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Applications of Logic in Philosohpy and the Foundations of Mathematics

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Part–Whole Theory

APPLICATIONS OF LOGIC
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Editorial note

(EN) means that the talk is presented in English, (PL) — in Polish.

The Problem of Singularity for Weighted Graphs

Anna Bień (EN)
University of Silesia, Katowice
Institute of Mathematics
Poland
anna.bien@us.edu.pl

We consider the problem of singularity and present methods of reducing weighted graphs. In our approach we take into consideration the set $S(G)$ of all sesquivalent spanning graphs of a weighted graph G. Every component of a graph $\Gamma \in S(G)$ is either a P_2 path or a cycle. Let $P_2(\Gamma)$ denote a set of all components, which are P_2 paths, and $C(\Gamma)$ a set of all components which are cycles. We denote $sg(\Gamma) := |V(\Gamma)| - c(\Gamma)$ and $k(\Gamma) := |E(\Gamma)| - |V(\Gamma)| + c(\Gamma)$ Notice, that $k(\Gamma) = |C(\Gamma)|$ and $sg(\Gamma)$ is a number of all components of Γ, which have even number of vertices.

If G is a weighted graph, then we can calculate $detA(G)$ applying the following theorem.

Theorem 1.

$$detA(G) = \sum_{\Gamma \in S(G)} \left[(-1)^{sg(\Gamma)} 2^{k(\Gamma)} \prod_{e \in P_2(\Gamma)} w(e)^2 \prod_{e \in C(\Gamma)} w(e) \right]$$

where $w(e)$ denotes the weight of the edge e.

It is a generalised version of a theorem presented by Harary in [1]. We can apply this formula to calculate determinants of adjacency matrices of weighted paths and cycles. We obtain

$$detA(P_n) = \begin{cases} 0 & \text{if } 2 \not| n \\ \prod_{e \in E_1(P_n)} w(e)^2 & \text{if } 2|n \end{cases}$$

and

$$detA(C_n) = \begin{cases} 2\prod_{e\in E(C_n)} w(e) & \text{if } 2 \not| n \\ -\left[\prod_{e\in E_1(C_n)} w(e) + \prod_{e\in E_2(C_n)} w(e)\right]^2 & \text{if } 2|n \wedge 4 \not| n \\ \left[\prod_{e\in E_1(C_n)} w(e) - \prod_{e\in E_2(C_n)} w(e)\right]^2 & \text{if } 4|n \end{cases}$$

where $E_1(C_{2k}) = \{[v_1, v_2], [v_3, v_4], \ldots [v_{2k-1}, v_{2k}]\}$ and $E_2(C_{2k}) = E(C_{2k}) \setminus E_1(C_{2k})$.

As long as we have the formula to calculate the determinant of a weighted graph, we can define the operation of adding and subtracting vertices. We might obtain weighted graphs, when we apply these operations to simple graphs. However, it does not change the determinant of the adjacency matrix of the graph.

We will present examples of simple graphs, such as hexagonal grids or three dimensional products of paths, which can be reduced, by subtracting or adding vertices, to uncomplicated weighted graphs. The determinant of obtained weighted graphs can be easily calculated by applying the main formula.

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Model Sets and Model Complete Theories

Janusz Czelakowski (EN)
Opole University, Opole
Institute of Mathematics and Informatics
Poland
jczel@math.uni.opole.pl

Let L be a *countable* language, i.e., L is a countable set of predicates, operation symbols and constants. $Var = \{v_n : n \in \mathbb{N}\}$ is a (countable) set of individual variables. $For(L)$ is the set of first-order formulas of L.

If Σ is a set of formulas, then $FreeVar(\Sigma)$ is the set of individual variables that occur free in the formulas of Σ. Thus $FreeVar(\Sigma) = \bigcup_{\sigma\in\Sigma} FreeVar(\sigma)$.

CL is the set of theses of classical logic. By definition, ***CL*** consists of formulas derivable from the standard logical axioms (including the quantifier and equality axioms) by means of *Modus Ponens* and the rule of generalization.

C is the 'provability' consequence operation of classical logic on the set $For(L)$. Thus, for any $\Sigma \subseteq For(L)$, $C(\Sigma)$ is the set of all formulas derivable from the set $\Sigma \cup \boldsymbol{CL}$ by means of *Modus Ponens.*

An *ultraset* (alias a *Lindenbaum set*) is an arbitrary maximal and consistent set Δ of formulas from $For(L)$. An ultraset Δ is called a *model set* if it additionally has the property that for every individual variable x and every formula φ of $For(L)$,

$$(1) \qquad (\exists x)\varphi \in \Delta \quad \Leftrightarrow \quad \begin{array}{l}\textit{for some variable } y \textit{ such that the substitution } x/y \\ \quad \textit{is free for } \varphi, \textit{ the formula } \varphi(x/y) \textit{ belongs to } \Delta.\end{array}$$

If $(\exists x)\varphi \in \Delta$, then any variable y such that the right hand side of (1) holds is called a *witness* for $(\exists x)\varphi$ with respect to Δ.

$Modset(L)$ is the family of model sets. Model sets exist:

Theorem *Let Σ be an arbitrary consistent set of formulas of L such that the set of variables $Var \backslash FreeVar(\Sigma)$ is infinite.* (*In particular, let Σ be a set of sentences.*) *Then $C(\Sigma)$ is the intersection of all model sets Δ such that $\Sigma \subseteq \Delta$.*

The above theorem (after some modifications) implies the Completeness Theorem in the strong sense: for any set of formulas Σ and any formula σ, σ is derivable form Σ if and only if σ logically follows from Σ. In other words, model sets determine a sufficiently broad class of models that yield the Completeness Theorem.

In the talk, the significance of model sets in various model-theoretic contexts is examined.

Almost Structural Completeness in some Many-valued Logics

Wojciech Dzik (EN)
Silesian University, Katowice
Institute of Mathematics
Poland
dzikw@silesia.top.pl

Key words: structural completeness, almost structural completeness, unification, projective unifiers, Łukasiewicz logics, MV_n-algebras.

Unifiers of special kind are applied to show that some many–valued logics, including many–valued Łukasiewicz logics are almost structurally complete.

We consider structural (i.e. closed under substitutions) rules of inference of the form $A \ /B$ instead of $A_1, \ldots, A_n \ /B$. A rule $r : A \ /B$ is *admissible* in a logic L, if for every substitution τ, whenever $\vdash_L \tau A$, than $\vdash_L \tau B$. A rule $r : A \ /B$ is *drivable* in a logic L, if $A \vdash_L B$.

Given a formula α, a *unifier for* A in a logic L is a substitution σ such that $\vdash_L \sigma(A)$. A *formula* A *is unifiable* in L, if such σ exists.A projective unifier for a unifiable formula A in a logic L is a unifier σ for A such that $A \vdash_L \sigma(x) \leftrightarrow x$. L has *projective unification* if every unifiable formula has a projective unifier.

Now a logic L is *structurally complete* iff ($\star$) every (structural) admissible rule in L is derivable in L, in terms of unifiers: iff every unifier for A is a unifier for B, $\vdash_L \sigma(A)$ implies $\vdash_L \sigma(B)$.

Some logics are not structurally complete due to the fact that some passive rules are not derivable. A rule $r : A/B$ is passive in L, if the premise A is not unifiable in L, i.e. for every substitution τ, $\nvdash_L \tau A$. A logic L is *almost structurally complete* iff every (structural) admissible rule in L which is not passive is derivable in L.

We show that some many–valued logics having projective unification are structurally complete or almost structurally complete.

Blocks of Skeleton Tolerances

Anetta Górnicka, Joanna Grygiel (EN)
Jan Długosz University, Częstochowa
Institute of Mathematics and Computer Science
Poland
a.gornicka@ajd.czest.pl, j.e.grygiel@gmail.com

Katarzyna Grygiel
Jagiellonian University, Cracow
Theoretical Computer Science Department
Poland
grygiel@tcs.uj.edu.pl

The notion of tolerance, which can be considered as a natural generalization of congruence for algebraic structures, was introduced by Chajda and Zelinka [1] and it is of growing importance nowadays. A binary relation R on a lattice L is said to be a tolerance relation iff it is reflexive, symmetric and compatible with joins and meets of the lattice. A block of R is every maximal subset of L in which every two elements are in the relation R. It is known [2] that in a finite case blocks are intervals and they themselves form a lattice, called a factor lattice by R. The skeleton tolerance of a lattice, i.e., the tolerance generated by the set of all prime quotients in the lattice, and its factor lattice called a skeleton play a special role in the lattice theory. It was proved by Herrmann [5] that every finite lattice is a skeleton of a finite distributive lattice. However, it turns out that there are lattices which cannot be blocks of a skeleton tolerance of any finite lattice. It is clear for modular lattices, in particular, as it is known that their blocks of the skeleton tolerance are maximal complemented intervals of such lattices [3]. Our goal is to characterize lattices which can be blocks of the skeleton tolerance in the general case.

