2,\ldots,\alpha_n)\qquad \Phi/\Phi(\mathrm{N}(\mathrm{N}(\alpha))/\alpha),$$

$$\text{IN}\quad \mathrm{N}(\alpha_1),\mathrm{N}(\alpha_2),\ldots,\mathrm{N}(\alpha_n)/\mathrm{N}(\alpha_1,\alpha_2,\ldots,\alpha_n)\qquad \Phi/\Phi(\alpha/\mathrm{N}(\mathrm{N}(\alpha))),$$

its *list interpretation* can be proposed:

$$\text{(ON)}\quad \mathrm{N}[\alpha_1,\alpha_2,\ldots,\alpha_n]/\mathrm{N}[\alpha_1]\quad \mathrm{N}[\alpha_1,\alpha_2,\ldots,\alpha_n]/\mathrm{N}[\alpha_2,\ldots,\alpha_n]\qquad \Phi/\Phi(\mathrm{N}[\mathrm{N}[\alpha]]/\alpha),$$

$$\text{(IN)}\quad \mathrm{N}[\alpha_1],\mathrm{N}[\alpha_2],\ldots,\mathrm{N}[\alpha_n]/\mathrm{N}[\alpha_1,\alpha_2,\ldots,\alpha_n]\qquad \Phi/\Phi(\alpha/\mathrm{N}[\mathrm{N}[\alpha]]),$$

in which the operator has one argument in the form of a list.

The analysis of this interpretation makes it possible to move on to the third interpretation – *interpretation with generalised disjunction*, where the expression $[\alpha_1, \alpha_2, \ldots, \alpha_n]$ is treated as a shortened notation of generalised disjunction consisting of n arguments: $\alpha_1 \wedge \alpha_2 \wedge \ldots \wedge \alpha_n$.

The paper gives proof for the inferential equivalence of the last of the above constructions to the classical propositional calculus. The first and second interpretations can be seen as alternative formulations of the classical propositional calculus. The second interpretation, namely the list one, can be interesting in the context of the question of artificial intelligence.

Projective Unification and Structural Completeness in Superintuitionistic Predicate Logics. Part II

Piotr Wojtylak (EN)
University of Opole
Institute of Mathematics and Computer Science
Poland
piotr.wojtylak@gmail.com

Wojciech Dzik (EN)
Silesian University, Katowice
Institute of Mathematics
Poland
wojciech.dzik@us.edu.pl

Projective unifiers were introduced by S. Ghilardi and successfully applied in propositional logic. Our aim is to lift projective unifiers to the first-order level. However extending results on projective unification of some intermediate propositional logics to their predicate counterparts is not immediate and it requires, in the first place, a proper definition of substitution. We prove, among others,

THEOREM. (i) Unifiable Harrop's formulas are *Q–INT* projective;
(ii) If A is projective in *Q–INT*, then $A|A$ (Kleene's slash).

Let L be a superintuitionistic predicate logic. Then

THEOREM. For any L-projective formula A, we have
(i) if $\vdash_L A \to B_1 \vee B_2$, then $\vdash_L (A \to B_1) \vee (A \to B_2)$;
(ii) if $\vdash_L A \to \exists_x C(x)$, then $\vdash_L \exists_x (A \to C(x))$.

On Applications of Modal Logic to Classical Philosophical Problems

Jan Woleński (EN)
University of Information Technology and Management in Rzeszów
Poland
wolenski@if.uj.edu.pl

Modal logic can be understood more or less widely. In the narrow sense, it covers alethic modal logic, but in a wider sense deontic logic, doxastic logic, epistemic logic, logic of action, etc. are its species. I will speak about applications of various systems belonging to the modal logic in the wide sense to classical philosophical problems. On the other hand, the formal apparatus employed in the further analysis is restricted to the octagon of modal sentences resulting from a generalization of the traditional logical square by its supplementing by additional axes-points. Although this apparatus is fairly simple, it allows to clarify various controversial questions, in particular:

1. The status of the principle of bivalence and the size of the world of logic;
2. The logic of truth;
3. The relation between necessary beings and accidental beings;
4. The problem of logical determinism;
5. The status of the classical definition of knowledge (knowledge is justified true belief);
6. The logic of skepticism;
7. The Hume thesis concerning the is/ought relation;
8. The problem of normativity of epistemology;
9. The thesis that every being is good;
10. The difference between omission and not-action.

The analysis via modal logic is is helpful in accounting what can be achieved in philosophy by formal methods and what requires additional substantial information.

Finitely Characterizable Models: the Case of Mosaics

Michał Zawidzki (EN)
University of Łódź
Department of Logic
Poland
michal.zawidzki@gmail.com

Among modal logics there exist ones that lack the finite model property despite the fact that they are decidable. Decidability theorems for such logics rest upon various properties: translatability of such a logic into a theory proven to be decidable or the existence of a finite approximation of a model for each satisfiable formula (known as quasi-models, mosaics or tableaux).

These approximations encode, in a finite form, all information necessary to restore the original model.

In my talk I will focus on the case of mosaics which usually serve as a tool of establishing decidability for temporal logics or, more generally, for logics defined over linear orderings. For each satisfiable formula φ mosaics are components of a larger structure providing a finite basis for a proper (possibly infinite) model for φ. Such a structure will be called a *saturated set of mosaics for* φ ($SSM(\varphi)$ in short). In the case of logics defined over linear orderings mosaics are named *bricks* and are pairs of Hintikka sets representing, respectively, an antecedent and a successor of an accessibility relation.

Using as a running example a modal logic with the universal modality, graded and graded inverse modalities, whose models are not necessarily linearly ordered, I will show that the mosaics method can be easily extended onto the wide class of (modal) logics with models of no particular characterization and serve as the basis for establishing decidability for these logics.

References

[1] Reynolds, Mark, *A Tableau for Temporal Logic over the Reals*, [in:] "Advances in Modal Logic", 2014, College Publications, 439–458.

[2] Reynolds, Mark, *Dense Time Reasoning via Mosaics*, [in:] "TIME", 2009, IEEE Computer Society, 3–10.

[3] Wolter, Frank and Zakharyaschev, Michael, *On the Decidability of Description Logics with Modal Operators*, [in:] KR, 1998, Morgan Kaufmann, 512–523.

ISBN 978-83-940690-2-5

9 788394 069025