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Finite Distributive Lattices and their Boolean Parts

Katarzyna Grygiel (EN)
Jagiellonian University, Cracow
Theoretical Computer Science Department
Poland
grygiel@tcs.uj.edu.pl

Joint work with Gábor Czédli and Joanna Grygiel

It is often difficult to maintain big, however finite, structures which emerge in practical applications of lattice theory. Therefore, there is a natural tendency to find their description as concise as possible. Finite distributive lattices are especially good from this point of view since they can be regarded as built from very regular parts — boolean intervals glued together according to some pattern being a lattice itself. Many representations of distributive lattices are known, however, none of them provides a possibility to store numerical features of the lattices.

We discuss another representation, introduced in 2006 by J. Grygiel, called the *weighted double skeleton.* Our goal is to show that this representation is unique for some classes of distributive lattices. We also formulate an algorithm for reconstructing the lattice on the basis of its weighted double skeleton and, in particular, for reconstructing the poset of its join-irreducible elements. It means that knowing only some facts concerning parts of a lattice we are able to describe most of its features and, in many cases, reconstruct it uniquely.

Our approach seems to offer the most concise known description of finite distributive lattices, especially in the situation when the size of a given lattice is much larger than the number of its maximal boolean intervals.[1]

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Some Result on Bimodal Logic (S5,S5)

Sławomir Kost (PL)
University of Silesia, Katowice
Institute of Mathematics
Poland
slawomir.kost@us.edu.pl

Mono-modal logic is one of the most popular areas of research in philosophy and mathematics. There is a fair number of high-powered results on (strong) completeness, decidability or finite model property ([2]), to name but a few. However, when we turn to the polymodal case, there are a lot of unanswered questions. In some cases of polymodal systems, to determine the completeness or the decidability problems, it suffices to confine our considerations to the independently axiomatizable bimodal logic.

In our talk we focus on independently axiomatizable bimodal system $(S5, S5)$ which is a fusion of two $S5$ systems. It means that this is the smallest multimodal system containing the following axioms

K $\quad \Box_i(\varphi \to \psi) \to (\Box_i\varphi \to \Box_i\psi)$,
T $\quad \Box_i\varphi \to \varphi$,
4 $\quad \Box_i\varphi \to \Box_i\Box_i\varphi$,
B $\quad \Diamond_i\Box_i\varphi \to \varphi$,

which is closed under the rules of detachment (MP: given φ and $\varphi \to \psi$, prove ψ) and necessity (RN_i: given a formula φ, we infer $\Box_i\varphi$), for $i = 1, 2$.

It is already known that completeness is preserved under the formation of fusions. Namely, the system $(S5, S5)$ is complete with respect to the class of the following 2-frames (see [1])

$$\{\langle W, R_1, R_2\rangle \colon R_i \text{ is an equivalence relation on } W, 1 \leq i \leq 2\}.$$

In the mono-modal logic case, it is easy to show that $S5$ is complete with respect to one particular frame. Thus, our aim is to distinguish only one frame with that property for the system $(S5, S5)$. To that end, first we have to consider notions such as p-morphism or finite model property. Then, we obtain our main result which states that the system $(S5, S5)$ is complete with respect to the frame $\mathcal{F} = \langle U, R_1, R_2 \rangle$, where $U = \{(a_1, \ldots, a_n) \in \mathbb{N}^n : n \in \{2, 3, \ldots\}\}$ and R_1, R_2 are some particular equivalence relations.

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Blocks of Tolerance in Reflexive and Symmetric Kripke Frames

Zofia Kostrzycka (EN)
Opole University of Technology, Opole
Faculty of Production Engineering and Logistics
Poland
z.kostrzycka@po.opole.pl

We examine the Brouwer logic $\mathbf{KTB} := \mathbf{K} \oplus T \oplus B$, where the special axioms are the following: $T := \Box p \to p$ and $B := p \to \Box\Diamond p$. The set of rules consists of modus ponens, substitution and the Gödel rule.

Let $\mathfrak{F} = \langle W, R \rangle$ be a KTB-Kripke frame (R is reflexive and symmetric relation on W). Then R is called a tolerance on $\mathfrak{F}$. A non-empty subset $U \subseteq W$ is called a block of the tolerance R, if U is a maximal subset with $U \times U \subseteq R$ (if $U \subseteq V$ and $V \times V \subseteq R$, then $U = V$). Each reflexive and symmetric frame $\langle W, R \rangle$ can be divided into blocks of tolerance.

Definition 1. *It is said that a frame $\langle W, R \rangle$ consists of linearly ordered blocks of tolerance if the following two conditions hold:*

$$(L1)\quad (B_1 \cap B_2 \cap B_3) = \emptyset,$$

$$(L2)\quad (B_1 \cap B_2 \neq \emptyset \ \wedge\ B_2 \cap B_3 \neq \emptyset) \ \Rightarrow\ (B_1 \cap B_2) \cup (B_2 \cap B_3) = B_2$$

for any three blocks B_1, B_2, B_3

The aim of the paper is to characterize logics determined by such frames. The motivation for our research are the results obtained by R. Bull for extensions of **S4.3** logic (see [2]). Semantically, all such extensions are determined by chains of clusters (the relation in a frame is reflexive and transitive). It is proved that all normal extensions of **S4.3** have finite model property (f.m.p.) and they are finitely axiomatizable.

The following question arises: is the linear order of blocks of tolerance in frames reflected by any modal formula? The answer to this question is positive. We consider the formula

$$(3') := \Box p \vee \Box(\Box p \to \Box q) \vee \Box((\Box p \wedge \Box q) \to r)$$

and the logic $\mathbf{KTB.3'} := \mathbf{KTB} \oplus (3')$.

We prove that:

Theorem 2. *The logic* $\mathbf{KTB.3'}$ *is complete with respect to the class of reflexive and symmetric frames with linearly ordered blocks of tolerance.*

Theorem 3. *The logic* $\mathbf{KTB.3'}$ *has f.m.p.*

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Bisimulations of Finite Kripke Models

Małgorzata Kruszelnicka (EN)
University of Silesia, Katowice
Institute of Mathematics
Poland
m.kruszelnicka@math.us.edu.pl

Given two classical structures, one of the most important questions is whether they validate the same formulae. Since the notion of logical equivalence involves language, our aim is to find a suitable condition that is defined directly in terms of structural properties. When we discard the notion of structure isomorphism as a too strong and too restrictive one, we have to look for another condition for logical equivalence.

In classical model theory the problem is to find a structural description for the notion of elementary equivalence. That problem was first stated by Alfred Tarski; the solution was found by Fraïssé and then by Ehrenfeucht, and referred to well-known Ehrenfeucht–Fraïssé games. One of the versions of Ehrenfeucht–Fraïssé Theorem states that two classical first-order structures $\mathcal{A}$ and $\mathcal{B}$ are elementarily equivalent with respect to all sentences with quantifier complexity not greater than n, $\mathcal{A} \equiv_n \mathcal{B}$, whenever there exists a winning strategy for Duplicator in Ehrenfeucht–Fraïssé game of length n on structures $\mathcal{A}$ and $\mathcal{B}$ ([1]). It turns out that the inverse implication also holds, but it requires some additional assumptions.

We consider the case of Kripke semantics for intuitionistic first-order theories. In that case the problem is to find a structural description for the notion of logical equivalence of two Kripke models. As a counterpart of Ehrenfeucht–Fraïssé game of length n on structures $\mathcal{A}$ and $\mathcal{B}$ we consider the notion of bounded bisimulation between nodes α and β of Kripke models $\mathcal{K}$ and $\mathcal{M}$ respectively, which fulfills particular *'zig'* and *'zag'* conditions.

Since quantifiers $\forall$ and $\exists$ are not mutually definable, and implication refers to all nodes accessible from a certain node, as a measure of formula's complexity we consider the *characteristic* of a formula. We say that characteristic of a formula φ, $char(\varphi)$, equals $(^{\to}p, ^{\forall}q, ^{\exists}r)$ whenever there are p nested implications, q nested universal quantifiers and r nested existential quantifiers in φ.

In the case of Kripke models for intuitionistic first-order logic it is already known (see [3]) that the existence of p, q, r-bisimulation between nodes α and β, $\alpha \sim_{p,q,r} \beta$, implies their logical equivalence with respect to all formulae of characteristic not greater than $(^{\to}p, ^{\forall}q, ^{\exists}r)$, $\alpha \equiv_{p,q,r} \beta$.

The subject of our research is, however, the reverse implication. The question is under what conditions it holds. In the case of propositional intuitionistic logic we have to restrict our considerations to the class of finite Kripke models. The theorem which states that the logical equivalence of nodes of two finite Kripke models implies a bisimulation between them was proved by Albert Visser ([4]), and, subsequently, another version of this proof was presented by Anna Paterson ([2]).

We encounter some difficulties when we turn into the first-order case and have to deal with quantifiers. Having analysed the inverse implication of Ehrenfeucht–Fraïssé Theorem in classical model theory, and the propositional intuitionistic logic case, we can expect that some additional assumptions on Kripke models, language L or first-order structures will be needed.

We say that Kripke model $\mathcal{K}$ is *strongly finite* if and only if both the frame and first-order structures assigned to the nodes are finite. We will consider those Kripke models in which formulae of characteristic not greater than $(^{\to}p, ^{\forall}q, ^{\exists}r)$ fulfil the law of excluded middle. Moreover, the finite signature of language L will be considered with no function symbols. Under those assumptions we obtain the theorem which states that the logical equivalence of nodes α and β of two strongly finite Kripke models implies a bisimulation between them, with respect to all formulae of a certain characteristic.

In order to generalise the above theorem, we must establish which assumptions are possible to remove, and which are necessary to maintain the theorem. In particular, to apply the theorem to a specific class of Kripke models, we should weaken the assumption of the strongly finite frame to the finite frame and include function symbols in the language L under consideration.

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Horn Supersystems of the Quantifier-free Fragment of Leśniewski's Ontology

Piotr Kulicki (EN)
John Paul II Catholic University of Lublin, Lublin
Department of Foundations of Computer Science
Poland
kulicki@l3g.pl

One of the ways of getting a better understanding of a formal system is to investigate its surroundings (systems that are weaker, stronger or 'neighbouring' in some other way). In the talk I apply that strategy to Leśniewski's ontology. The system, although relatively simple and already investigated in details, still attracts interest from the metalogical and practical points of view.

More precisely I shall present extensions of the quantifier free fragment of Leśniewski's ontology (presented by A. Ishimoto), that are obtained by adding to it axioms that are Horn formulas. A formula is a Horn formula if it takes one of the three forms: (a) it is an implication with the conjunction of atoms as its antecedent and an atom as its consequent, (b) it is an atom or (c) it is a negation of a conjunction of atoms. The formulas of that type are particularly interesting for several reasons. Firstly, all axioms of the quantifier-free fragment of ontology provided by Ishimoto are Horn formulas. Secondly, Horn formulas, especially of the form (a), can be straightforwardly transformed into rules of deduction. Thirdly, Horn formulas can be used as elements of logic programs.

The investigations of the possible extensions of Ishimoto's system lead to the conclusion that there exist exactly 11 of them (including the inconsistent system). I shall present their axioms, sketch the proof that they are the only possible systems taking Horn formulas as axioms and show the models that can be used to prove that each of the systems is different from the others.

The interesting feature of the discovered space of systems is that it is finite, though there exist infinitely many quantifier-free extensions of Ishimoto system in general.

Relations Between Buddhist Logic and Ontology

Michał Lipnicki (EN)
Adam Mickiewicz University, Poznań
Department of Applied Logic
Poland
lelipou@wp.pl

This paper investigates the influence of the ontological views on the logical theory developed by the prominent Buddhist logician — Dharmakirti (*Dharmakīrti*) who lived circa VI — VII century AD in India.

Our inquiry is being done on the two levels:

The first one is more general. Here the relation between the rules (*trairūpya*) and laws (*vyāpti*) that enable the validity of a reasoning on the logical level and the laws that characterised in what way things exist in reality (*anvaya*, *vyatireka*) is analysed. Thus, our first task is to show that Dharmakirti's logic and its "metatheory", the very rules on which he based his view on what constitutes the valid reasoning, have their foundation in the postulated nature of reality.

Secondly, the specific ontological views held by Dharmakirti which had influenced his philosophical views on logic are to be presented. Talking about Buddhism one must keep in mind that there is no such thing as "the general Buddhist's ontology" or "the general Buddhist's epistemology". Over the ages Buddhism has developed quite a few philosophical standpoints which, in terms of ontology, may spread from a „cautious" realism of the early schools (such as: *vaibhāṣika*, *pudgalavāda*), to the idealism of the *yogācāra* school. Dharmakirti's views can be classified as belonging to the latter one. Although there is no agreement between Buddhists when it comes to the general ontological issues, one can point at some ideas common to most of the Buddhist sects, such as: eventism (*arthakriyāsamartha*) related with permanent momentariness (*kṣaṇikavāda*) and anti-essencialism also called „interdependent co-arising" (*pratītyasamutpāda*). According to the former, the reality consists of series of causally related and absolutely individual moments (*kṣaṇa*); the latter says that nothing exists independently or has the being on its own (*svabhāva*). At this point Buddhists' view on the part (*avayava*) and the whole (*avayavin*) relation becomes very significant. Since they claim that there is no whole existing independently of its parts, every event can be deconstructed in more and more subtle particles until it utterly disappears. Such idea is crucial for Dharmakirti's consideration of what is the nature of the relations between an actual subject of reasoning and a logical sign. Hence finally we are going to show that from the three types of reasoning distinguished by Dharmakirti the main two can be described in the categories that correspond to the aforementioned ontological views.

On $\rightarrow$-irreducibility in Finite Heyting Lattices

Marcin Łazarz & Krzysztof Siemieńczuk (EN)
Wrocław University, Wrocław
Department of Logic and Methodology of Science
Poland
lazarzmarcin@poczta.onet.pl, logika6@gmail.com

According to familiar concepts of $\vee$- and $\wedge$-irreducibility we introduce the notion of $\rightarrow$-irreducibility defined as follows: an element a is called *$\rightarrow$-irreducible*, whenever for all $x, y \in L$ holds:

$$a = x \rightarrow y \quad \Rightarrow \quad a = y.$$

In our talk we consider the issue of $\rightarrow$-irreducibility in finite Heyting lattices. The main result is the following

Theorem. *Let L be the finite Heyting lattice. An element $a \in L$ is $\rightarrow$-irreducible iff a is the least element in some maximal Boolean interval in L.*

Furthermore we discuss the problem of the *$\rightarrow$-representability* of a given element a by means of $\rightarrow$-irreducible elements.

Modal Ontologic: Parts and Qualities

Marek Magdziak (PL)
Wrocław University, Wrocław
Department of Logic and Methodology of Science
Poland
mmagdziak@tlen.pl

The name *ontologic* was suggested by Polish logician Jerzy Perzanowski for theoretical or formal part of ontology. By *modal ontologic* I understand the formal logical study of ontological concepts within the framework of propositional modal logic, especially a study of logical interconnections between modal concepts as applied to propositions or some proposition-like entities, on the one hand, and ontological concepts of existence, possibility, localness, well-foundation, fusion, heteronomy and merger, used in reference to objects, on the other hand. It is shown, that a slight modification to contemporary semantic analysis of modal terms can capture some ontological intuitions. There is one key difference in comparison to the standard approach. I assume that modal concepts as applied to propositions or proposition-like entities may be relativized to the objects in a fixed ontological universe. So, instead of contexts like *it is so and so that* **A**, where **A** stands for a proposition, I will study contexts like *for* **b** *it is so and that* **A**, where **A** stands for a proposition and **b** stands for an object. An object can be treated like a simple quality or a merger of qualities and like an aggregate or a fusion of aggregates. An object can exist or not, can be possible or not, and can be local or not. An object can be heteronomous to another object and can be well-founded in another object. Let me adopt an informal notation. For object $\mathbf{a}$ and proposition $\mathbf{B}$ let $\mathbf{a}^{\bullet}\mathbf{B}$ mean that for $\mathbf{a}$ it is necessary that $\mathbf{B}$ and let $\mathbf{a}^{\bullet\bullet}\mathbf{B}$ mean that for $\mathbf{a}$ it is essentially that $\mathbf{B}$. For object $\mathbf{a}$ let $\mathrm{Ex}\mathbf{a}$ mean that $\mathbf{a}$ exist, let $\mathrm{Pos}\mathbf{a}$ mean that $\mathbf{a}$ is possible and let $\mathrm{Loc}\mathbf{a}$ mean that $\mathbf{a}$ is local. For objects $\mathbf{a}$ and $\mathbf{b}$ let $\mathbf{a}/\mathbf{b}$ mean that $\mathbf{a}$ is well-founded in $\mathbf{b}$ and let $\mathbf{a}\backslash\mathbf{b}$ mean that $\mathbf{a}$ is heteronomous to $\mathbf{b}$. Let $(\mathbf{a}{*}\mathbf{b})$ stand for the fusion of $\mathbf{a}$ and $\mathbf{b}$ and let $(\mathbf{a}\#\mathbf{b})$ stand for the merger of $\mathbf{a}$ and $\mathbf{b}$. The signs $\sim, \&, \vee, \to$ and $\equiv$ will then be used respectively as symbols for negation, conjunction, disjunction, material implication and material equivalence. The following formulas express some basic insights.

(1)	$\mathrm{Ex}\mathbf{a} \to (\mathbf{a}^{\bullet}\mathbf{B} \to \mathbf{B})$,	$\mathbf{a}^{\bullet\bullet}\mathbf{B} \to (\mathbf{B} \to \mathrm{Ex}\mathbf{a})$,
(2)	$\mathbf{a}^{\bullet}\mathbf{B} \to ({\sim}\mathbf{B} \to {\sim}\mathrm{Ex}\mathbf{a})$,	${\sim}\mathrm{Ex}\mathbf{a} \to (\mathbf{a}^{\bullet\bullet}\mathbf{B} \to {\sim}\mathbf{B})$,
(3)	$(\mathbf{a}^{\bullet}\mathbf{B} \;\&\; \mathbf{a}^{\bullet}({\sim}\mathbf{B})) \to {\sim}\mathrm{Pos}\mathbf{a}$,	$(\mathbf{a}^{\bullet\bullet}\mathbf{B} \;\&\; \mathbf{a}^{\bullet\bullet}({\sim}\mathbf{B})) \to {\sim}\mathrm{Loc}\mathbf{a}$,
(4)	$\mathrm{Pos}\mathbf{a} \to (\mathbf{a}^{\bullet}\mathbf{B} \to {\sim}\mathbf{a}^{\bullet}({\sim}\mathbf{B}))$,	$\mathrm{Loc}\mathbf{a} \to (\mathbf{a}^{\bullet\bullet}\mathbf{B} \to {\sim}\mathbf{a}^{\bullet\bullet}({\sim}\mathbf{B}))$,
(5)	$\mathrm{Ex}\mathbf{a} \to \mathrm{Pos}\mathbf{a}$,	${\sim}\mathrm{Loc}\mathbf{a} \to \mathrm{Ex}\mathbf{a}$,
(6)	$\mathbf{a}/\mathbf{b} \to (\mathbf{b}^{\bullet}\mathbf{C} \to \mathbf{a}^{\bullet}\mathbf{C})$,	$\mathbf{a}\backslash\mathbf{b} \to (\mathbf{a}^{\bullet\bullet}\mathbf{C} \to \mathbf{b}^{\bullet\bullet}\mathbf{C})$,
(7)	$\mathbf{a}^{\bullet}(\mathrm{Ex}\mathbf{b}) \equiv \mathbf{a}/\mathbf{b}$,	$\mathbf{b}^{\bullet\bullet}(\mathrm{Ex}\mathbf{a}) \equiv \mathbf{a}\backslash\mathbf{b}$,
(8)	$(\mathbf{a}^{\bullet}\mathbf{C} \vee \mathbf{b}^{\bullet}\mathbf{C}) \to (\mathbf{a}{*}\mathbf{b})^{\bullet}\mathbf{C}$,	$(\mathbf{a}{*}\mathbf{b})^{\bullet\bullet}\mathbf{C} \to (\mathbf{a}^{\bullet\bullet}\mathbf{C} \;\&\; \mathbf{b}^{\bullet\bullet}\mathbf{C})$,
(9)	$\mathrm{Ex}(\mathbf{a}{*}\mathbf{b}) \to (\mathrm{Ex}\mathbf{a} \;\&\; \mathrm{Ex}\mathbf{b})$,	$\mathrm{Ex}(\mathbf{a}\#\mathbf{b}) \to (\mathrm{Ex}\mathbf{a} \;\&\; \mathrm{Ex}\mathbf{b})$,
(10)	$(\mathbf{a}{*}\mathbf{b})/\mathbf{a}$ and $(\mathbf{a}{*}\mathbf{b})/\mathbf{b}$,	$(\mathbf{a}\#\mathbf{b})\backslash\mathbf{a}$ and $(\mathbf{a}\#\mathbf{b})\backslash\mathbf{b}$.

The formal counterparts of these formulas are in fact theorems of two sentential multimodal logics which are to be syntactically and semantically (in Kripke style) described in the lecture.

The Connective "ani" and the Connective "i" in Polish

Elżbieta Magner (PL)
Wrocław University, Wrocław
Department of Logic and Methodology of Science
Poland
dr.em@wp.pl

The connective "ani" in Polish is equivalent to "neither..., nor...", while "i" is equivalent to "and". In our paper we discuss the possibilities of replacing the connective "i" in various interpretative contexts with the connective "ani".

A Contribution to a Part-Whole Theory with the Welding Relation

Krystyna Misiuna (EN)
Warsaw University, Warsaw
Faculty of Philosophy and Sociology
Poland
krystyna.simons@uw.edu.pl

The property of unrestricted composition characteristic of the classical mereology implies that for any collection of objects there exists the object composed by the collection which is its mereological sum. The property of unrestricted composition is not a consequence of the very formalism of the part-whole relation, but a substantial claim. The classical mereology makes a difference between two operations: sums and fusions, both having the property of unrestricted composition. Generally, the property may be expressed in the following way:

For all collections of objects a: a compose if and only if there is at least one a.

Kit Fine (2010) proffers the principles satisfied by the operation of summation and fusion. He mentions four principles: the Principle of Absorption, the Principle of Collapse, the Principle of Leveling, and the Principle of Permutation. Peter Simons (2006) claims that there are limitations on the universality of composition if there is more than one category. He argues for the Principle of Intracategoriality of Composition (IC) which states that composition is always intracategorial, and in consequence there cannot be a whole whose parts are of different ontological categories, or wholes which are different categories than their parts. Pairwise mereologically disjoint parts of a whole w partition w, therefore an equivalence relation between the pairwise disjoint parts exists, but the existence of the equivalence relation is not sufficient for the existence of a real whole. Peter Simons (2006) argues that the equivalence relation must be also a welding relation which welds a collection of individuals into a whole. The problem with the principle (IC) is that we do not exactly know how many such categories we need to embrace all what has been discovered by science. We consider this problem in the case of quantum mechanics, that is, in the case of different theories explaining the measurement problem, and ontological commitments of these theories. Our main question is formulated in the following way:

Is the entity represented by a wave-function of quantum mechanics a whole with its own welding relation?

Roger Penrose (2004) argues that the measurement problem should be explained by taking into account the phenomenon of gravitation. Quantum mechanics leads us to a conjecture that there is a general welding relation. The welding relation is the gravitational field of an object which welds the object with its quantum wave.

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Spatial Reasoning in Heyting Mereology

Thomas Mormann (EN)
University of the Basque Country, San Sebastian
Department of Logic and Philosophy of Science
Spain
ylxmomot@sf.ehu.es

Key words: Mereology, Heyting algebras, co-Heyting algebras, Topology, Boundary, Interior Parts, Contact Relation, Representations.

In this paper it is shown that Heyting and Co-Heyting mereological systems provide convenient conceptual frameworks for spatial reasoning. More precisely, in these frameworks spatial concepts such as connectedness, interior parts, (exterior) contact, and concepts of boundaries can be defined in natural and intuitively appealing ways. This fact refutes the wide-spread contention that mereology cannot deal with the more advanced aspects of spatial reasoning and therefore has to be enhanced by further non-mereological concepts to overcome its congenital limitations. Actually, the relation between mereology and topology turns out to be much more complex than is usually thought.

The Many Facets of Boundaries: Mereology, Topology, and Measure Theory

Thomas Mormann (EN)
University of the Basque Country, San Sebastian
Department of Logic and Philosophy of Science
Spain
ylxmomot@sf.ehu.es

Key words: Boundary, Mereology, Topology, Measure Theory, Boolean Algebras with Operators, Fat Cantor Sets.

The concept of boundary has many different facets that bring it into the focus of many different theories, among them mereology, topology and measure theory. The aim of this paper is to single out a convenient class of systems of spatial regions with well-behaved boundaries. It is shown that the class BW of „boundary-well-behaved“ regions of Euclidean space having topologically and measure-theoretically „nice“ boundaries is a (non-complete) Boolean subalgebra of the Boolean algebra of Borel sets of Euclidean space. With the aid of „fat Cantor sets“ it is proved that not all regular open (or regular closed) Euclidean regions have „nice“, i.e. topologically and measure-theoretically thin, boundaries.

On the Ontological Status of Substantial Wholes and their Parts

Marek Piwowarczyk (EN)
John Paul II Catholic University of Lublin, Lublin
Faculty of Philosophy
Poland
piwowar1@wp.pl

In my talk I want to consider the old problem of the ontological status of substantial wholes and their parts.

Contemporary analytic substantialists are inclined to accept the thesis that substances can be composed of other substances. Usually such a view contains also some restrictions concerning this composition: it is said that no substance can be a simple sum of its parts, or that substances are not identity-dependent on their parts (E.J. Lowe). Anyway on the ground of these positions there is a possibility that some artifacts and natural formations like rocks, stones, minerals etc. are genuine substances.

This is the point of great difference between analytic and classical (Aristotelian and scholastic) substantialism. According to Aristotle and his ancient and medieval followers parts of substantial wholes cannot be substances. There are at least three reasons used in arguments for this claim:

1. Substances are genuine unities unified by substantial form. The factor unifying many substances into one object can be only accidental form, thus such an object has less degree of unity than its parts.

2. No actual substance can contain two or more actual things.

3. No substance can be existentially dependent on its parts in such a way that parts exist before the substance.

Those reasons also imply theses about the ontological status of parts of substances: they do not have their own forms, but their forms are dependent on the form of substance, they are only potential substances, they become during the becoming of substance.

I would like to analyze problems connected with those theses. My text will be based on metaphysics of Thomas Aquinas and ontology of Roman Ingarden. In order to describe the ontological status of parts of substances more precisely I will give a typology of composed objects where the criterion of distinguishing different types is a kind of parts' properties which must be changed during a process of composition.

Bisimulation Reducts of Kripke Models for Intuitionistic Predicate Logic

Tomasz Połacik (EN)
University of Silesia, Katowice
Institute of Mathematics
Poland
polacik@us.edu.pl

We consider Kripke semantics for intuitionistic predicate logic. Confining ourselves to the class of tree models, we introduce the notion of bisimulation reduct $\mathcal{K}^\rho$ of a given model $\mathcal{K}$. Roughly speaking, a bisimulation reduct of $\mathcal{K}$ arises from the careful selection of the world of each equivalence class with respect to the given bisimulation on $\mathcal{K}$.

We prove that a bisimulation reduct of a given Kripke model $\mathcal{K}$ preserves forcing of all formulas, i.e., for every formula $\varphi(\bar{x})$ and a tuple of elements $\bar{a}$ of the common root of the models $\mathcal{K}$ and $\mathcal{K}^\rho$ we have $\mathcal{K} \Vdash \varphi(\bar{a})$ iff $\mathcal{K}^\rho \Vdash \varphi(\bar{a})$. Thus, since $\mathcal{K}^\rho$ arises from removing some worlds from $\mathcal{K}$, it is clearly an elementary submodel of $\mathcal{K}$ in the sense of [3].

Next, from resuts of [2] it follows that under some additional assumptions on $\mathcal{K}^\rho$, we can replace (some of) the worlds of $\mathcal{K}^\rho$ by their suitable elementary substructures in such a way that the resulting model $\mathcal{K}^{\rho\varepsilon}$ still preserves forcing of all formulas. Consequently, $\mathcal{K}^{\rho\varepsilon}$ is an elementary submodel of the model $\mathcal{K}$ in the general sense, as defined in [1].

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Parts and Wholes in Philosophy and Philosophy of Parts and Wholes

Marek Rosiak (EN)
University of Łódź, Łódź
Department of Logic and Methodology of Science
Poland
rosiak@filozof.uni.lodz.pl

The part-whole notions were present, albeit implicitly, already in considerations of first philosophers. **Tales'** *hydor* or **Heraclitus'** *pyr* had the status of a basic part of reality but this status itself was not taken into consideration by thinkers of this time. The part-whole notions worked in presocratic theories of reality, but they had not been worked upon – it was too early for the reflection on them.

These notions became more prominent in the ancient atomism. In the system of **Democritus** atoms are explicit parts of sensuous things. **Parmenides**, on the other hand, advanced a kind of a negative part-whole theory in his radical doctrine of *hen*.

In **Plato's** theory of ideas the awareness of the part-whole relationship reached still higher level: he tested a hypothesis that *methexis* of an individuum in an idea is just a part-whole relation or its reverse – with a negative result in both cases. In his late dialectic he developed some particular PW-theory of the division of notions.

Although **Aristotle** developed the *organon* – an advanced theory of science – part-whole notions did not win there a status of primary importance: they remained hidden behind categorial notions of substance and its various accidents. Their actual importance is evident from Aristotelian theory of definition however.

In the modern era **Descartes** reworked classical metapysics in a radical manner and this resulted in his dualistic metaphysics of mind (*res cogitans*) and body (*res extensa*). As the unity of a human person was seriously endangered, Descartes tried – in vain – to make one whole of these two independent components. In this trouble the need of a philosophical part-whole theory became prominent. But, alas, it was still too early for one: both **Spinoza** and **Leibniz** tried to overcome difficulties of dualistic metaphysics by developing rival monistic theories: numeric vs. generic monism.

From the point of view of parts and wholes **Kant** retreated to a dualistic standpoint of *Dinge an sich* and *Erscheinungen*, which from systematic point of view is not so different from the Cartesian dualism. Of course, he developed much the theory of mind and its phaenomenal correlates: the problems of the constitution of phaenomenal things came into light and with them – problems of objects of higher orders – founded on constituents which are not their parts in a proper sense.

Hegel and his dialectical construction is just a mockery of all reason, but nevertheless his dynamic monism was an attempt at building one universal whole of all there is, which has a fractal structure. This futile effort of making one whole of just everything with the help of a magical operation of *Aufhebung* (an echo of classical infinitation operation on branches of the Porphyrean tree), reaching the verge of absurd, led at last to a formulation of a demand of a general part-whole theory.

Franz **Brentano**, rejecting heglism as an eruption of irrationalism, postulated a reneval of metaphysics on the basis of a sound general part-whole theory. This wish was fulfilled by his pupils who proposed interesting and quite elaborate theories. **Twardowski's** theory, although built on a practical purpose of clarifying relations between a content and an object of thinking, has nevertheless the air of true universality. **Husserl** took Twardowski's theory as a starting point of his own one, trying to avoid some of its awkward consequences. He also tried to pass from a purely descriptive shape of the theory of his predecessor to a more formal („*more geometrico*") formulation of his own, although without a spectacular effect.

Ingarden, justly famous of his scrupulous descriptive analyses, returned to a properly phaenomenological, descriptive layout of a part-whole theory, supplying us with numerous illuminative insights.

Recently such imposing formally developed theories of parts and wholes as the set theory on one hand and Leśniewski's mereology on the other are in use. They assume quite different concepts of a **set**: **distributive** and **collective** one. Their particularity, as well as their mutual differences deserve closer consideration.

The Role of Reasoning Types in Argumentation

Marcin Selinger (PL)
Wrocław University, Wrocław
Department of Logic and Methodology of Science
Poland
marcisel@uni.wroc.pl

One of the original achievements of the Lvov-Warsaw School in methodology was the development of the so-called "classification of reasonings". It was primarily introduced by Łukasiewicz. Then, improved by Czeżowski, it was finally modified and elaborated by Ajdukiewicz in [1], which seems to be the most advanced approach. The aim of this talk is to use this classification in the theory of argumentation, particularly to associate the reasoning types with some argumentation schemes and diagrams that are usual tools for representing and analyzing arguments in informal logic (see, *e.g.* [2], [3]).

Firstly, we recall the definitions of the basic reasoning types (inference, derivation, demonstration, verification and explanation) as they were distinguished by Ajdukiewicz. In opposition to reasoning, which is a mental act (or process), we understand argumentation as an act (or process) of communication, *i.e.* a speech act, in which, however, some agent's mental acts (in particular those of reasoning) are presumed and, so to say, submitted for acceptance. On the other hand, if we disregard the persuasive and performative function of argumentation, we can obtain the propositional structures, which are the same in arguments and in the presumed acts of reasoning. Thus the schemes and diagrams assigned to some reasoning types and to the corresponding arguments are the same too. We point out such schemes and diagrams in the second part of our talk.

Single arrows in the usual diagrams represent simple inferences that are submitted for acceptance. Whole diagrams consist of one or many arrows, and they represent more or less complex inferences and proofs as well. However, the case of derivation can be represented only by extended diagrams, in which not only premises, but also assumptions may be taken into account. Thus we introduce a suitable notation for diagramming derivations to consider the process of verification. There are two kinds of verification, namely confirmation and falsification, so we show how to represent arguments corresponding to confirmation and (*ad absurdum*) arguments corresponding to falsification. We also indicate some argumentation schemes associated with the discussed reasoning types, in particular, some meta-schemes concerning the process of reasoning itself. Finally, we consider explanation, which seems to be essentially different from argumentation (according to Ajdukiewicz it may have no conclusion at all). However, explanation plays the crucial role in the abductive reasoning, which obviously has a conclusion. So, in the end we also discuss schemes that can be assigned to abductive arguments.

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Toward a General Theory of Object and its Parts

Bartłomiej Skowron (EN)
Wrocław University, Wrocław
Department of Logic and Methodology of Science
Poland
bartlomiej.skowron@gmail.com

I will present a general topological theory of the object and its parts. I used in this theory the tools and concepts of topology. This theory is the development of the ideas contained in Husserl's third investigation in *Logical Investigations*. The basic representation of object is a set in the sense of ZFC set–theory.

In every object I distinguish at least four kinds of parts: **appearances**, **formal parts**, **pieces** and **elements**.

For every object, we can look at them from different sides. From different sides we can see different appearances of the object. Appearances are parts of the object, they are not a parts of experience, as Husserl wanted. We want to talk about the appearances of objects, about their extent, continuity, connectedness, density, compactness, etc. Thus, the appearances are represented by the topologies that can be built on this object X. Despite the fact that the object has a lot of appearances, it is—after all—one an the same object. How to express this fact in my theory? The moment that gives unity is the Tychonoff topology (product topology). The object it is therefore X together with all his appearances, joined together by Tychonoff topology.

Between the appearances (topologies) a fixed object X there are a certain relations. These relations form the complete non-distributive lattice (for $|X| > 3$) with complementation.

Pieces of the object are simply subspaces of the object. If X is an object that satisfies the axioms of separation T_0, T_1, T_2, T_3, each its subspace also satisfies these axioms. It is a fact which we interpret as a theorem: piece of a piece is a piece. The type of piece is the same as a type of a whole, which is a piece of.

The existence of elements is the ontological postulate. They—as Jerzy Perzanowski would say—pre-exist.

The Application of Aristotelian Theory of Topoi to the Theory of Argument Schemes

Joanna Skulska (PL)
Military University of Technology, Warsaw
Cardinal Stefan Wyszyński University, Warsaw
Poland
joannaskulska@o2.pl

The aim of the paper is to propose a hierarchy of argument schemes using the theory of topoi introduced by Aristotle. The strength of Aristotelian list of topoi is that it is robust and applicable in rhetorical practice and computational models of arguments.

Common topoi are an overarching category of topoi. Common topoi for (im)possible properties and common topoi for time can be specified to obtain the pattern of formal topoi, which, in turn, have material topoi as their subcategory. Because of their less abstract form, the specification of common topoi for magnitude is material topoi. Formal topoi defines the "abstract" relation between objects and gives some information about properties that can be inferred in a given argument (in the case of subcategory common topoi for (im)possible properties) or gives some information about the time (in the case of subcategory common topoi for time). Material topoi further specifies the objects and provides more information about the properties.

The hierarchy of argument schemes has important applications both in persuasion practice and in computational models of argument. From the practical point of view, the hierarchy would extend Aristotle's idea that led him to develop the art of rhetoric, i.e. to equip a speaker with tools which minimize an effort of memorizing the rhetorical techniques, i.e. to create a list of argument patterns (topoi) rather than argument instances. As a result, a relatively small list will allow the rhetor to create a variety of many arguments useful in different contexts. A hierarchy makes the process of learning how to build an argument even easier, i.e. knowing the small amount of very generic and abstract "superschemes" and knowing what specifications of those schemes are the most common for rhetorical practice would allow the speaker to learn and produce instances of arguments in a more effective manner.

Similarly, the process of computing the properties of arguments becomes more efficient, since the hierarchy allows for using the procedure of inheritance, i.e. the specific argument schemes (and their properties) can be inferred from more generic schemes. The instances of a specific argument scheme will inherit not only the properties of the most direct super-class but also the properties of the indirect ones.

On ostensive definitions

Kazimierz Świrydowicz (PL)
Adam Mickiewicz University, Poznań
Department of Mathematical Logic
University of Arts in Poznań
Poland
swirydow@amu.edu.pl

My talk will be devoted to the so-called ostensive definitions. Such definitions are specified as an explanation of the meaning of an expression x by showing in the world an object, which is denoted by this expression x, i.e. the definition shows a connection between words and objects or their pictures. These definitions have been rather rarely investigated in the literature. I present examples of such definitions in encyclopedias and dictionaries and I analyze them. In such a way the types of the definitions can be identified and the types of typical mistakes can be found.

Deontic Operators Meet Compound Actions

Robert Trypuz & Piotr Kulicki (EN)
John Paul II Catholic University of Lublin, Lublin
Poland
trypuz@kul.pl, kulicki@l3g.pl

A Deontic Logic of Action, an article published by K. Segerberg in 1982 was a milestone in the development of deontic logic based on action theory since the 1950s when G. H. von Wright and J. Kalinowski had published their innovative deontic systems. Segerberg's work was further developed in the similar style by J. Czelakowski and R. Trypuz and P. Kulicki in deontic first-order theories (cf. works of G.-J. C. Lokhorst or SETNA theory of R. Trypuz) and deontic logics of action built in connection with Propositional Dynamic Logic (PDL). In the latter class of systems two approaches can be distinguished. In one of them deontic operators are introduced with the use of dynamic operators and the notion of violation (the approach initiated by J. -J. Ch. Meyer), in the other one at least some of them are taken as primitive (cf. works of L. T. McCarty, R. van der Meyden and recently P. F. Castro and T. S. E. Maibaum). In the systems of the latter kind we can further distinguish those having a two-layered construction (PDL and logic for deontic operators) and those having a three-layered construction (Boolean or Kleene algebra for actions, PDL, logic for deontic operators).

The main aim of our presentation is (i) to analyse the combination of atomic Boolean action algebra with deontic operators (in particular, the relations between the systems of deontic action logic without obligation, closely related to Segerberg's systems B.O.D. and B.C.D. and the deontic and Boolean layers of Castro and Maibaum's **DPL** logic are taken into account) and (ii) to analyse the combination of an algebraic structure with parallel execution, free choice operator and sequential execution of actions with deontic operators (in particular we shall refer to the recent results of C. Prisacariu and G. Schneider).

Taking into account deontic action logics based on Boolean algebra, we shall point out that differences among them lie in two aspects that are intuitively significant: the *level of closedness* of a deontic action logic and the *possibility of performing no action at all*. Another specific feature of the systems of deontic action logic based on Boolean algebra is that they take the notions of permission and forbiddance (or weak permission) as primitive introducing obligation by definition. We shall show that the existing definitions of obligation in those systems are not acceptable. As a solution we propose an axiomatic characterisation of obligation with an adequate class of models.

Considering deontic action logic with sequential composition of actions, it is important to note that they pose very interesting and challenging questions concerning the deontic characteristics of sequences of actions. For instance, what does it really mean to impose a regulation on someone "you ought to do

α and then β" or "you are allowed to do α and then β", taking into account that action α can be carried out in several ways, some of which may exclude the possibility of executing β afterwards. Sequences of actions (as fragments of computer programs) are widely studied in computer science by means of such formalisms as Kleene algebra, propositional dynamic logic (PDL) (in which Boolean and Kleene algebras are the essential components) or Hoare logic. Those systems show their usefulness the in analysis of algorithms, programs and more general phenomena of agents' behaviour. They were also connected with deontic notions.

Especially recent work of C. Prisacariu and G. Schneider presents an approach similar to the one we shall refer to in our presentation. Authors build a deontic action logic on the foundation of an algebraic structure which they call synchronous Kleene algebra. The structure combines parallel and sequential execution of actions and a free choice operator on actions. The deontic characteristics of complex actions, including multi-step actions, is defined on the basis of deontic values of simple state-to-state transitions. The model they use we find at some points unintuitive. Also the fact that they assume algebra of actions independently from model-theoretic structure and then define the model for algebraically defined normal form is unusual in logic. Moreover, they use the notion of compensation which is inseparably connected with prohibition and obligation, which makes it difficult to extract the fundamental properties of prohibition and obligation. Thus, we shall propose an alternative system based on our earlier works. We put forward the recursive definitions of metalogical counterparts of deontic operators and discuss the validity of formulas constructed in a minimal language with a finite number of basic actions, parallel and sequential compositions of actions, a free choice operator and the standard deontic operators of obligation, strong permission and prohibition. We propose the interpretation function of actions taking into account their terminated and non-terminated executions.[1]

Parts and Constituents of Language Expressions

Jacek Waldmajer & Urszula Wybraniec-Skardowska
(PL with the slides in EN)
University of Opole, Opole
Poland
jwaldmajer@uni.opole.pl, uws@uni.opole.pl

Making a philosophical analysis connected with mental searching, differentiating, distinguishing an object and its parts, is related with a thought analysis of creations that are language expressions. Such an analysis entails separating their *parts* up to the most elementary (atomic) ones. A syntactic analysis of

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language expressions, treated as some *wholes* built of parts, plays also an important role in formulating fundamental semantic principles of compositionality of meaning and denotation, which in a loose formulation reads: the meaning (denotation) of a composed expression is a function of meanings (denotations) of its parts.

In linguistic research, logical language analysis, and also in philosophy of language, language expressions could be considered once as expression *tokens*, and another time as expression *types*, according to the *token-type* distinction by Ch.S. Pierce. Expression *tokens* are understood as concrete, sensorily perceivable physical ontological objects spread in time and space, i.e., ontological concretes, while expression *types* are treated as certain classes of the first ones – composed of expression *tokens*. *Types* of expressions are most often understood as abstract ontological objects, although they can also be treated as mereological collections (see Leśniewski [1929, 1930]) or multitudes (in the framework of Peter Simons [2011]) of expression *tokens*.

Formalizing syntactically any language we must choose either linguistic *tokens* or *types* as the starting point of considerations. If we start from *token level* as the first level of language formalization, then we accept the nominalistic (*concretistic*) approach to the formal analysis of language in the Leśniewski's spirit and language of expression *types* are formalized on the second, *types* level.

The paper concerns the relationships between *whole* and *parts* in logical the syntactic analysis of language formalized in the spirit of nominalistic approach given by the second co-author in e.g. [1991, 2006]. *Well-formed expression-tokens* (briefly: *wfes*) are understood here as some mereological objects. Composed expression-*tokens* have the so-called functor-argument structure. Every composed *wfe* is compounded from a part (its main *functor*) and remaining parts (*arguments* of that functor). All such parts of the *wfe* (treated as a *whole*) are its *1-st order constituents*. *Constituents of the higher k* ($k > 1$) *order* of the *wfe* are parts of its constituent of the $k - 1$ order but they are not proper constituents of the *wfe*. The relation 'being a (proper) constituent of', in the opposition to the relation 'being a (proper) part of', is not partial ordering.

Every *wfe* is a generalized *concatenation* of simple or composed *words* of language. These words are *componential parts* of the *wfe*. They can consist of some simpler ones. Parts of the *wfe* do not have to be its constituents. The simplest parts of the *wfe* and the concatenation from which it is built are words of language *vocabulary*. They decide of the length of the *wfe*.

So, the starting point of the formal-logical considerations on the first, *concretistic*, *token level* is an axiomatic theory $\mathbf{T^1}$ of concatenation and word *tokens*. It serves as a basis for formalization of all the above-mentioned concepts, including notions of 'part' and 'constituent' denoting label-*tokens* as ontological objects that are *concreta*. Important primitive notions of the theory are: *vocabulary*, the ternary relation of *concatenation* of label-*tokens* and the relation of *identifiability* of label-*tokens*. The notions of a '*generalized concatenation*' and its '*part*' are defined ones. It should be mentioned that in $\mathbf{T^1}$ the relation of concatenation is not a set-theoretical function: concatenations of two label-*tokens* can be different, but identifiable label-*tokens*. Identifiability of label *tokens* depends on pragmatic aspects or purposes. In *tokens*, we formulate not only some basic properties of concatenation but also some basic properties of the relation of '*being a part of*'.

The theory of language expression *tokens*, the theory $\mathbf{LT^1}$, is built as an extension of $\mathbf{T^1}$. The basic notions of $\mathbf{LT^1}$ are a '*wfe token*' and a '*constituent of* the *wfe*'. *Wfes* are here some special generalized concatenations of word *tokens* determined by means of their structure defined by a categorial grammar originating from Kazimierz Ajdukiewicz [1935]. It to formulate an algorithm of checking correctness of well-formedness of language expressions. In $\mathbf{LT^1}$, we describe some basic properties of parts and constituents of *wfes* and relations between them.

On the *type level* the theory $\mathbf{LT^2}$ of language syntax is constructed as an extension of $\mathbf{T^1}$ and $\mathbf{LT^1}$. It is a theory of expression *types* treated as some classes of identifiable *wfe tokens*. Their *parts* and *constituents* are some classes of identifiable word-*tokens*, thus some word *types* that can be, but do not have to be, understood as abstract, set-theoretical collections. They can be understood as mereological collections or multitudes. We can also speak about collections of parts or collections of constituents, not necessarily in the set-theoretical sense. It is worth mentioning that all properties of part *tokens* and constituent *tokens* valid on the *token level* are valid for their counterparts on the *type level*.

The theory $\mathbf{TL^2}$ can be developed to a semantic theory of language in which the principles of compositionality are formulated by means of the notion of 'constituent *types*', hence the notion of 'part *types*'.

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Elementary Ontology and the Classical Calculus of Relations

Eugeniusz Wojciechowski (PL with the slides in EN)
Hugo Kołłątaj University of Agriculture, Cracow
Poland
rlwojcie@cyf-kr.edu.pl

The notion of relation is one of the most important notions in our language. Therefore it should come as no surprise that it was considered as one of the categories by Aristotle. The calculus of relations was further consolidated by Ch. S. Peirce (1839–1914) and E. Schröder (1841–1902). It is also present in Whitehead and Russell's *Principia*. The theory of relations figures prominently in the logic textbooks from the first half of the 20$^{\text{th}}$ century. Later the theory of relations has been given less attention. It has been presented, as if by the way, in the textbooks on the classical predicate calculus or set theory. On the other hand, it is given hardly any attention in the logical structures which are nominal calculi. The traditional syllogistics has some serious difficulties with relative names (the problem of the so-called *oblique syllogisms*), whereas in the sophisticated nominal calculus—Leśniewski's ontology—relations (like in set theory) can be introduced by definition.

There are some sound arguments in favour of seeing elementary ontology as an appropriate tool in the analysis of natural language. We also have strong linguistic intuitions about seeing the phrases of natural language with relations as being primary to expressions with relative names.

This study proposes some extension of elementary ontology aiming at including those linguistic intuitions.

Structural Completeness in Modal Logics

Piotr Wojtylak (EN)
University of Opole, Opole
Institute of Mathematics
Poland
Piotr.Wojtylak@math.uni.opole.pl

We characterize all structurally complete consequence relations which are extensions of the modal logic S4.3. It turns out, in particular, that all modal logics in $Next(S4.3)$ are almost structurally complete (see Dzik W., Wojtylak P., *Projective unification in modal logic*, Logic Journal of the IGPL (2012) 20 (1), 121–153) which means that all admissible rules with unifiable premisses are derivable on the ground of a given logic. Derivability of the so-called *passive* *rules*, that is rules with non-unifiable premisses), is a more complicated matter. We characterize all consequence operations over $S4.3$ in terms of algebraic operations such as products of their matrix semantics. Structurally complete relations are given thus by products of matrices (which are topological boolean algebras) with the degenerate algebra.

Applying Logic in Modelling Communication

Olena Yaskorska (PL)
Cardinal Stefan Wyszynski University, Warsaw
Institute of Philosophy
Poland
oyaskorska@gmail.com

Key words: formal fallacy; propositional fallacy; fallacy in a dialogue; dialogue game; Lorenzen game

An important motivation for the interest in studying dialogues in game-theoretical style was Hamblin's program [1] of designing a protocol which rules out the use of fallacies during a dialogue. This approach has resulted in many formal systems exploring different informal fallacies occurring in natural dialogues.

Yet in real-life communication the speakers commit not only informal, but also formal fallacies. A formal fallacy is understood as an argument invalid according to some logical system. Amongst fallacies which do not follow the rules of propositional logic and are claimed to be common in real-life practice are, e.g., fallacies of incorrect operations on implication, i.e. *denying the antecedent* ($\alpha \to \beta$, $\neg\alpha$, therefore $\neg\beta$) and *affirming the consequent* ($\alpha \to \beta$, β, therefore α), or fallacies of incorrect operations on disjunction, i.e. *affirming a disjunct* ($\alpha \vee \beta$, α, therefore $\neg\beta$).

The aim of this talk is to propose a dialogue protocol that allows formal fallacies to be dealt with in a game-theoretic framework of natural discourse. While there was a lot of attention paid to the study of informal fallacies, the formal fallacies typical for real-life communication remained ignored in the game-theoretical exploration of dialogues.

In order to model the execution and elimination of formal fallacies in natural dialogues, we need a system for representing *natural dialogue* and a system for representing *formal dialogue* (i.e. the dialogue in which the validity of argument is the topic of the dialogue). In the first case, we use the framework proposed by Prakken [2], since it provides a generic and formal specification of the main elements of dialogue systems for persuasion. For handling formal fallacies in a dialogue, we use the dialogical logic introduced by Lorenzen [3].

The strength of Lorenzen logic is that it is already expressed in the game-theoretical style. His dialogue game allows the players to prove if the formula is a tautology of propositional logic. Yet, the communication language and the structure of the system is different than in games designed to simulate natural discourse. Using Prakken's specification of dialogue systems for persuasion, the talk proposes a game in which the players can persuade each other not only about facts, but also about the validity of propositional formulas. The main contribution is the translation of Lorenzen dialogical logic from original description to the specification proposed by Prakken.

Specifically, in case of locution rules (rules which describe which speech acts the player can use during the game), Prakken allows six speech acts: *claim* α, *why* α, α *since* S, *concede* α, *question* α and *retract* α. In Lorenzen's game there are only two possible acts to perform: X *attacks* A and X *defends* A. Still the Lorenzen's rules can be reconstructed into Prakken's generic language such as for example:

- If X *attacks* $A \wedge B$, X makes *question* ϕ, where ϕ is formula A or formula B, because according to Lorenzen's dialogue game, to attack conjunction the player has to contest the validity of the formula which is an element of the attacked conjunction,
- If X *defends* $A \wedge B$ with assertion of validity of A, X makes *claim* ϕ where ϕ is the formula A, respectively, if X *defends* $A \wedge B$ with assertion B, X makes *claim* ψ where ψ is the formula B, because according to the Lorenzen's dialogue game the defence of the conjunction is made by the assertion of the validity of the formula which was questioned in the attack.

The project has important applications for modelling communication in both natural contexts and Artificial Intelligence, since the system proposed allows representing and eliminating formal fallacies in dialogue.

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Tableau-based Decision Procedures and Blocking Mechanisms

Michał Zawidzki (EN)
University of Łódź, Łódź
Department of Logic
Poland
The University of Manchester, Manchester
School of Computer Science
United Kingdom
michal.zawidzki@gmail.com

Modal and description logics are a widely investigated area these days. Many of these logics are known to be decidable. However, traditional Hilbert-style calculi for deciding the validity of formulae are becoming rather obsolete in comparison to other deductive systems, like tableau-based, resolution-based or sequent-style decision procedures.

Tableau calculi gained much popularity in the field of modal and description logics. The reason is that they often provide efficient decision procedures in terms of complexity. Nevertheless, sole tableau rules, which are usually directly synthesised from a semantic specification of a given logic, are insufficient for ensuring termination of the procedure in the case of many modal and description logics. The simplest example of such non-terminating decision procedure is a tableau calculus for the logic $S4$, containing rules for the connectives and rules specifying frame conditions. It turns out that applying these rules to the simple formula of the form $\Diamond\varphi \wedge \Diamond\Box\varphi$ ends up in an infinite derivation which is caused by the transitivity condition. That is the point where we must bring into force *blocking mechanisms* which consist in identifying worlds that are initially treated in a tableau as distinct.

We can distinguish several kinds of blocking techniques, namely *subset blocking, equality blocking, pairwise blocking, pattern-based blocking* or *unrestricted blocking.* They differ in the criteria of worlds identification they use. It results in differences in efficiency of a particular tableau-based decision procedure, depending on a blocking technique chosen. However, it turns out that not every blocking mechanism is applicable to every decidable logic. By applying a wrong blocking mechanism we can easily loose completeness of the whole calculus.

Introduced in [4], the *unrestricted blocking* mechanism is an explicit tableau rule of the form:

$$(\mathsf{ub}) \quad \frac{}{x = y \mid x \neq y},$$

where x and y are labels of worlds. Obviously, (ub) is a sound rule which is a variant of *the analytical cut rule.* Intuitively, (ub) allows comparing any pair of worlds that occurred on a tableau branch and checking which applications of the rule lead to the clash and as such, it subsumes any other blocking mechanisms. Admittedly, it causes many superfluous tableau performances and branchings, however, one of the results of exploiting (ub) is obtaining minimal model for a given, satisfiable formula. Moreover, in the case of the description logic $\mathcal{ALBO}_{id}$, which is proven to be equivalent to $\mathcal{L}^2$, unrestricted blocking is the only known mechanism that ensures termination of the tableau calculus for this logic. It raises a question of the importance of analytical cut rule for termination of tableau calculi.

In my talk I will describe in a more detailed manner the aforementioned blocking mechanisms. I will also discuss a question of links between particular blocking techniques and logics that are liable to these techniques and consider whether applicability of a particular blocking mechanism to a particular logic is a matter of computational efficiency or whether it has more intrinsic nature. In latter case I will briefly explain a meaning of this intrinsicness.

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My Contribution to an Anthology of Universal Logic: Comments on a Paper by Tarski (1928) and by Łoś and Suszko (1958)

Jan Zygmunt (PL)
Wrocław University, Wrocław
Department of Logic and Methodology of Science
Poland
Logika@uni.wroc.pl

I will speak about my two contributions to the book **Universal Logic: *An Anthology — From Hertz to Dov Gabbay***, edited by J.-Y. Béziau (Springer, 2012). The anthology is a retrospective on universal logic in the 20$^{\text{th}}$ century. It contains 15 papers from the period 1922 to 1996 authored by 19 logicians, mathematicians and philosophers, e.g. by P. Bernays, H. Curry, K. Gödel, D. Scott, A. Tarski, N. da Costa. Each of the 15 works is preceded by an introductory essay to explain its origin, import and impact on the development of the science of logic.

I have written two introductory essays. First, to Tarski's *Remarks on Fundamental Concepts of the Methodology of Mathematics* (a short note originally published in French in 1928), and, second, to Łoś and Suszko's *Remarks on Sentential Logics* (1958). The main ideas of both these essays will be presented during the lecture.

A Gap in Kant's Division of Judgments

Zbigniew Żurakowski (EN)
Opole University, Opole
Institute of Philosophy
Poland
zurakowskizbigniew@gmail.com

In this speech, I present the possibility of the independent existence of analytic a posteriori judgments, whose existence has been ruled out by Immanuel Kant. The foundation of my analysis is based on the phrase *cogito ergo sum*, written by Descartes in the *Discourse on Method.* This sentence is necessary to prove the possibility of analyzing a posteriori judgments. The documentation is also used in the classical understanding of the nominal definition by Kazimierz Ajdukiewicz. We can may see a similarity between this type of nominal definition to Kant's analytical judgments.

Essentially, the existence of analytical a posteriori judgments becomes apparent by careful examination of Kant's detailed analysis in *Prolegomena to Any Future Metaphysics* and *The Critique of Pure Reason.* In order to provide evidence, one may transform Descartes' *cogito ergo sum* to fit into the division of judgments in order to show how the judgment is possible.

Additionally, I present other thinkers theses: Kant's critique of the semantics of *cogito ergo sum*, Friedrich Nietzsche's critique in *Beyond Good and Evil* and Charles H. Kahn's examination of how the word *is* presents a problem in the reasoning (in *The Greek Verb 'to be' and the Concept of Being*).

To conclude, I make reference to Saul Kripke's more contemporary position on this issue and then present my views and the difference in this case.